

# Natural Language Processing

## Syntactic parsing

Yulia Tsvetkov

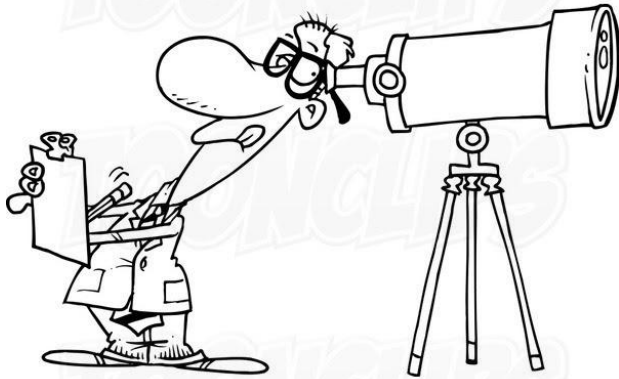
[yuliats@cs.washington.edu](mailto:yuliats@cs.washington.edu)

# Announcements

-

# Ambiguity

- I saw a girl with a telescope



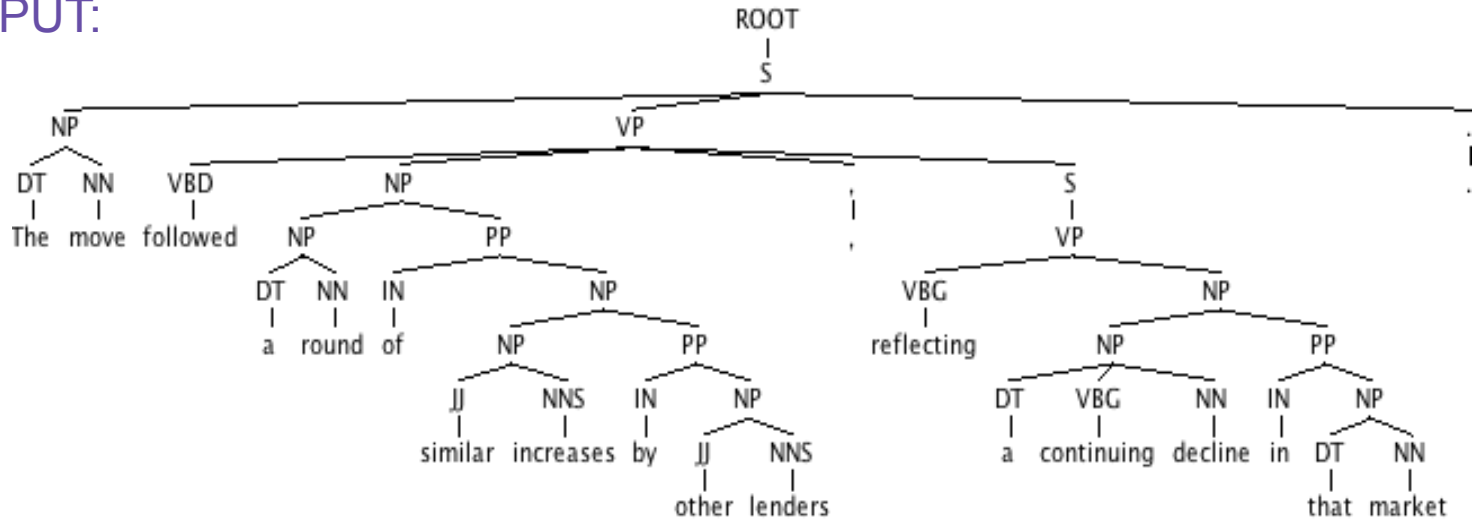
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# Syntactic Parsing

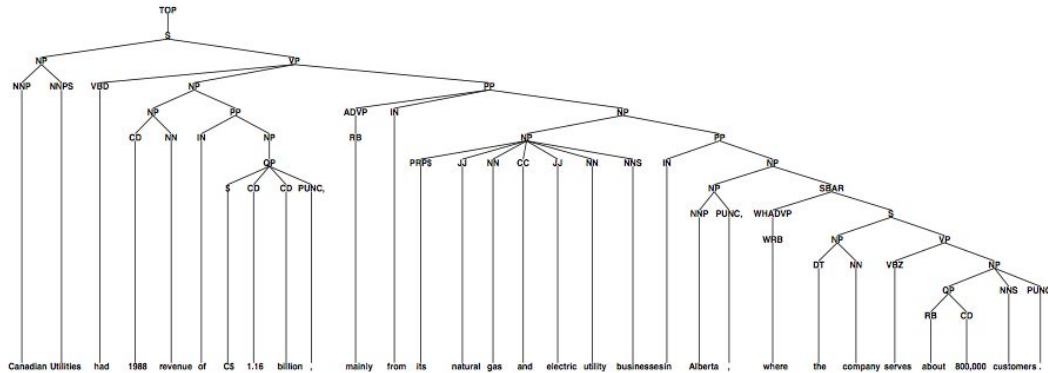
- INPUT:
  - The move followed a round of similar increases by other lenders, reflecting a continuing decline in that market

- OUTPUT:



# A Supervised ML Problem

- Data for parsing experiments:
  - Penn WSJ Treebank = 50,000 sentences with associated trees
  - Usual set-up: 40,000 training, 2,400 test



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers [from Michael Collins slides]

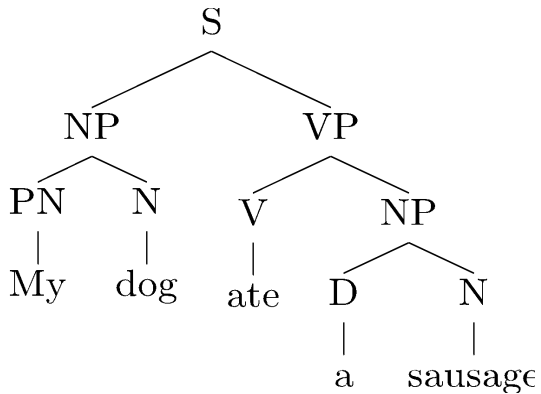
# Syntax

# Syntax

- The study of the patterns of formation of sentences and phrases from words
  - my dog                      Pron N
  - the dog                     Det N
  - the cat                     Det N
  
  - and                         Conj
  
  - the large cat             Det Adj N
  - the black cat            Det Adj N
  
  - ate a sausage            V Det N

# Parsing

- The process of predicting **syntactic representations**
- Different types of syntactic representations are possible, for example:



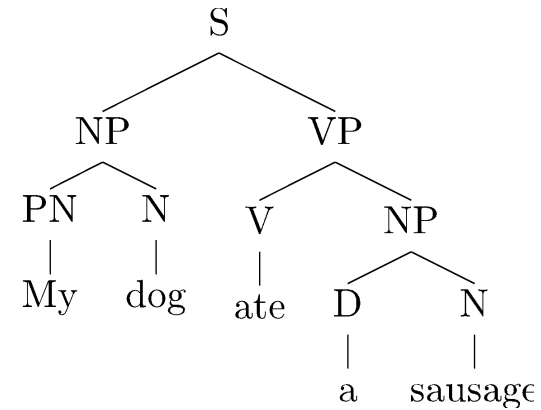
Constituent (a.k.a. phrase-structure) tree



# Constituent trees

- Internal nodes correspond to phrases

- **S** – a sentence
- **NP** – Noun Phrase: My dog, a sandwich, lakes,...
- **VP** – Verb Phrase: ate a sausage, barked, ...
- **PP** – Prepositional phrases: with a friend, in a car, ...

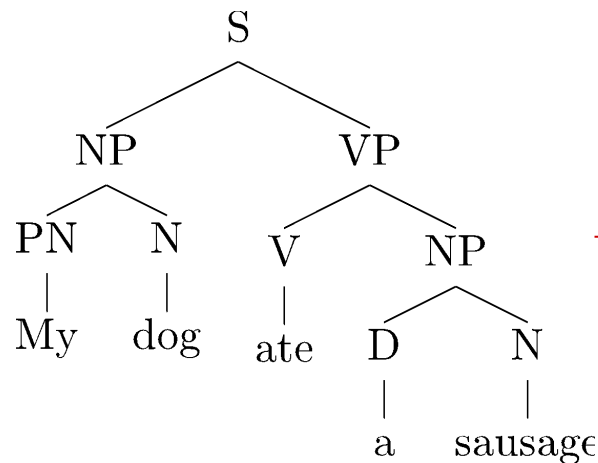


- Nodes immediately above words are PoS tags (aka preterminals)

- **PN** – pronoun
- **D** – determiner
- **V** – verb
- **N** – noun
- **P** – preposition

# Bracketing notation

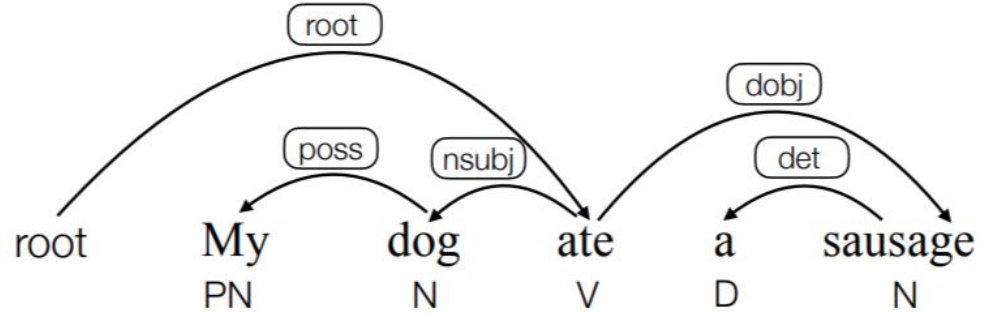
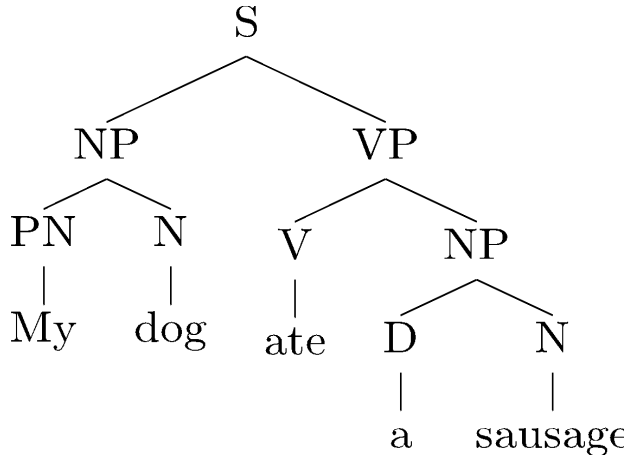
- It is often convenient to represent a tree as a bracketed sequence



(S  
 (NP (PN My) (N dog) )  
 (VP (V ate)  
     (NP (D a) (N sausage) )  
 )  
 )

