

# CSE 446: Machine Learning

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Natasha Jaques

# Course Website

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<https://courses.cs.washington.edu/courses/cse446/25sp/>

## **Everything you need to know is there:**

- Lecture slides (blank and annotated versions will be posted)
- Links to places to get help (Ed Discussion, staff email)
- Homeworks and due dates
- Past exams to study from
- Textbook and other references
- Sections, office hours
- Grading, FAQ, etc.

# Course Staff - Instructor

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**Natasha Jaques**

Assistant Professor in CSE

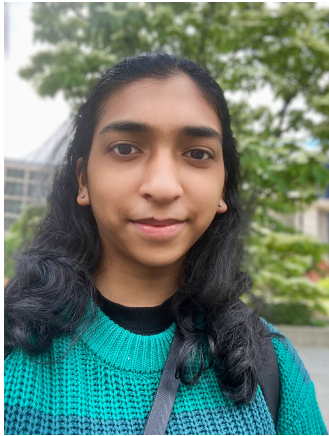
Senior Research Scientist at Google DeepMind



# Course Staff - Teaching Assistants



Donovan Clay  
(Head TA)



Cleah Taryn  
Winston



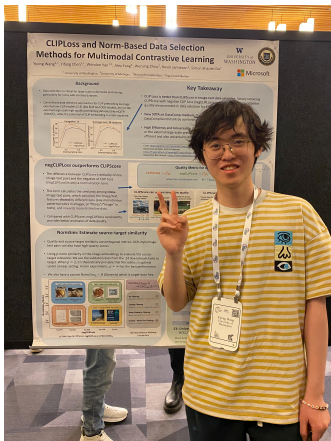
Emmanuel Azuh  
Mensah



Sankar Vaishnav  
Harilal



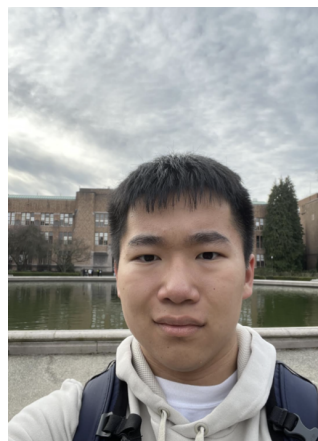
Leo Maynard-  
Zhang



Yiping Wang



Saket Gollapudi



Anthony Xing



Varun Ananth



Marco Hendra  
Dani

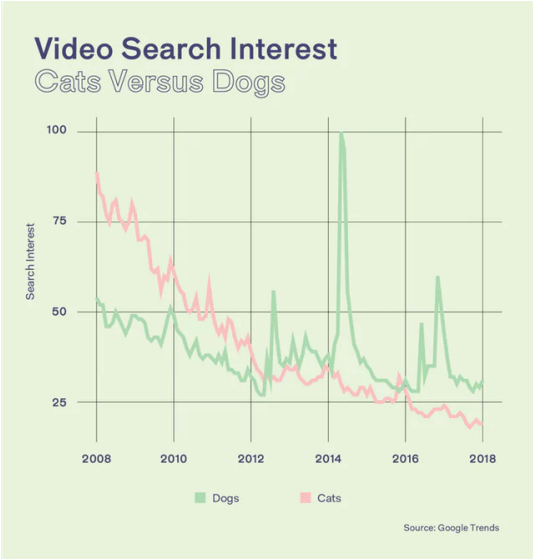
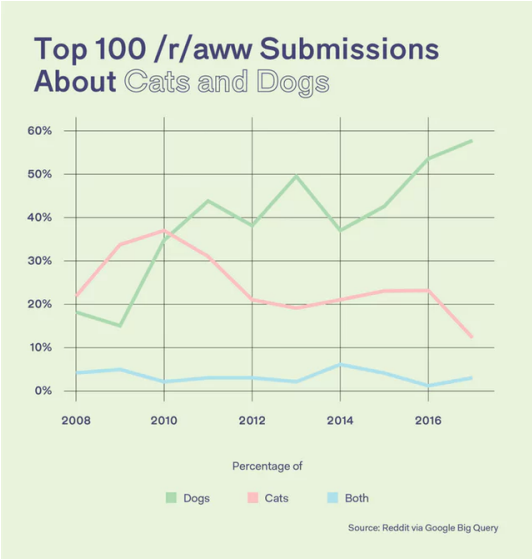
# Traditional Algorithms

## Social media mentions of Cats vs. Dogs

Reddit

Google

Twitter?



