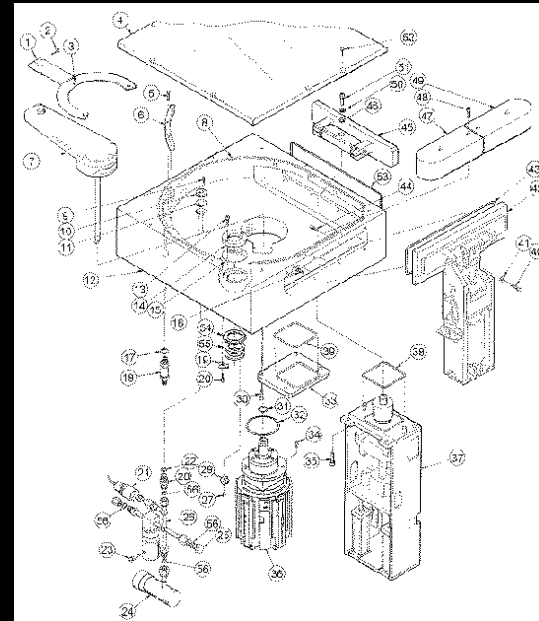# Edge Operators

a kind of filtering that leads to useful features

Lines and Arcs Segmentation

In some image sets, lines, curves, and circular arcs are more useful than regions or helpful in addition to regions.

Lines and arcs are often used in

• object recognition

• stereo matching

• document analysis

Edge Detection

Basic idea: look for a neighborhood with strong signs of change.

Problems:

• neighborhood size

• how to detect change

81	82	26	24
82	33	25	25
81	82	26	24

Differential Operators

Differential operators

• attempt to approximate the gradient at a pixel via masks

• threshold the gradient to select the edge pixels

Example: Sobel Operator

$$Sx = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad Sy = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$
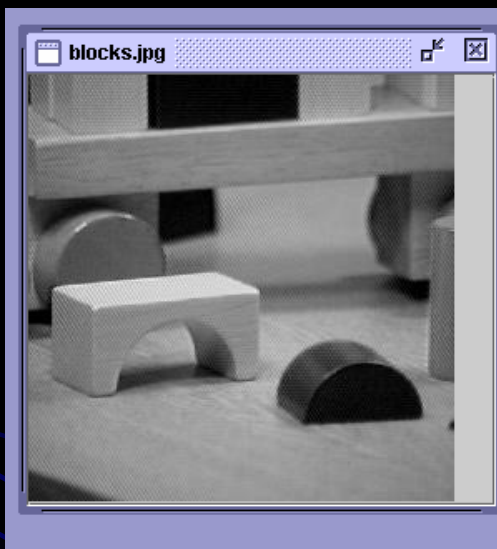
On a pixel of the image I
- let gx be the response to Sx
- let gy be the response to Sy
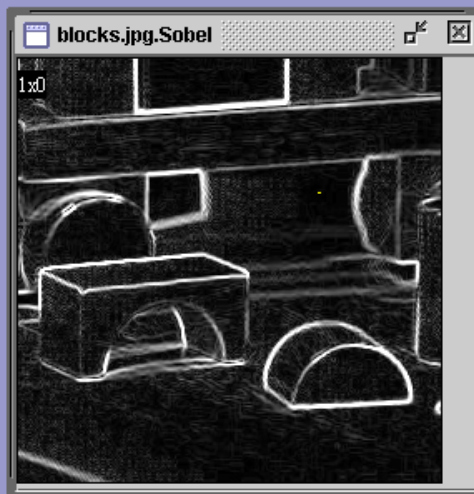
Then the gradient is
$$\nabla I = [gx \ \ gy]^T$$

And $g = (gx^2 + gy^2)^{1/2}$ is the gradient magnitude.

$\theta = atan2(gy,gx)$ is the gradient direction.

Sobel Operator on the Blocks Image

original image

gradient magnitude

thresholded gradient magnitude

Common Masks for Computing Gradient

- Sobel:

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- Prewitt:

