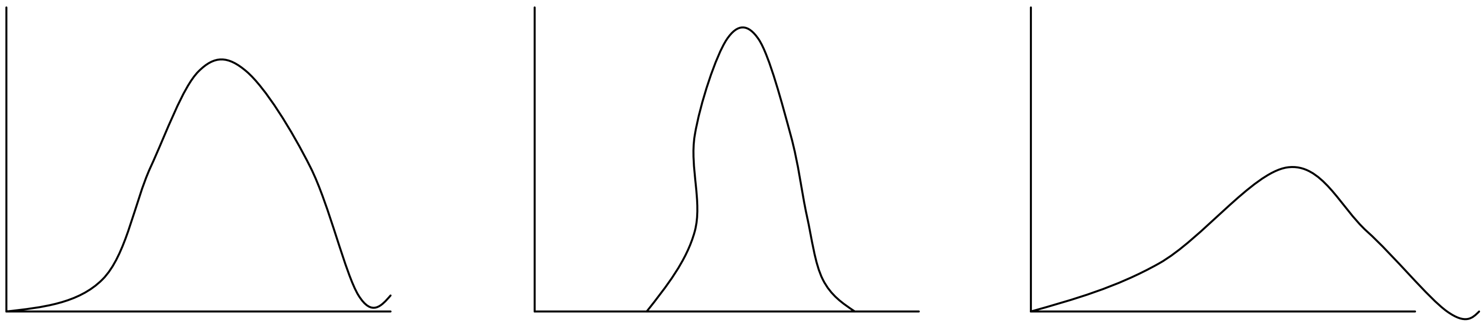
EM Algorithm in 1D



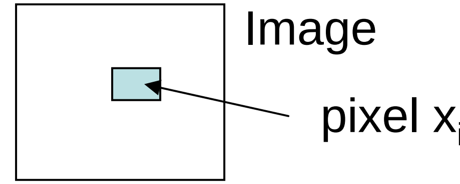
C_1
$N(\mu_1,\sigma_1)$
$\alpha_1 = P(C_1)$

C_2
$N(\mu_{2,}\sigma_2)$
$\alpha_2 = P(C_2)$

C_3
$N(\mu_3,\sigma_3)$
$\alpha_3 = P(C_3)$

Initialization Step: for each of the K clusters C_j, initialize its mean μ_j, its variance σ_j, and its weight $\alpha_j = P(C_j)$.

Expectation Step



- General Formula

$$p(C_j \mid x_i) = \frac{p(x_i \mid C_j) \cdot p(C_j)}{p(x_i)} = \frac{p(x_i \mid C_j) \cdot p(C_j)}{\sum_j p(x_i \mid C_j) \cdot p(C_j)}$$

  for cluster C_j

- Explanation: find the probability $P(C_j \mid x_i)$ for each pixel x_i and each cluster C_j. The formula requires

  - $P(C_j)$ (the current α_j weight for C_j)

  - $P(x_i \mid C_j) = \dfrac{1}{\sqrt{2\pi}\,\sigma_j} e^{-\frac{(x_i - \mu_j)^2}{2\sigma_j^2}}$

We have all the needed parameters from the initialization step.

Maximization Step

General Formulas

$$\mu_j = \frac{\sum_i p(C_j \mid x_i) \cdot x_i}{\sum_i p(C_j \mid x_i)}$$

$$\Sigma_j = \frac{\sum_i p(C_j \mid x_i) \cdot \overbrace{(x_i - \mu_j) \cdot (x_i - \mu_j)^T}^{(x_i - u_j)^2}}{\sum_i p(C_j \mid x_i)}$$

$$p(C_j) = \frac{\sum_i p(C_j \mid x_i)}{N}$$

Explanation: Now that we have computed $P(C_j \mid x_i)$ for every cluster C_j and pixel x_i, we just use them to update the mean, the variance, and the weight (probability) of each cluster.

The process repeats the expectation and maximization steps till some stopping criterion is reached.