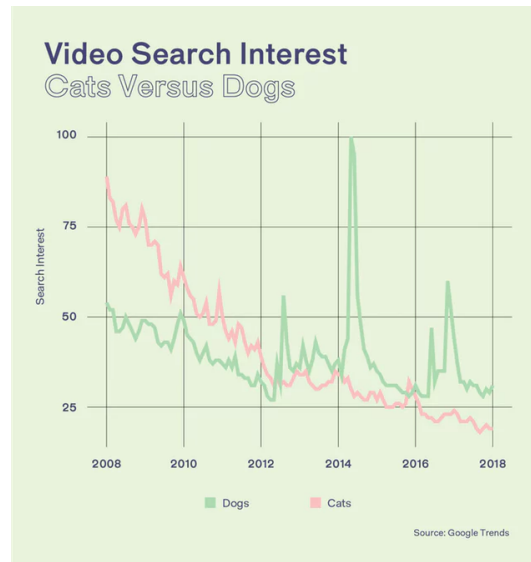
# Traditional Algorithms

Social media mentions of Cats vs. Dogs

Reddit

Google

Twitter?



**Write a program that sorts tweets into those containing “cat”, “dog”, or *other***

# Traditional Algorithms

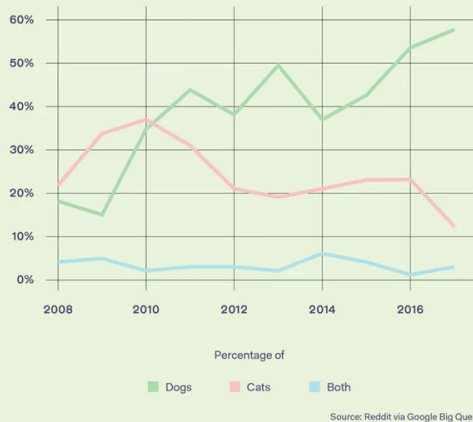
## Social media mentions of Cats vs. Dogs

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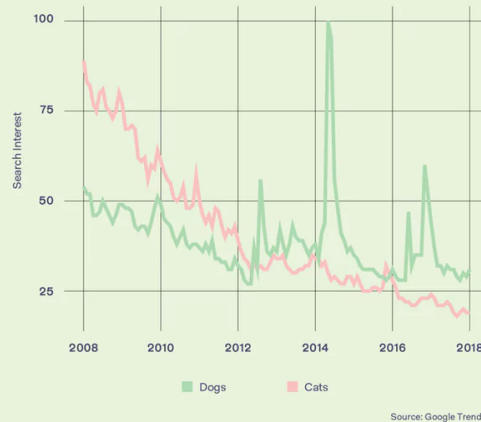
Google

Twitter? 

Top 100 /r/aww Submissions About Cats and Dogs



Video Search Interest Cats Versus Dogs

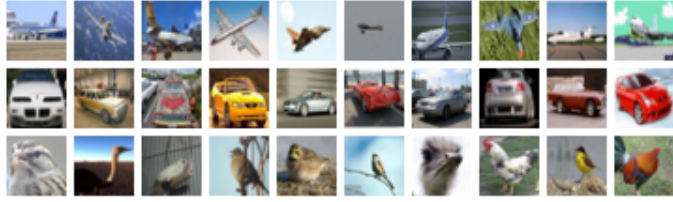


Write a program that sorts tweets into those containing "cat", "dog", or other

```
cats = []
dogs = []
other = []
for tweet in tweets:
    if "cat" in tweet:
        cats.append(tweet)
    elif "dog" in tweet:
        dogs.append(tweet)
    else:
        other.append(tweet)
return cats, dogs, other
```

# Machine Learning Algorithms

Write a program that sorts images into those containing “birds”, “airplanes”, or *other*.