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

- Roberts
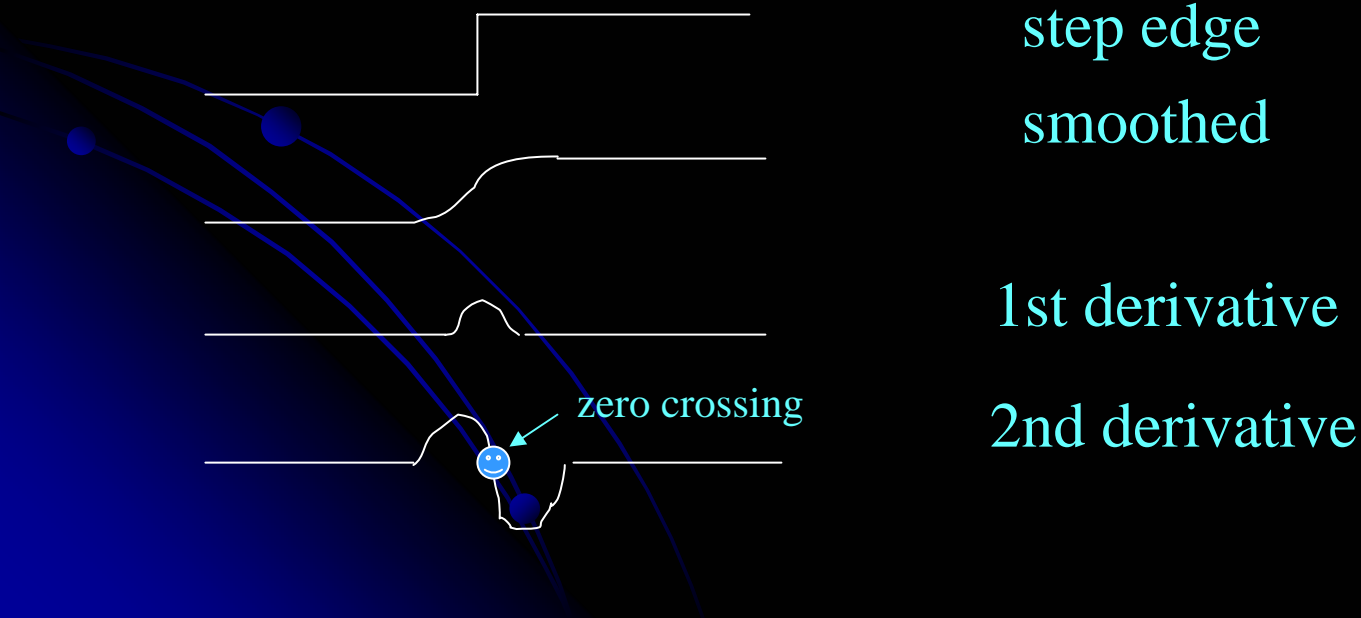
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Sx　　　　　　**Sy**

Zero Crossing Operators

Motivation: The zero crossings of the second derivative
of the image function are more precise than
the peaks of the first derivative.

step edge

smoothed

1st derivative

zero crossing

2nd derivative

How do we estimate the Second Derivative?

- **Laplacian Filter:** $\nabla^2 f = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Standard mask implementation

- Derivation: In 1D, the first derivative can be computed with mask [-1 0 1]

- The 1D second derivative is [1 -2 1]*

- The Laplacian mask estimates the 2D second derivative.

* How can we derive this?

Properties of Derivative Masks

- Coordinates of derivative masks have opposite signs in order to obtain a high response in regions of high contrast.

- The sum of coordinates of derivative masks is zero, so that a zero response is obtained on constant regions.

- First derivative masks produce high absolute values at points of high contrast.

- Second derivative masks produce zero-crossings at points of high contrast.

Marr/Hildreth Operator

• First smooth the image via a Gaussian convolution.

• Apply a Laplacian filter (estimate 2nd derivative).

• Find zero crossings of the Laplacian of the Gaussian.

This can be done at multiple resolutions.

Haralick Operator

- Fit the gray-tone intensity surface to a piecewise cubic polynomial approximation.

- Use the approximation to find zero crossings of the second directional derivative in the direction that maximizes the first directional derivative.

The derivatives here are calculated from direct mathematical expressions wrt the cubic polynomial.

Canny Edge Detector

- Smooth the image with a Gaussian filter with spread σ.

- Compute gradient magnitude and direction at each pixel of the smoothed image.

- Zero out any pixel response ≤ the two neighboring pixels on either side of it, along the direction of the gradient.

- Track high-magnitude contours.

- Keep only pixels along these contours, so weak little segments go away.

Canny Examples from Text
(see Chapter 10)

Canny $\sigma=1$ Canny $\sigma=4$
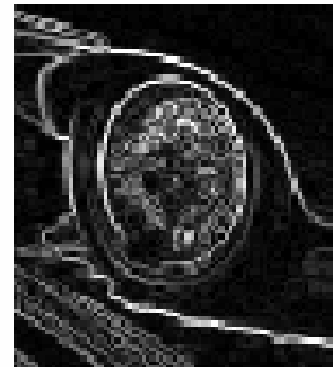
Canny $\sigma=1$ Roberts 2X2

Canny on Kidney from Hw1

Canny on the Blocks image

Canny Characteristics

- The Canny operator gives single-pixel-wide images with good continuation between adjacent pixels

- It is the most widely used edge operator today; no one has done better since it came out in the late 80s. Many implementations are available.

- It is very sensitive to its parameters, which need to be adjusted for different application domains.