# Parsing

- The process of predicting syntactic representations
- Different types of syntactic representations are possible, for example:

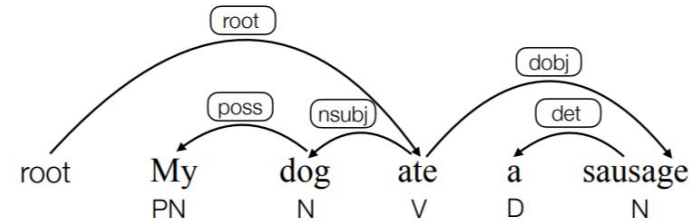


Constituent (a.k.a. phrase-structure) tree

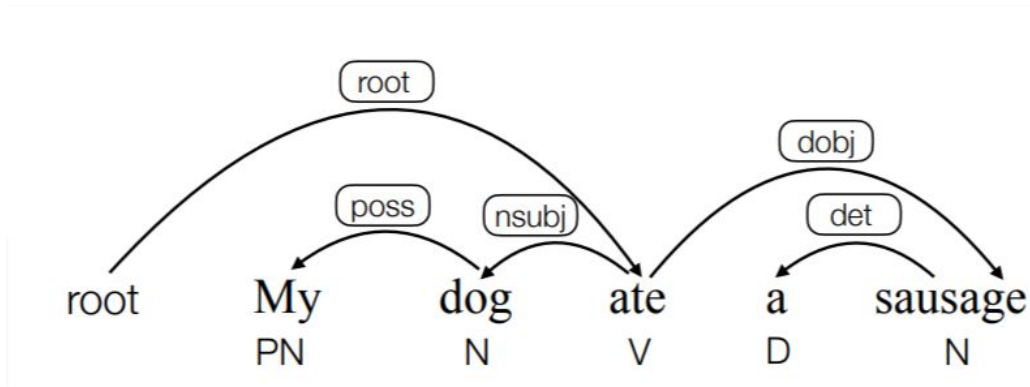
Dependency tree

# Dependency trees

- Nodes are **words** (along with part-of-speech tags)
- Directed arcs encode **syntactic dependencies** between them
- Labels are types of relations between the words
  - **poss** – possessive
  - **dobj** – direct object
  - **nsubj** - subject
  - **det** - determiner

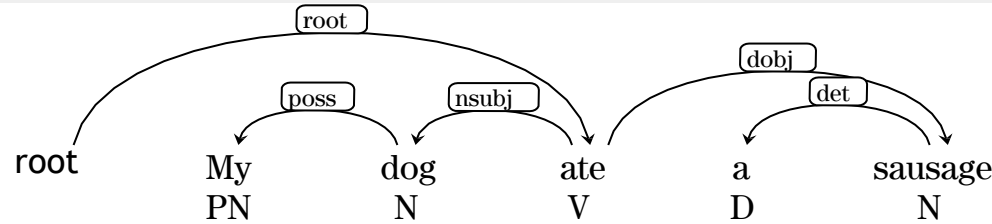


# Recovering shallow semantics



- Some semantic information can be (approximately) derived from syntactic information
  - Subjects (**nsubj**) are (often) **agents** ("initiator / doers for an action")
  - Direct objects (**dobj**) are (often) **patients** ("affected entities")

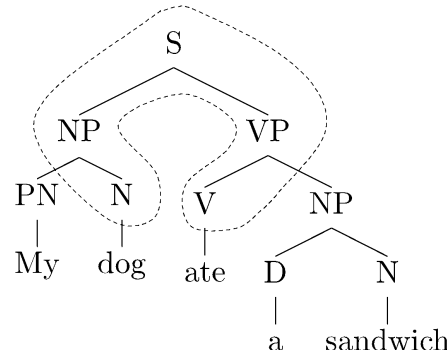
# Recovering shallow semantics



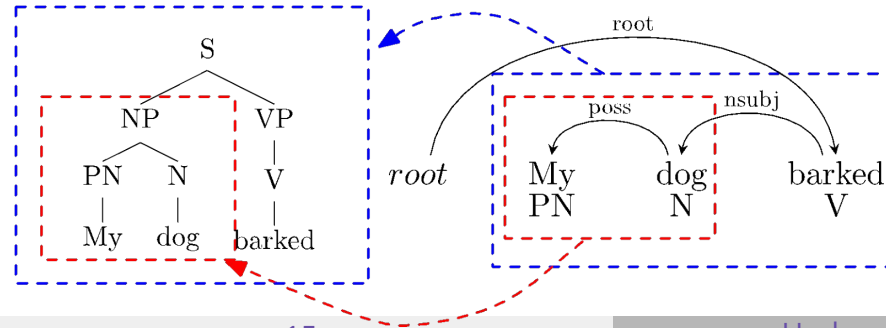
- Some semantic information can be (approximately) derived from syntactic information
  - Subjects (**nsubj**) are (often) **agents** ("initiator / doers for an action")
  - Direct objects (**dobj**) are (often) **patients** ("affected entities")
- But even for agents and patients consider:
  - Mary is baking a cake in the oven
  - A cake is baking in the oven
- In general it is not trivial even for the most shallow forms of semantics
  - E.g., consider prepositions: *in* can encode direction, position, temporal information, ...

# Constituent and dependency representations

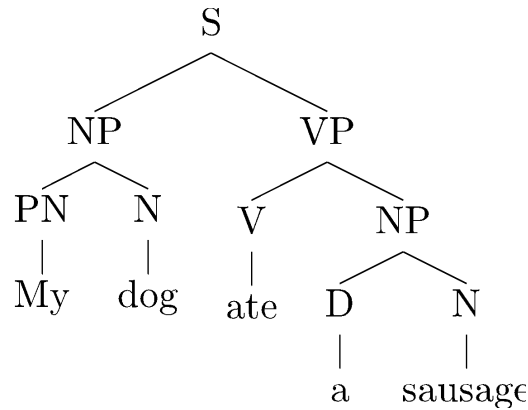
- Constituent trees can (potentially) be converted to dependency trees



- Dependency trees can (potentially) be converted to constituent trees



# Constituent trees



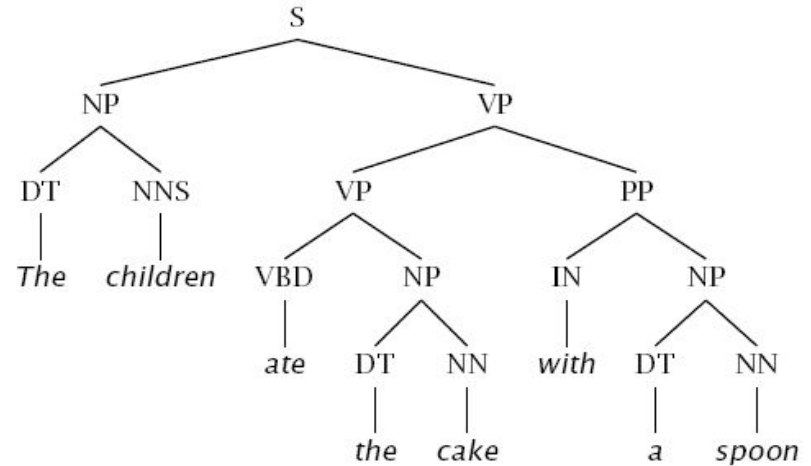
- Internal nodes correspond to phrases
  - S – a sentence
  - NP (Noun Phrase): My dog, a sandwich, lakes,...
  - VP (Verb Phrase): ate a sausage, barked, ...
  - PP (Prepositional phrases): with a friend, in a car, ...
- Nodes immediately above words are PoS tags (aka preterminals)

- PN – pronoun
- D – determiner
- V – verb
- N – noun
- P – preposition



# Constituency Tests

- How do we know what nodes go in the tree?
- Classic constituency tests:
  - Replacement
  - Movement
    - Passive
    - Clefting
    - Preposing
  - Substitution by *proform*
  - Modification
  - Coordination/Conjunction
  - Ellipsis/Deletion



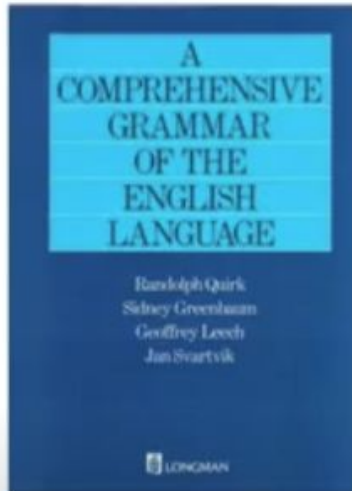
# Morphology/Syntax/Semantics

- **Syntax:** The study of the patterns of formation of sentences and phrases from word
  - Borders with **semantics** and **morphology** sometimes blurred

***Afyonkarahisarlılaştırabildiklerimizdenmişsinizcesine***

in Turkish means "as if you are one of the people that we thought to be originating from Afyonkarahisar" [[wikipedia](#)]

# English grammar



## Product Details (from Amazon)

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Shipping Weight: 4.6 pounds

# Context Free Grammar (CFG)

# Context Free Grammar (CFG)

## Grammar (CFG)

ROOT  $\rightarrow$  S  
S  $\rightarrow$  NP VP  
NP  $\rightarrow$  DT NN  
NP  $\rightarrow$  NN NNS  
NP  $\rightarrow$  NP PP  
VP  $\rightarrow$  VBP NP  
VP  $\rightarrow$  VBP NP PP  
PP  $\rightarrow$  IN NP

## Lexicon

NN  $\rightarrow$  interest  
NNS  $\rightarrow$  raises  
VBP  $\rightarrow$  interest  
VBZ  $\rightarrow$  raises  
...

Other grammar formalisms: LFG, HPSG, TAG, CCG...