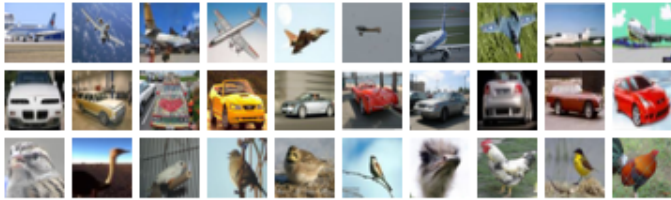
airplane

other

bird

# Machine Learning Algorithms

Write a program that sorts **images** into those containing “**birds**”, “**airplanes**”, or ***other***.



airplane

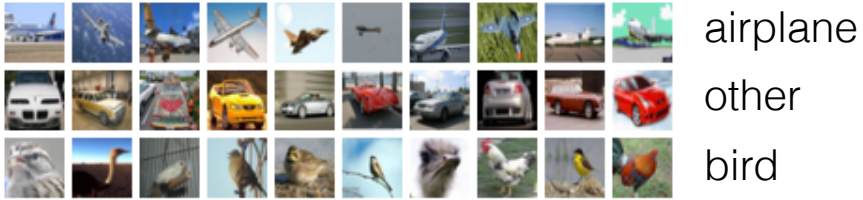
other

bird

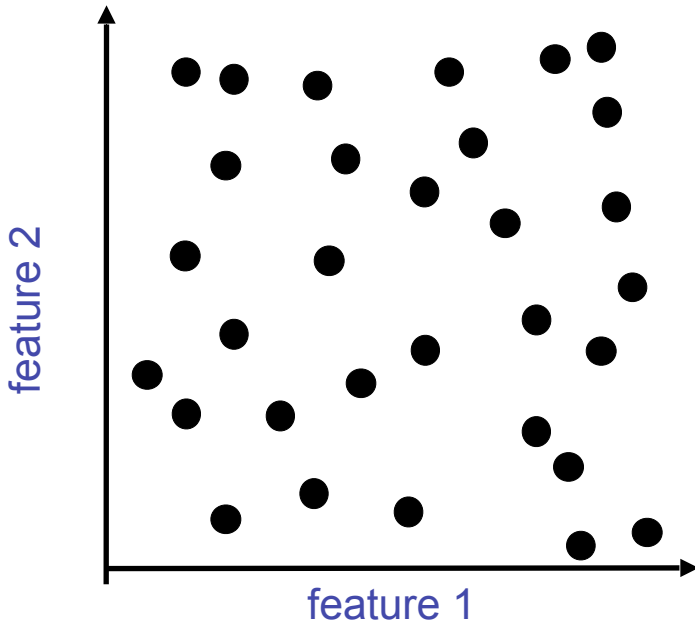
```
birds = []
planes = []
other = []
for image in images:
    if bird in image:
        birds.append(image)
    elif plane in image:
        planes.append(image)
    else:
        other.append(image)
return birds, planes, other
```

# Machine Learning Algorithms

Write a program that sorts **images** into those containing “**birds**”, “**airplanes**”, or **other**.

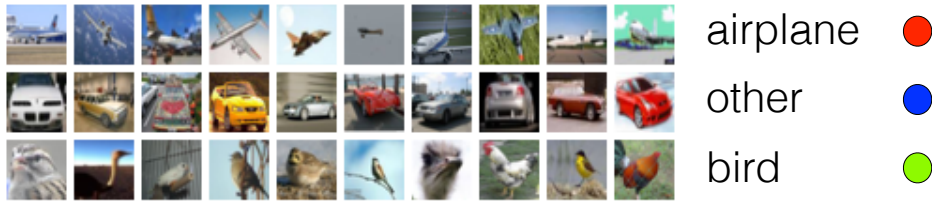


```
birds = []  
planes = []  
other = []  
for image in images:  
    if bird in image:  
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        planes.append(image)  
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return birds, planes, other
```

