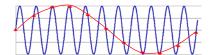


Sampling and the Nyquist rate

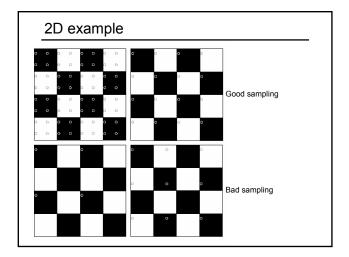


Aliasing can arise when you sample a continuous signal or image

- occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an alias
- formally, the image contains structure at different scales
- called "frequencies" in the Fourier domain
- the sampling rate must be high enough to capture the highest frequency in the image

To avoid aliasing:

- sampling rate ≥ 2 * max frequency in the image
 - said another way: ≥ two samples per cycle
- This minimum sampling rate is called the Nyquist rate



Subsampling with Gaussian pre-filtering



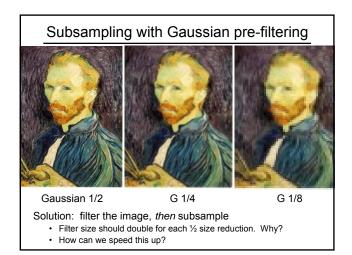


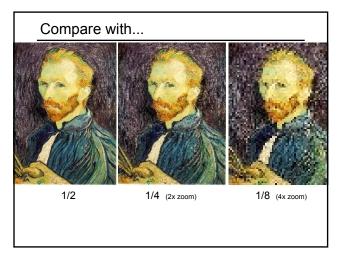


Gaussian 1/2

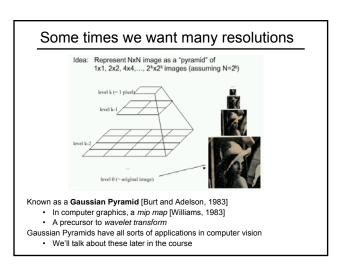
Solution: filter the image, $\it then$ subsample

- Filter size should double for each $\frac{1}{2}$ size reduction. Why?

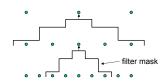








Gaussian pyramid construction



Repeat

- Filter
- Subsample

Until minimum resolution reached

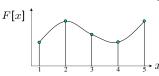
• can specify desired number of levels (e.g., 3-level pyramid)

The whole pyramid is only 4/3 the size of the original image!

Image resampling

So far, we considered only power-of-two subsampling

- · What about arbitrary scale reduction?
- · How can we increase the size of the image?



d = 1 in this example

Recall how a digital image is formed

 $F[x, y] = quantize\{f(xd, yd)\}$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

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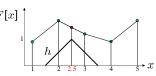
$$F[x, y] = quantize\{f(xd, yd)\}$$

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Image resampling

So what to do if we don't know f

- Answer: guess an approximation $\,\tilde{f}\,$
- · Can be done in a principled way: filtering



d = 1 in this example

Image reconstruction

- Convert ${\cal F}$ to a continuous function

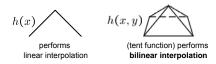
 $f_F(x) = F(\frac{x}{d})$ when $\frac{x}{d}$ is an integer, 0 otherwise

Reconstruct by cross-correlation:

$$\tilde{f} = h \otimes f_F$$

Resampling filters

What does the 2D version of this hat function look like?



Better filters give better resampled images

- · Bicubic is common choice
 - fit 3rd degree polynomial surface to pixels in neighborhood
 - can also be implemented by a convolution

Bilinear interpolation

A simple method for resampling images