# CFGs

S

$S \rightarrow NP VP$

$N \rightarrow girl$

$N \rightarrow telescope$

$VP \rightarrow V$

$N \rightarrow sandwich$

$VP \rightarrow V NP$

$PN \rightarrow I$

$VP \rightarrow VP PP$

$V \rightarrow saw$

$V \rightarrow ate$

$NP \rightarrow NP PP$

$P \rightarrow with$

$NP \rightarrow D N$

$P \rightarrow in$

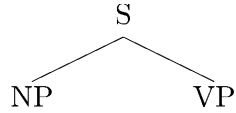
$NP \rightarrow PN$

$D \rightarrow a$

$PP \rightarrow P NP$

$D \rightarrow the$

# CFGs



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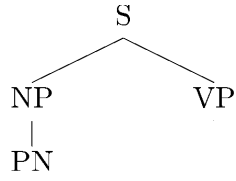
$P \rightarrow in$

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# CFGs



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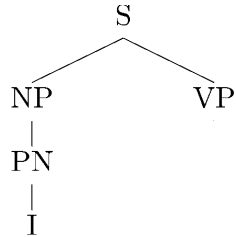
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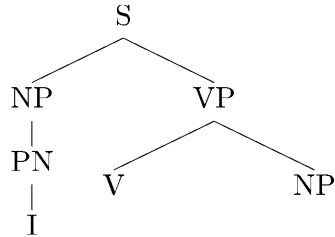
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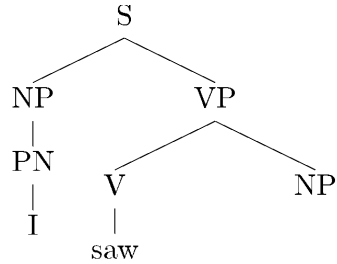
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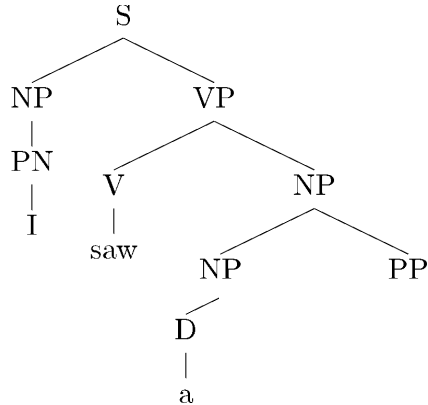
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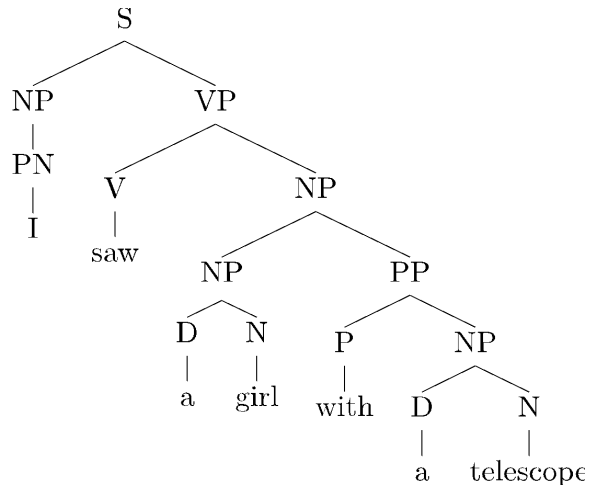
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$P \rightarrow in$

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$D \rightarrow a$

$D \rightarrow the$

# Trebank Sentences

```
( (S (NP-SBJ The move)
  (VP followed
    (NP (NP a round)
      (PP of
        (NP (NP similar increases)
          (PP by
            (NP other lenders))
          (PP against
            (NP Arizona real estate loans))))))
    ,
    (S-ADV (NP-SBJ *)
      (VP reflecting
        (NP (NP a continuing decline)
          (PP-LOC in
            (NP that market))))))
  .))
```

# Context-Free Grammars

- A context-free grammar is a 4-tuple  $\langle N, T, S, R \rangle$ 
  - $N$  : the set of **non-terminals**
    - **Phrasal categories**: S, NP, VP, ADJP, etc.
    - **Parts-of-speech** (pre-terminals): NN, JJ, DT, VB
  - $T$  : the set of **terminals** (the words)
  - $S$  : the **start** symbol
    - Often written as ROOT or TOP
    - Not usually the sentence non-terminal S
  - $R$  : the set of **rules**
    - Of the form  $X \rightarrow Y_1 Y_2 \dots Y_k$ , with  $X, Y_i \in N$
    - Examples:  $S \rightarrow NP VP$ ,  $VP \rightarrow VP CC VP$
    - Also called rewrites, productions, or local trees

# An example grammar

$N = \{S, VP, NP, PP, N, V, PN, P\}$

$T = \{girl, telescope, sandwich, I, saw, ate, with, in, a, the\}$

$S = \{S\}$

$R :$

$S \rightarrow NP VP$  (NP A girl) (VP ate a sandwich)

$VP \rightarrow V$

$VP \rightarrow V NP$  (V ate) (NP a sandwich)

$VP \rightarrow VP PP$  (VP saw a girl) (PP with a telescope)

$NP \rightarrow NP PP$  (NP a girl) (PP with a sandwich)

$NP \rightarrow D N$  (D a) (N sandwich)

$NP \rightarrow PN$

$PP \rightarrow P NP$  (P with) (NP with a sandwich)

Called Inner rules

Preterminal rules

$N \rightarrow girl$

$N \rightarrow telescope$

$N \rightarrow sandwich$

$PN \rightarrow I$

$V \rightarrow saw$

$V \rightarrow ate$

$P \rightarrow with$

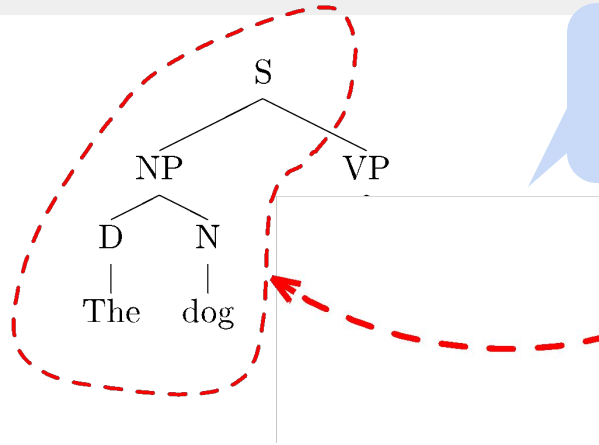
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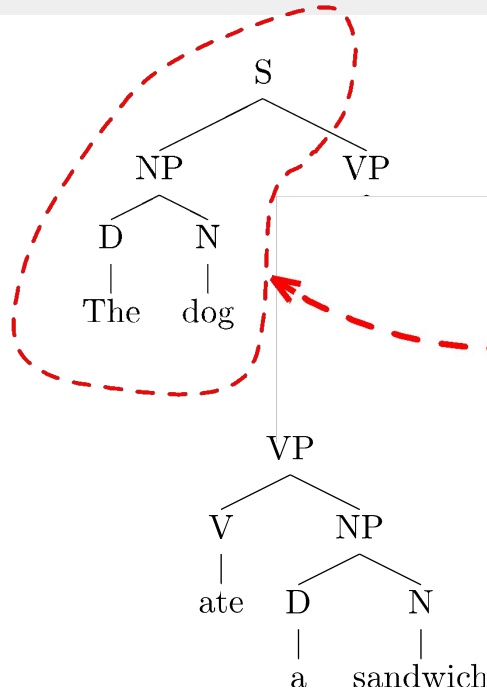


# Why context-free?

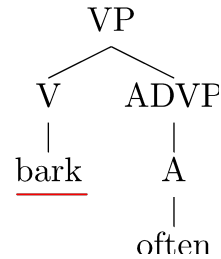


What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**

# Why context-free?



What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**

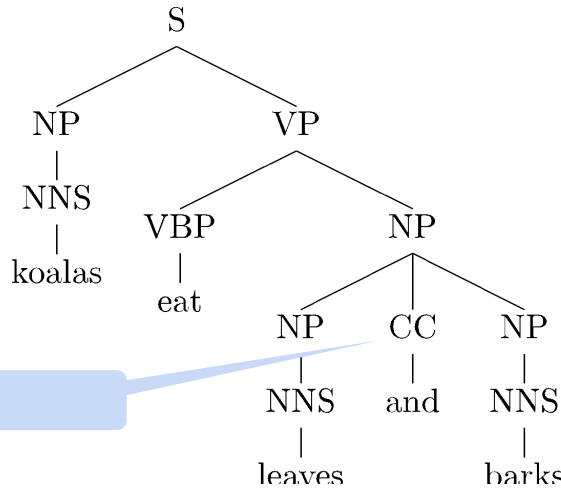


Not grammatical

# Ambiguities

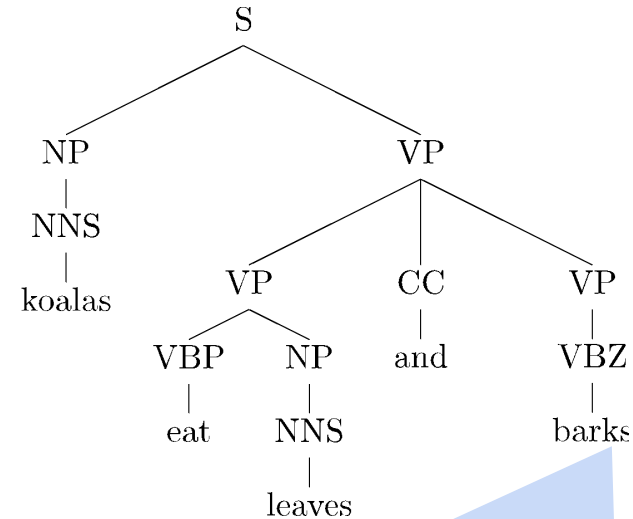
# Coordination ambiguity

- Here, the coarse VP and NP categories cannot enforce subject-verb agreement in number resulting in the coordination ambiguity



Coordination

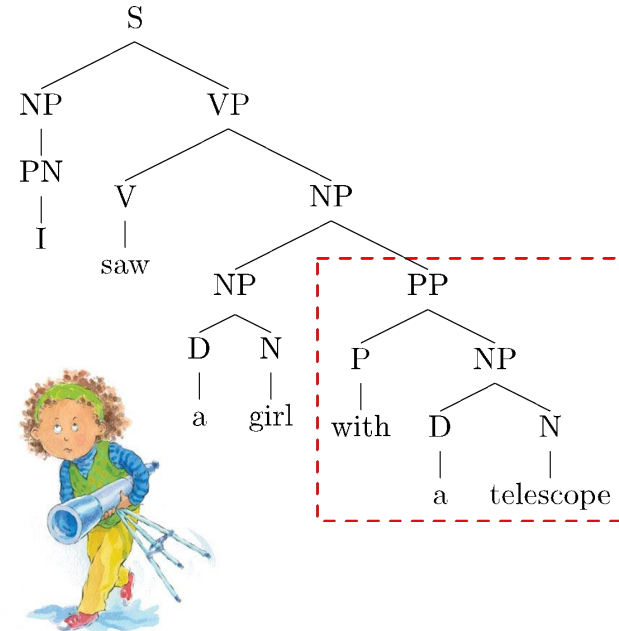
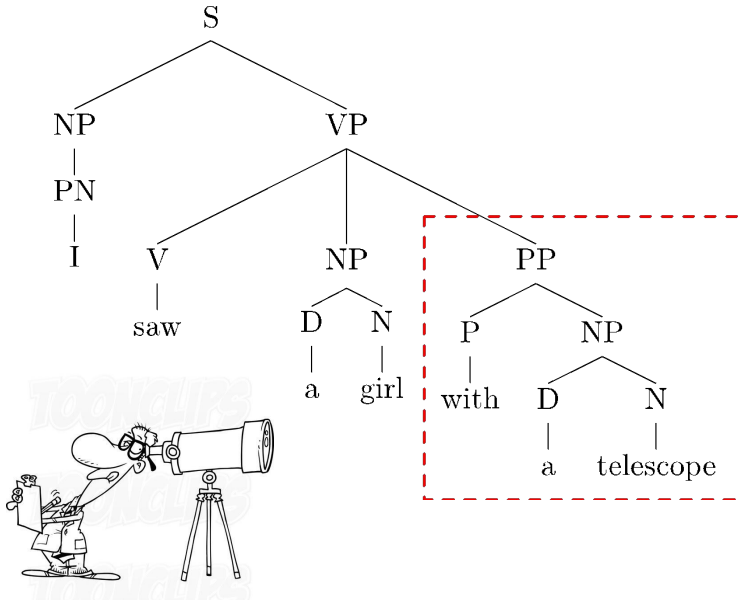
"Bark" can refer both to a noun or a verb



This tree would be ruled out if the context would be somehow captured (subject-verb agreement)

# Why parsing is hard? Ambiguity

- Prepositional phrase attachment ambiguity



# PP Ambiguity

*Put the block in the box on the table in the kitchen*

3 prepositional phrases, 5 interpretations:

- Put the block ((in the box on the table) in the kitchen)
- Put the block (in the box (on the table in the kitchen))
- Put ((the block in the box) on the table) in the kitchen.
- Put (the block (in the box on the table)) in the kitchen.
- Put (the block in the box) (on the table in the kitchen)

# PP Ambiguity

***Put the block in the box on the table in the kitchen***

3 prepositional phrases, 5 interpretations:

- Put the block ((in the box on the table) in the kitchen)
- Put the block (in the box (on the table in the kitchen))
- ...

A general case:

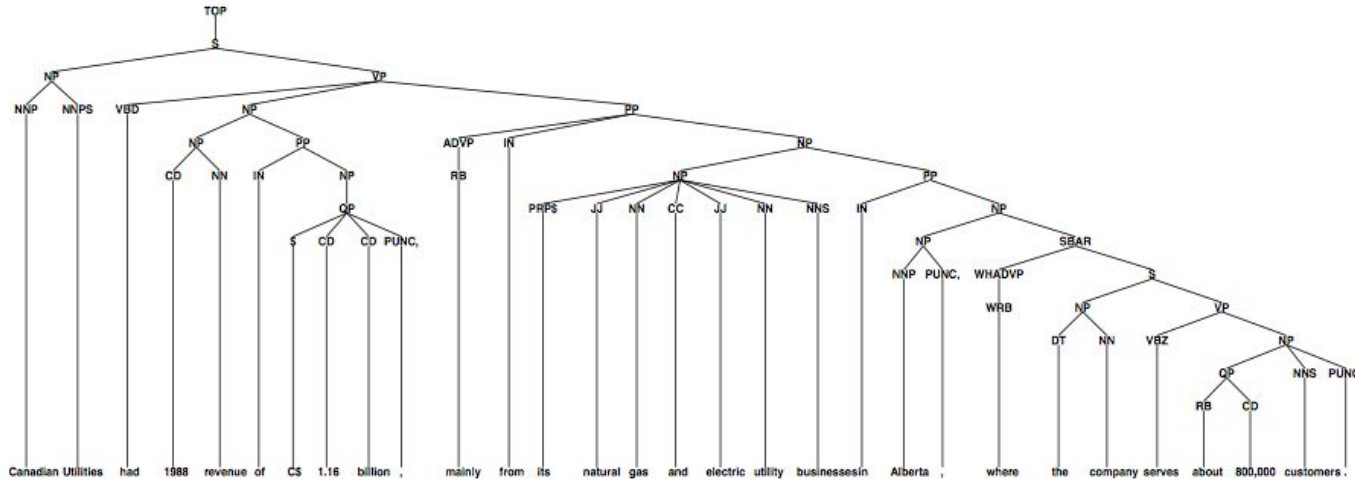
- ((( )))    ()(())    ()(())    (()())    (()())

$$Cat_n = \binom{2n}{n} - \binom{2n}{n-1} \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$

Catalan numbers

1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...

# A typical tree from a standard dataset (Penn treebank WSJ)



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers .

[from Michael Collins slides]



# Syntactic Ambiguities I

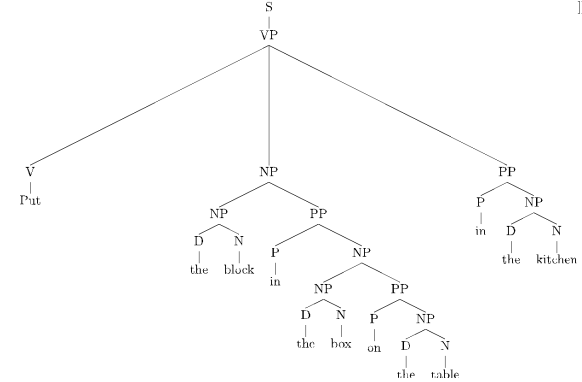
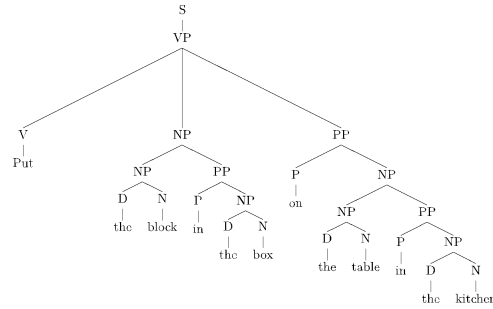
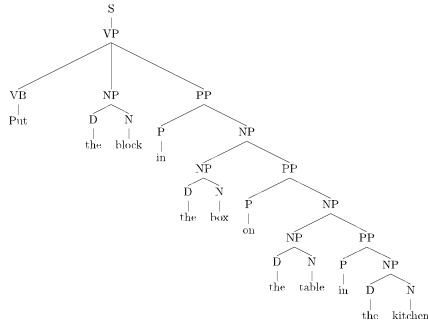
- Prepositional phrases:
  - They cooked the beans in the pot on the stove with handles.
  
- Particle vs. preposition:
  - The puppy tore up the staircase.
  
- Complement structures
  - The tourists objected to the guide that they couldn't hear.  
She knows you like the back of her hand.
  
- Gerund vs. participial adjective
  - Visiting relatives can be boring.  
Changing schedules frequently confused passengers.

# Syntactic Ambiguities II

- Modifier scope within NPs
  - impractical design requirements  
plastic cup holder
- Multiple gap constructions
  - The chicken is ready to eat.  
The contractors are rich enough to sue.
- Coordination scope:
  - Small rats and mice can squeeze into holes or cracks in the wall.

# How to Deal with Ambiguity?

- We want to **score all the derivations** to encode how plausible they are



*Put the block in the box on the table in the kitchen*

# Probabilistic Context Free Grammar (PCFG)

# Probabilistic Context-Free Grammars

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  - $N$  : the set of **non-terminals**
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    - **Parts-of-speech** (pre-terminals): NN, JJ, DT, VB
  - $T$  : the set of **terminals** (the words)
  - $S$  : the **start** symbol
    - Often written as ROOT or TOP
    - Not usually the sentence non-terminal S
  - $R$  : the set of **rules**
    - Of the form  $X \rightarrow Y_1 Y_2 \dots Y_k$ , with  $X, Y_i \in N$
    - Examples:  $S \rightarrow NP VP$ ,  $VP \rightarrow VP CC VP$
    - Also called rewrites, productions, or local trees
- A PCFG adds:
  - A top-down **production probability** per rule  $P(Y_1 Y_2 \dots Y_k \mid X)$

# PCFGs

Associate probabilities with the rules :  $p(X \rightarrow \alpha)$

$$\forall X \rightarrow \alpha \in R : 0 \leq p(X \rightarrow \alpha) \leq 1$$

$$\forall X \in N : \sum_{\alpha: X \rightarrow \alpha \in R} p(X \rightarrow \alpha) = 1$$

Now we can score a tree as a product of probabilities corresponding to the used rules

$S \rightarrow NP VP$	1.0	(NP A girl) (VP ate a sandwich)	$N \rightarrow girl$	0.2
$VP \rightarrow V$	0.2		$N \rightarrow telescope$	0.7
$VP \rightarrow V NP$	0.4	(VP ate) (NP a sandwich)	$N \rightarrow sandwich$	0.1
$VP \rightarrow VP PP$	0.4	(VP saw a girl) (PP with ...)	$PN \rightarrow I$	1.0
$NP \rightarrow NP PP$	0.3	(NP a girl) (PP with ....)	$V \rightarrow saw$	0.5
$NP \rightarrow D N$	0.5	(D a) (N sandwich)	$V \rightarrow ate$	0.5
$NP \rightarrow PN$	0.2		$P \rightarrow with$	0.6
$PP \rightarrow P NP$	1.0	(P with) (NP with a sandwich)	$P \rightarrow in$	0.4
			$D \rightarrow a$	0.3
			$D \rightarrow the$	0.7

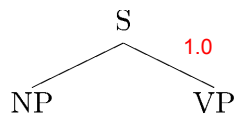
# PCFGs

S

$S \rightarrow NP VP$	1.0	$N \rightarrow girl$	0.2
$VP \rightarrow V$	0.2	$N \rightarrow telescope$	0.7
$VP \rightarrow V NP$	0.4	$N \rightarrow sandwich$	0.1
$VP \rightarrow VP PP$	0.4	$PN \rightarrow I$	1.0
$NP \rightarrow NP PP$	0.3	$V \rightarrow saw$	0.5
$NP \rightarrow D N$	0.5	$V \rightarrow ate$	0.5
$NP \rightarrow PN$	0.2	$P \rightarrow with$	0.6
$PP \rightarrow P NP$	1.0	$P \rightarrow in$	0.4
		$D \rightarrow a$	0.3
		$D \rightarrow the$	0.7

$p(T) =$

# PCFGs



$S \rightarrow NP VP$  1.0

$VP \rightarrow V$  0.2

$VP \rightarrow V NP$  0.4

$VP \rightarrow VP PF$  0.4

$NP \rightarrow NP PF$  0.3

$NP \rightarrow D N$  0.5

$NP \rightarrow PN$  0.2

$PP \rightarrow P NP$  1.0

$N \rightarrow girl$  0.2

$N \rightarrow telescope$  0.7

$N \rightarrow sandwich$  0.1

$PN \rightarrow I$  1.0

$V \rightarrow saw$  0.5

$V \rightarrow ate$  0.5

$P \rightarrow with$  0.6

$P \rightarrow in$  0.4

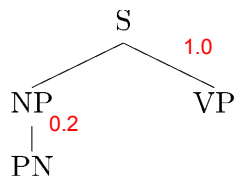
$D \rightarrow a$  0.3

$D \rightarrow the$  0.7

$$p(T) = 1.0 \times$$



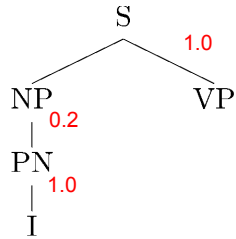
# PCFGs



$S \rightarrow NP VP$	1.0	$N \rightarrow girl$	0.2
$VP \rightarrow V$	0.2	$N \rightarrow telescope$	0.7
$VP \rightarrow V NP$	0.4	$N \rightarrow sandwich$	0.1
$VP \rightarrow VP PP$	0.4	$PN \rightarrow I$	1.0
$NP \rightarrow NP PP$	0.3	$V \rightarrow saw$	0.5
$NP \rightarrow D N$	0.5	$V \rightarrow ate$	0.5
$NP \rightarrow PN$	0.2	$P \rightarrow with$	0.6
$PP \rightarrow P NP$	1.0	$P \rightarrow in$	0.4
		$D \rightarrow a$	0.3
		$D \rightarrow the$	0.7

$$p(T) = 1.0 \times 0.2 \times$$

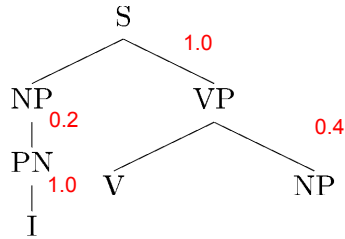
# PCFGs



- |                            |                               |
|----------------------------|-------------------------------|
| $S \rightarrow NP VP$ 1.0  | $N \rightarrow girl$ 0.2      |
| $VP \rightarrow V$ 0.2     | $N \rightarrow telescope$ 0.7 |
| $VP \rightarrow V NP$ 0.4  | $N \rightarrow sandwich$ 0.1  |
| $VP \rightarrow VP PP$ 0.4 | $PN \rightarrow I$ 1.0        |
| $NP \rightarrow NP PP$ 0.3 | $V \rightarrow saw$ 0.5       |
| $NP \rightarrow D N$ 0.5   | $V \rightarrow ate$ 0.5       |
| $NP \rightarrow PN$ 0.2    | $P \rightarrow with$ 0.6      |
| $PP \rightarrow P NP$ 1.0  | $P \rightarrow in$ 0.4        |
|                            | $D \rightarrow a$ 0.3         |
|                            | $D \rightarrow the$ 0.7       |

$$p(T) = 1.0 \times 0.2 \times 1.0 \times$$

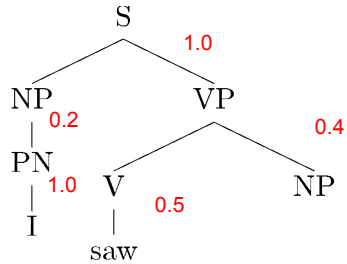
# PCFGs



- |                            |                               |
|----------------------------|-------------------------------|
| $S \rightarrow NP VP$ 1.0  | $N \rightarrow girl$ 0.2      |
| $VP \rightarrow V$ 0.2     | $N \rightarrow telescope$ 0.7 |
| $VP \rightarrow V NP$ 0.4  | $N \rightarrow sandwich$ 0.1  |
| $VP \rightarrow VP PP$ 0.4 | $PN \rightarrow I$ 1.0        |
| $NP \rightarrow NP PP$ 0.3 | $V \rightarrow saw$ 0.5       |
| $NP \rightarrow D N$ 0.5   | $V \rightarrow ate$ 0.5       |
| $NP \rightarrow PN$ 0.2    | $P \rightarrow with$ 0.6      |
|                            | $P \rightarrow in$ 0.4        |
|                            | $D \rightarrow a$ 0.3         |
| $PP \rightarrow P NP$ 1.0  | $D \rightarrow the$ 0.7       |

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times$$

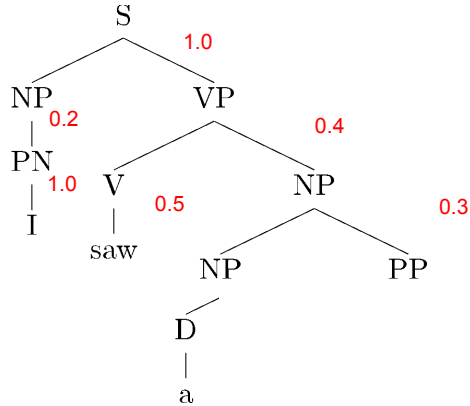
# PCFGs



- |                            |                               |
|----------------------------|-------------------------------|
| $S \rightarrow NP VP$ 1.0  | $N \rightarrow girl$ 0.2      |
| $VP \rightarrow V$ 0.2     | $N \rightarrow telescope$ 0.7 |
| $VP \rightarrow V NP$ 0.4  | $N \rightarrow sandwich$ 0.1  |
| $VP \rightarrow VP PP$ 0.4 | $PN \rightarrow I$ 1.0        |
| $NP \rightarrow NP PP$ 0.3 | $V \rightarrow saw$ 0.5       |
| $NP \rightarrow D N$ 0.5   | $V \rightarrow ate$ 0.5       |
| $NP \rightarrow PN$ 0.2    | $P \rightarrow with$ 0.6      |
| $PP \rightarrow P NP$ 1.0  | $P \rightarrow in$ 0.4        |
|                            | $D \rightarrow a$ 0.3         |
|                            | $D \rightarrow the$ 0.7       |

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times$$

# PCFGs



$$S \rightarrow NP VP \ 1.0$$

$$VP \rightarrow V \ 0.2$$

$$VP \rightarrow V NP \ 0.4$$

$$VP \rightarrow VP PF \ 0.4$$

$$NP \rightarrow NP PF \ 0.3$$

$$NP \rightarrow D N \ 0.5$$

$$NP \rightarrow PN \ 0.2$$

$$PP \rightarrow P NP \ 1.0$$

$$N \rightarrow girl \ 0.2$$

$$N \rightarrow telescope \ 0.7$$

$$N \rightarrow sandwich \ 0.1$$

$$PN \rightarrow I \ 1.0$$

$$V \rightarrow saw \ 0.5$$

$$V \rightarrow ate \ 0.5$$

$$P \rightarrow with \ 0.6$$

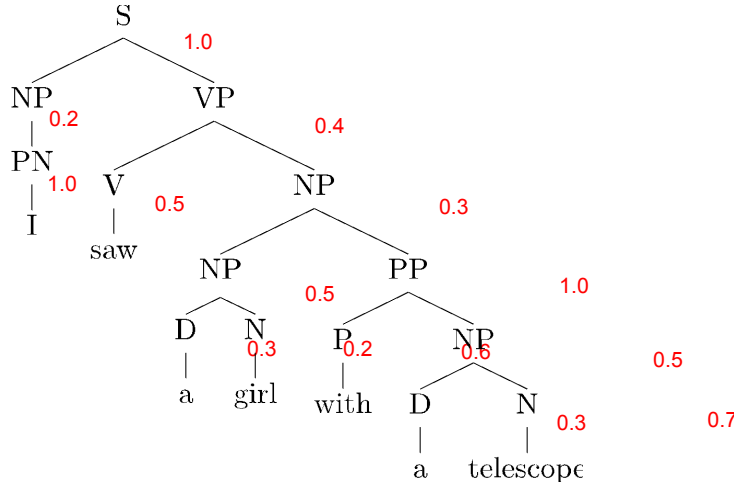
$$P \rightarrow in \ 0.4$$

$$D \rightarrow a \ 0.3$$

$$D \rightarrow the \ 0.7$$

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times$$

# PCFGs



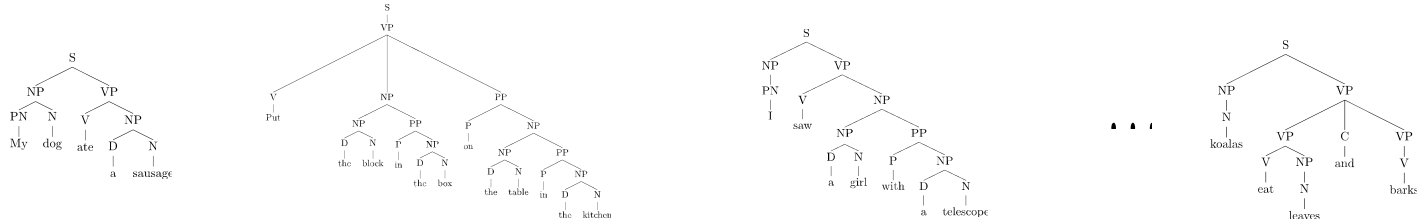
- $S \rightarrow NP VP$  1.0
- $VP \rightarrow V$  0.2
- $VP \rightarrow V NP$  0.4
- $VP \rightarrow VP PP$  0.4
- $NP \rightarrow NP PP$  0.3
- $NP \rightarrow D N$  0.5
- $NP \rightarrow PN$  0.2
- $PP \rightarrow P NP$  1.0
- $N \rightarrow girl$  0.2
- $N \rightarrow telescope$  0.7
- $N \rightarrow sandwich$  0.1
- $PN \rightarrow I$  1.0
- $V \rightarrow saw$  0.5
- $V \rightarrow ate$  0.5
- $P \rightarrow with$  0.6
- $P \rightarrow in$  0.4
- $D \rightarrow a$  0.3
- $D \rightarrow the$  0.7

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 = 2.26 \times 10^{-5}$$

# PCFG Estimation

# ML estimation

- A treebank: a collection sentences annotated with constituent trees



- An estimated probability of a rule (maximum likelihood estimates)

$$p(X \rightarrow \alpha) = \frac{C(X \rightarrow \alpha)}{C(X)}$$

The number of times the rule used in the corpus

The number of times the nonterminal X appears in the treebank

- Smoothing is helpful
  - Especially important for preterminal rules



# CKY Parsing

# Parsing

- **Parsing is search** through the space of all possible parses
  - e.g., we may want either any parse, all parses or the highest scoring parse (if PCFG):

$$\arg \max_{T \in G(x)} P(T)$$

- **Bottom-up:**
  - One starts from words and attempt to construct the full tree
- **Top-down**
  - Start from the start symbol and attempt to expand to get the sentence

# CKY algorithm (aka CYK)

- **Cocke-Kasami-Younger** algorithm
  - Independently discovered in late 60s / early 70s
- An efficient bottom up parsing algorithm for (P)CFGs
  - can be used both for the recognition and parsing problems
  - Very important in NLP (and beyond)
- We will start with the non-probabilistic version

# Constraints on the grammar

- The basic CKY algorithm supports only rules in the **Chomsky Normal Form (CNF)**:

$$C \rightarrow x$$

Unary **preterminal** rules (generation of words given PoS tags)

$$N \rightarrow \text{telescope} \quad D \rightarrow \text{the}$$

$$C \rightarrow C_1 C_2$$

Binary **inner** rules  $S \rightarrow NPVP \quad NP \rightarrow D N$

# Constraints on the grammar

- The basic CKY algorithm supports only rules in the **Chomsky Normal Form (CNF)**:

$$C \rightarrow x$$

$$C \rightarrow C_1 C_2$$

- Any CFG can be converted to an equivalent CNF
  - Equivalent means that they define **the same language**
  - However (syntactic) **trees will look differently**
  - It is possible to address it by defining such transformations that allows for easy **reverse transformation**

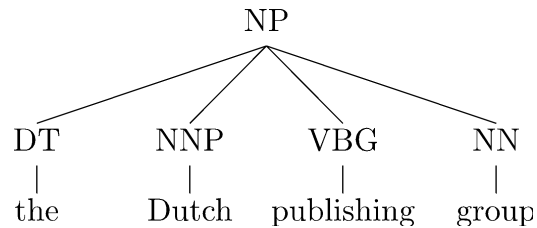
# Transformation to CNF form

- What one need to do to convert to CNF form
  - Get rid of rules that mix terminals and non-terminals
  - Get rid of unary rules:  $C \rightarrow C_1$
  - Get rid of N-ary rules:  $C \rightarrow C_1 C_2 \dots C_n \quad (n > 2)$

Crucial to process them, as required for efficient parsing

# Transformation to CNF form: binarization

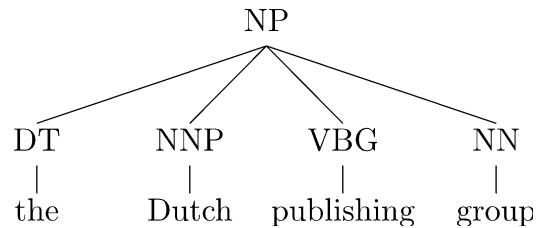
- Consider  $NP \rightarrow DT\ NNP\ VBG\ NN$



- How do we get a set of binary rules which are equivalent?

# Transformation to CNF form: binarization

- Consider  $NP \rightarrow DT \ NNP \ VBG \ NN$



- How do we get a set of binary rules which are equivalent?

$NP \rightarrow DT \ X$

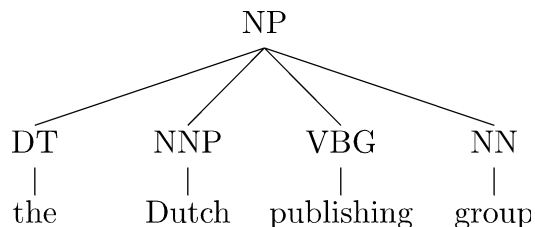
$X \rightarrow NNP \ Y$

$Y \rightarrow VBG \ NN$



# Transformation to CNF form: binarization

- Consider  $NP \rightarrow DT \ NNP \ VBG \ NN$



- How do we get a set of binary rules which are equivalent?

$NP \rightarrow DT \ X$

$X \rightarrow NNP \ Y$

$Y \rightarrow VBG \ NN$

- A more systematic way to refer to new non-terminals

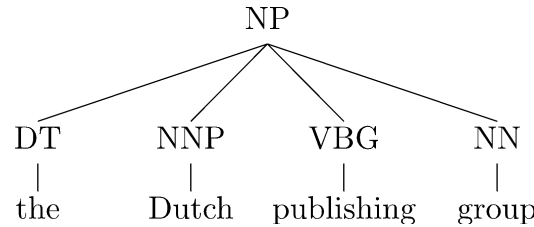
$NP \rightarrow DT \ @NP|DT$

$@NP|DT \rightarrow NNP \ @NP|DT\_NNP$

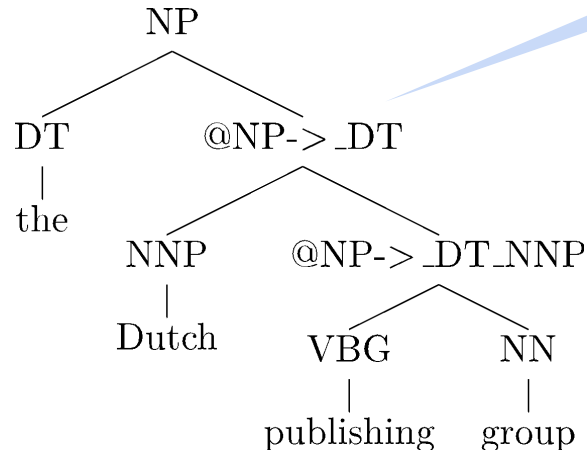
$@NP|DT\_NNP \rightarrow VBG \ NN$

# Transformation to CNF form: binarization

- Instead of binarizing tuples we can binarize trees on preprocessing:



Also known as **lossless Markovization** in the context of PCFGs



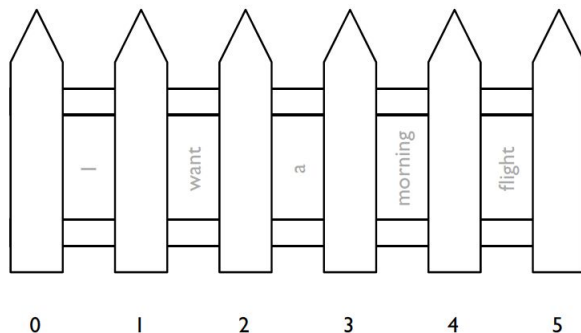
Can be easily reversed on postprocessing

# CKY: Parsing task

- We are given
  - a grammar  $\langle N, T, S, R \rangle$
  - a sequence of words  $\mathbf{w} = (w_1, w_2, \dots, w_n)$
  
- Our goal is to produce a parse tree for  $w$

# CKY: Parsing task

- We are given
  - a grammar  $\langle N, T, S, R \rangle$
  - a sequence of words  $w = (w_1, w_2, \dots, w_n)$
- Our goal is to produce a parse tree for  $w$
- We need an easy way to refer to substrings of  $w$



indices refer to fenceposts

**span**  $(i, j)$  refers to words between fenceposts  $i$  and  $j$

# Parsing one word

$$C \rightarrow w_i$$

$w_i$

# Parsing one word

$$C \rightarrow w_i$$

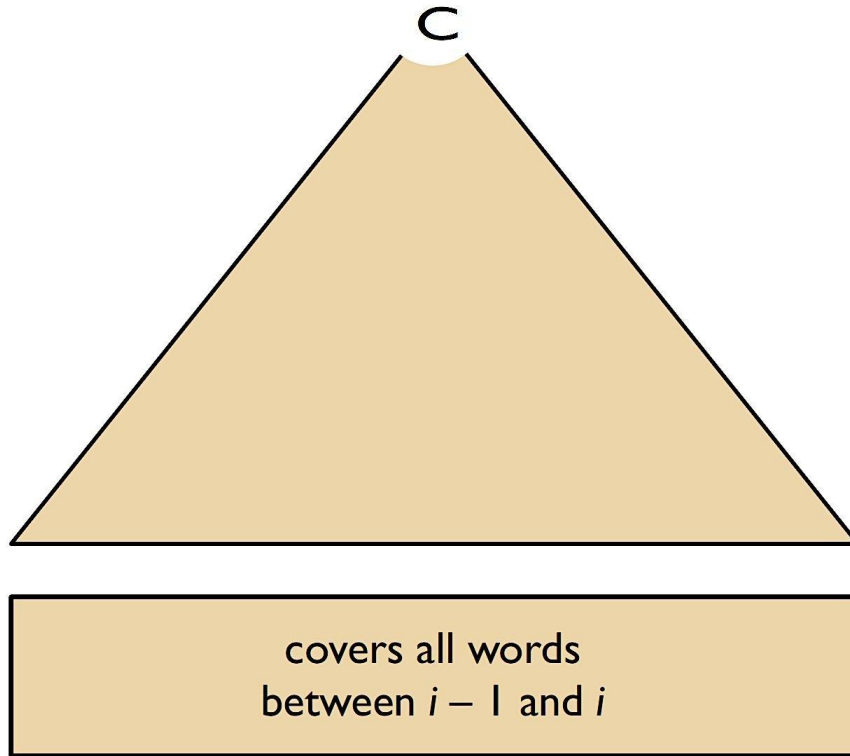
C



$w_i$

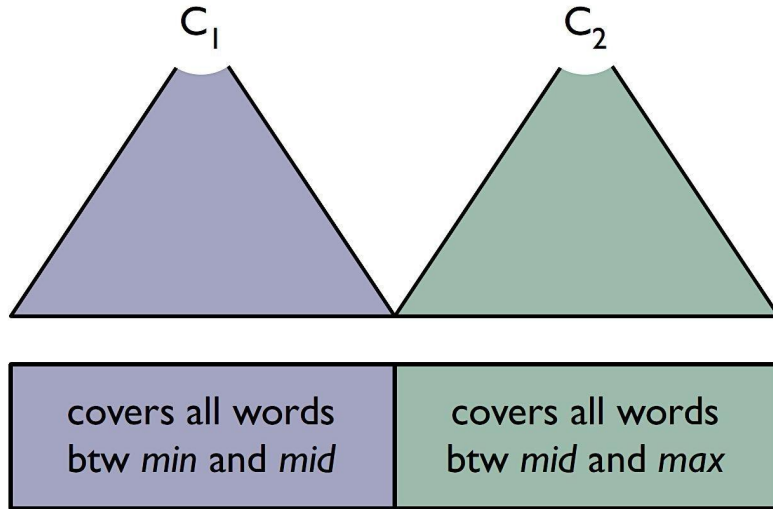
# Parsing one word

$$C \rightarrow w_i$$



# Parsing longer spans

$$C \rightarrow C_1 C_2$$

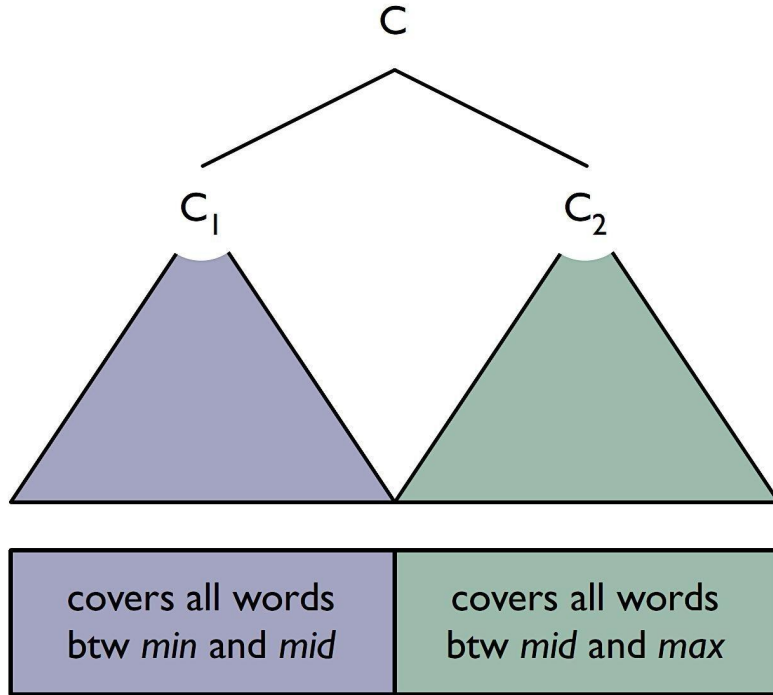


Check through all  
C1, C2, mid



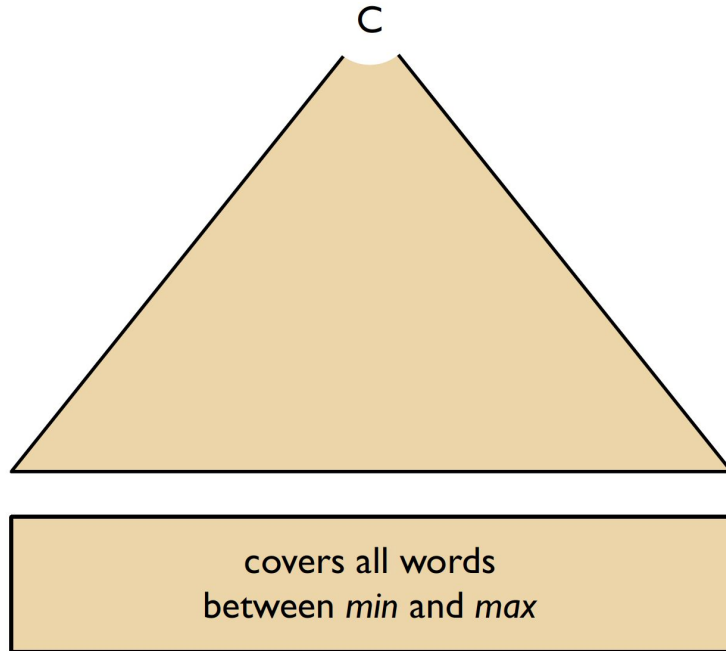
# Parsing longer spans

$$C \rightarrow C_1 C_2$$



Check through all  
 $C_1$ ,  $C_2$ , *mid*

# Parsing longer spans



lead	can	poison	
0	1	2	3

max = 1      max = 2      max = 3

min = 0			<i>S?</i>
min = 1			
min = 2			

Chart (aka parsing triangle)

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

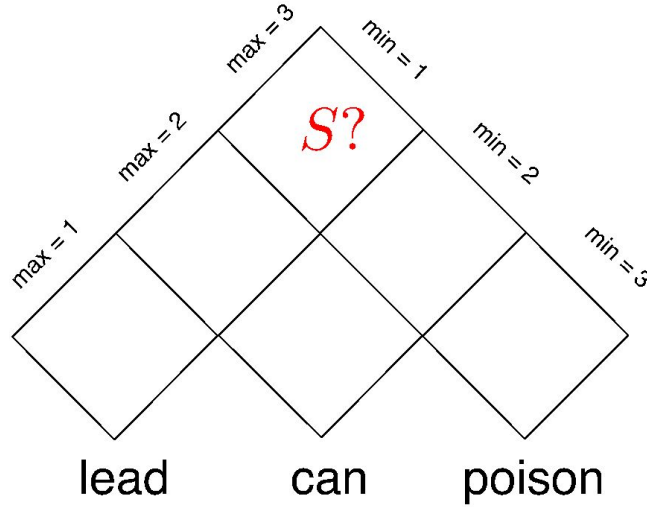
$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

---

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

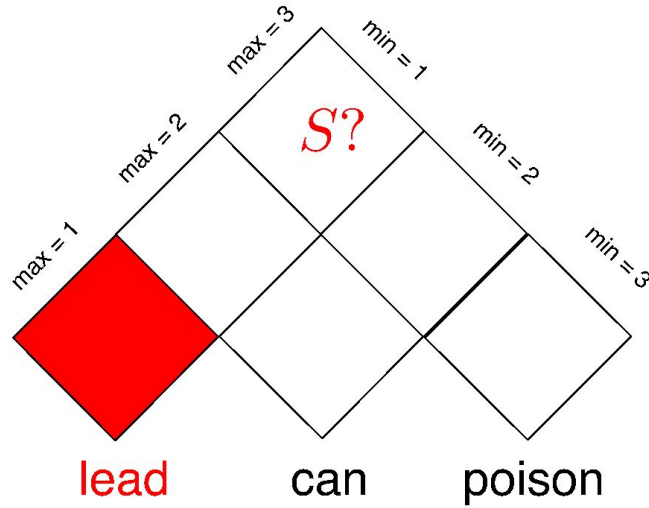
$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner  
rules

Preterminal  
rules

$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow \text{can}$   
 $N \rightarrow \text{lead}$   
 $N \rightarrow \text{poison}$

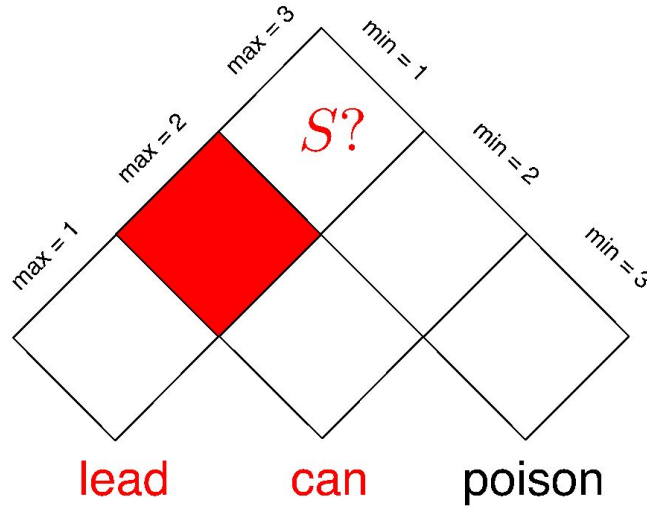
$M \rightarrow \text{can}$   
 $M \rightarrow \text{must}$

$V \rightarrow \text{poison}$   
 $V \rightarrow \text{lead}$

Inner  
rules

Preterminal  
rules

$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow \text{can}$   
 $N \rightarrow \text{lead}$   
 $N \rightarrow \text{poison}$

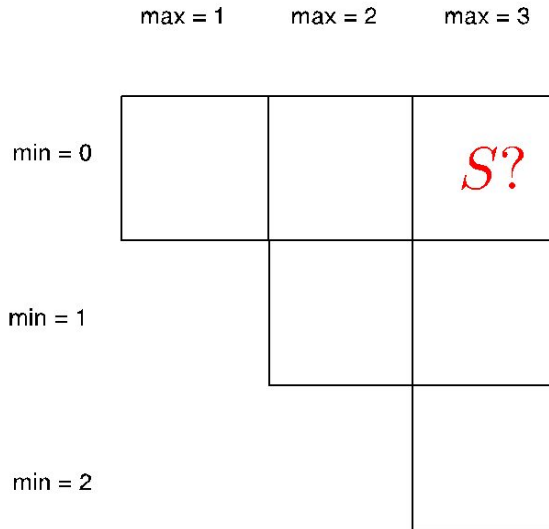
$M \rightarrow \text{can}$   
 $M \rightarrow \text{must}$

$V \rightarrow \text{poison}$   
 $V \rightarrow \text{lead}$

Inner  
rules

Preterminal  
rules

	lead		can		poison	
0		1		2		3



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

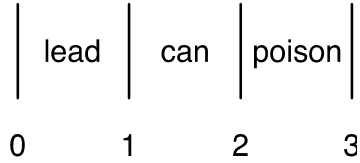
$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

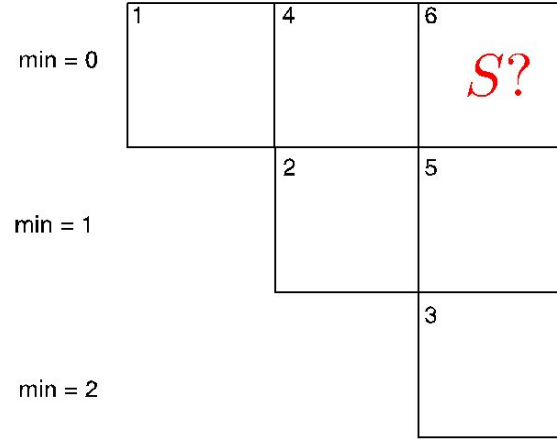
$V \rightarrow poison$   
 $V \rightarrow lead$

Inner  
rules

Preterminal  
rules



max = 1
max = 2
max = 3



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

---

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

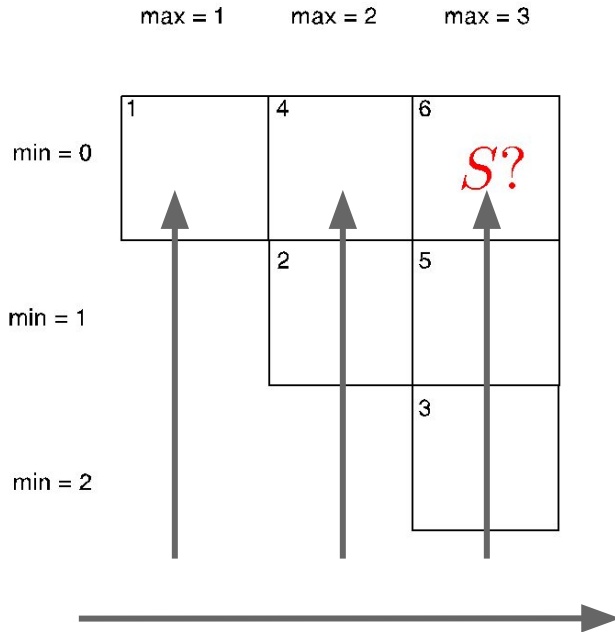
$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules



$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$



$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow \text{can}$

$N \rightarrow \text{lead}$

$N \rightarrow \text{poison}$

$M \rightarrow \text{can}$

$M \rightarrow \text{must}$

$V \rightarrow \text{poison}$

$V \rightarrow \text{lead}$

Inner  
rules

Preterminal  
rules

	lead		can		poison	
0	1	2	3			

max = 1      max = 2      max = 3

min = 0	1	?		
min = 1		2	?	
min = 2			3	?

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner  
rules

Preterminal  
rules

lead	can	poison	
0	1	2	3

max = 1
max = 2
max = 3

min = 0	<span style="font-size: 2em; color: red;">?</span>		
min = 1		<span style="font-size: 2em; color: red;">?</span>	
min = 2			<span style="font-size: 2em; color: red;">?</span>

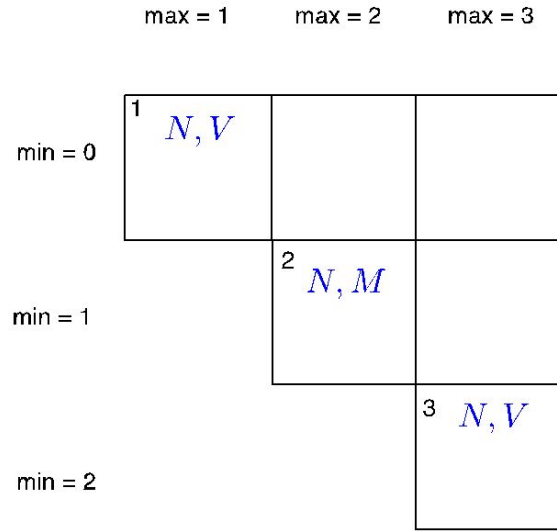
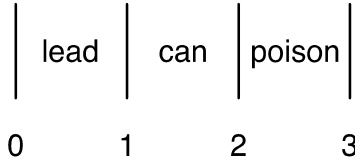
$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$   
  
 $M \rightarrow can$   
 $M \rightarrow must$   
  
 $V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules



$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

---

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$   
  
 $M \rightarrow can$   
 $M \rightarrow must$   
  
 $V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules

	lead		can		poison	
0	1	2	3			

max = 1      max = 2      max = 3

min = 0	1	$N, V$ $NP, VP$	4	?	
min = 1			2	$N, M$ $NP$	
min = 2					3
					$N, V$ $NP, VP$

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2
0	1	2
0	1	2

	max = 1	max = 2	max = 3
min = 0	<div style="display: flex; justify-content: space-between;"> <span>1</span> <span>4</span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>N, V</i></span> <span>?</span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>NP, VP</i></span> </div>		
min = 1		<div style="display: flex; justify-content: space-between;"> <span>2</span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>N, M</i></span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>NP</i></span> </div>	
min = 2			<div style="display: flex; justify-content: space-between;"> <span>3</span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>N, V</i></span> </div> <div style="display: flex; justify-content: space-between;"> <span><i>NP, VP</i></span> </div>

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison	
0	1	2	3

	max = 1	max = 2	max = 3
min = 0	1 $N, V$ $NP, VP$	4 $NP$	
min = 1		2 $N, M$ $NP$	
min = 2			3 $N, V$ $NP, VP$

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

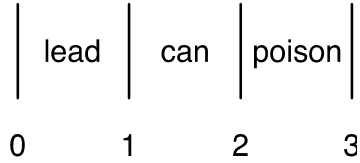
$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner  
rules

Preterminal  
rules



max = 1
max = 2
max = 3

min = 0	<div style="display: flex; flex-direction: column; justify-content: space-between;"> <span>1</span> <span><i>N, V</i></span> <span><i>NP, VP</i></span> </div>	<div style="display: flex; flex-direction: column; justify-content: space-between;"> <span>4</span> <span><i>NP</i></span> </div>	
min = 1		<div style="display: flex; flex-direction: column; justify-content: space-between;"> <span>2</span> <span><i>N, M</i></span> <span><i>NP</i></span> </div>	<div style="display: flex; flex-direction: column; justify-content: space-between;"> <span>5</span> <span style="color: red; font-size: 2em;">?</span> </div>
min = 2			<div style="display: flex; flex-direction: column; justify-content: space-between;"> <span>3</span> <span><i>N, V</i></span> <span><i>NP, VP</i></span> </div>

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules



$S \rightarrow NP VP$

$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$

max = 1      max = 2      max = 3

min = 0	1 $N, V$ $NP, VP$	4 $NP$	
min = 1		2 $N, M$ $NP$	5 $S, VP,$ $NP$
min = 2			3 $N, V$ $NP, VP$

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow \text{can}$   
 $N \rightarrow \text{lead}$   
 $N \rightarrow \text{poison}$

$M \rightarrow \text{can}$   
 $M \rightarrow \text{must}$

$V \rightarrow \text{poison}$   
 $V \rightarrow \text{lead}$

Inner rules

Preterminal rules

lead	can	poison
0	1	2
0	1	2
0	1	2

max = 1
max = 2
max = 3

min = 0	<div style="display: flex; justify-content: space-between;"> <span>1</span> <span>4</span> <span>6</span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>N, V</math></span> <span><math>NP</math></span> <span style="color: red; font-size: 2em;">?</span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>NP, VP</math></span> <span></span> <span></span> </div>
min = 1	<div style="display: flex; justify-content: space-between;"> <span>2</span> <span>5</span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>N, M</math></span> <span><math>S, VP,</math></span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>NP</math></span> <span><math>NP</math></span> </div>
min = 2	<div style="display: flex; justify-content: space-between;"> <span>3</span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>N, V</math></span> </div> <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <span><math>NP, VP</math></span> </div>

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

---

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison	
0	1	2	3

max = 1      max = 2      max = 3

min = 0	1 $N, V$ $NP, VP$	4 $NP$	6 ?
min = 1		2 $N, M$ $NP$	5 $S, VP,$ $NP$
min = 2			3 $N V$ $NP VP$

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

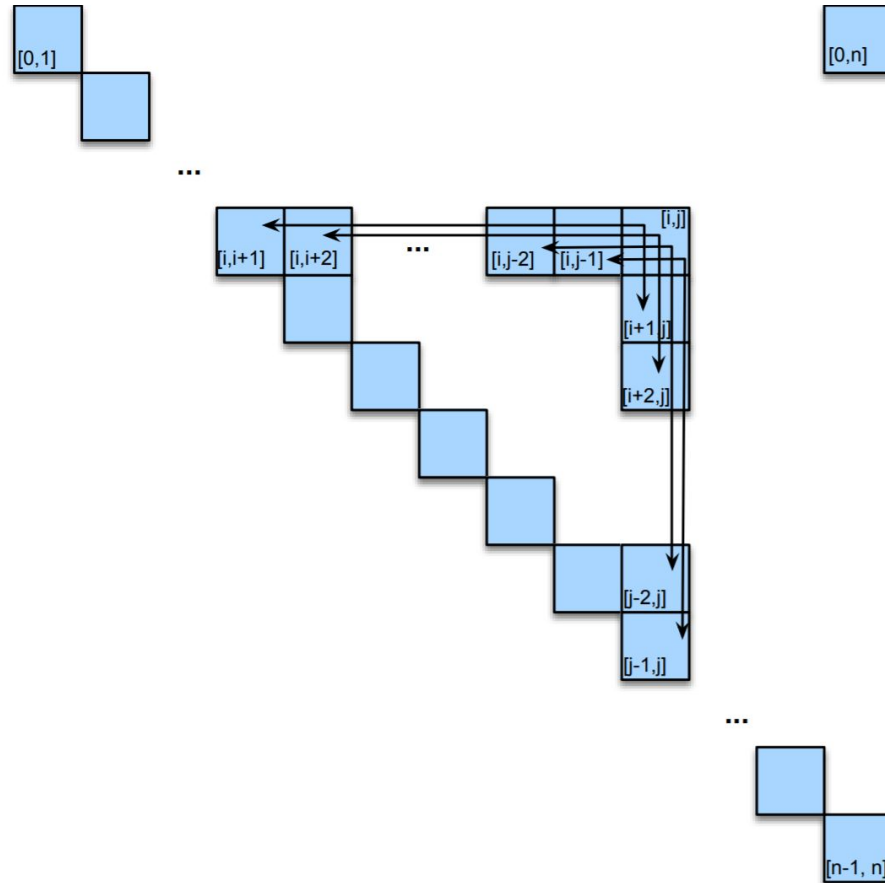
$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner  
rules

Preterminal  
rules



$S \rightarrow NP VP$

$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$

max = 1      max = 2      max = 3

*mid=1*

min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>	6 <i>S, NP</i>
min = 1		2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i>
min = 2			3 <i>N, V</i> <i>NP, VP</i>

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow \text{can}$   
 $N \rightarrow \text{lead}$   
 $N \rightarrow \text{poison}$

$M \rightarrow \text{can}$   
 $M \rightarrow \text{must}$

$V \rightarrow \text{poison}$   
 $V \rightarrow \text{lead}$

Inner rules

Preterminal rules

$S \rightarrow NP VP$

lead	can	poison	
0	1	2	3

max = 1      max = 2      max = 3

*mid=2*

min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>	6 <i>S, NP</i> <i>S(!?)</i>
min = 1		2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i>
min = 2			3 <i>N, V</i> <i>NP, VP</i>

$VP \rightarrow M V$   
 $VP \rightarrow V$

$NP \rightarrow N$   
 $NP \rightarrow N NP$

$N \rightarrow can$   
 $N \rightarrow lead$   
 $N \rightarrow poison$

$M \rightarrow can$   
 $M \rightarrow must$

$V \rightarrow poison$   
 $V \rightarrow lead$

Inner  
rules

Preterminal  
rules

$S \rightarrow NP VP$

$\left| \begin{array}{c} \text{lead} \\ 0 \end{array} \right| \left| \begin{array}{c} \text{can} \\ 1 \end{array} \right| \left| \begin{array}{c} \text{poison} \\ 2 \end{array} \right| \left| \begin{array}{c} \\ 3 \end{array} \right|$

max = 1      max = 2      max = 3

min = 0	1 $N, V$ $NP, VP$	4 $NP$	6 $S, NP$ $S(!?)$
min = 1		2 $N, M$ $NP$	5 $S, VP,$ $NP$
min = 2			3 $N, V$ $NP, VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

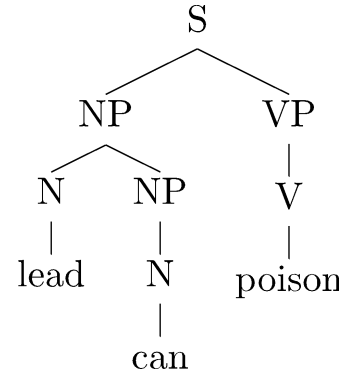
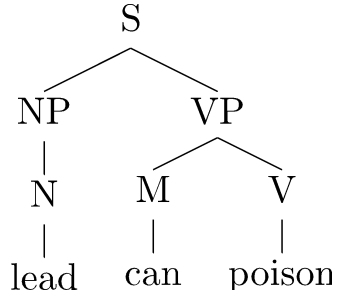
$V \rightarrow lead$

Inner rules

Preterminal rules

Apparently the sentence is ambiguous for the grammar: (as the grammar overgenerates)

# Ambiguity



No subject-verb agreement, and *poison* used as an intransitive verb