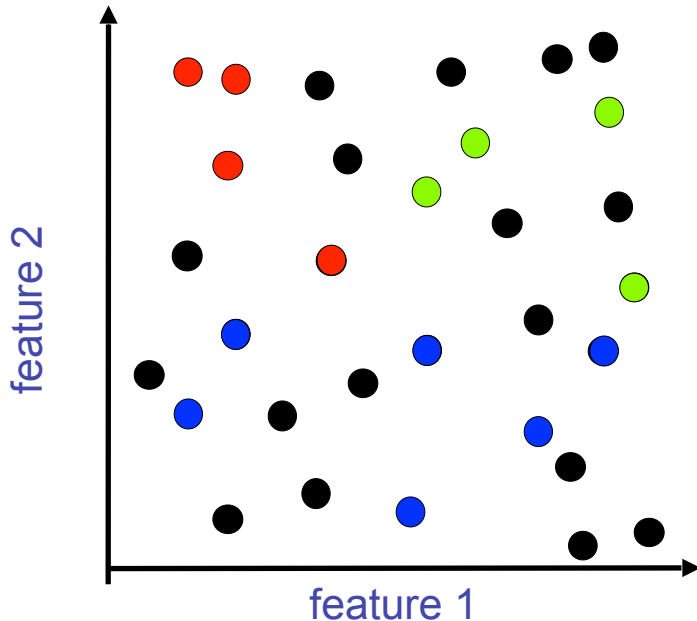


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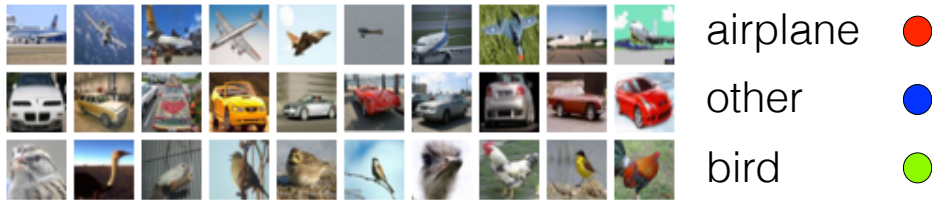


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return birds, planes, other
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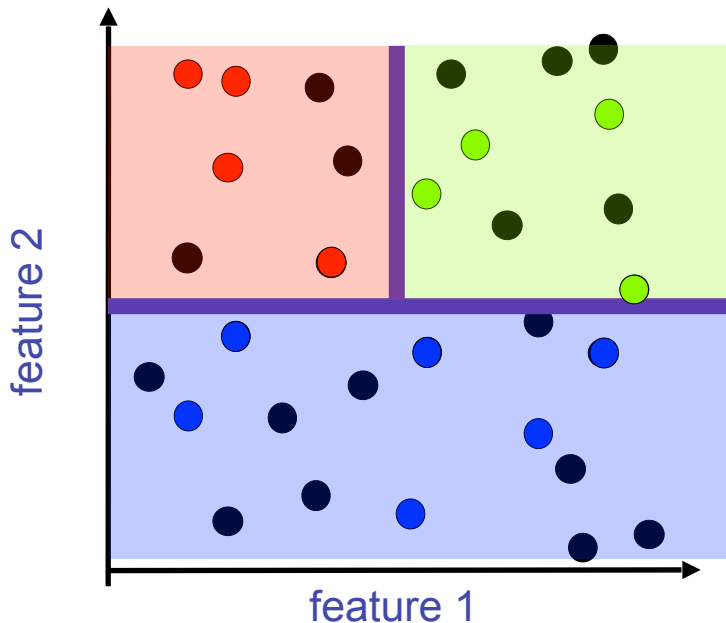


# Machine Learning Algorithms

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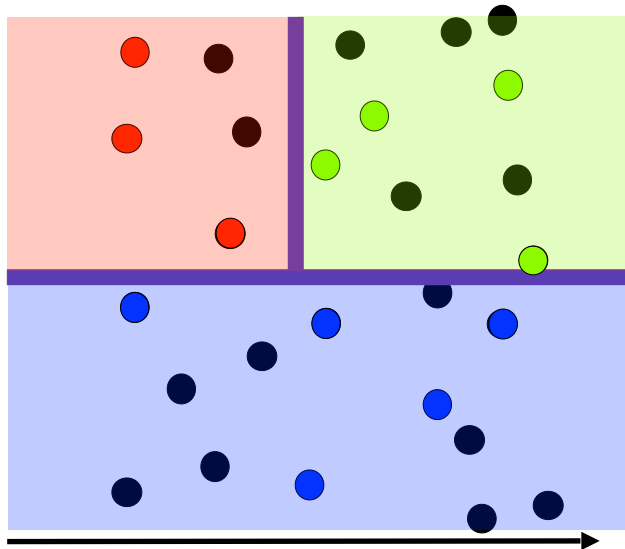
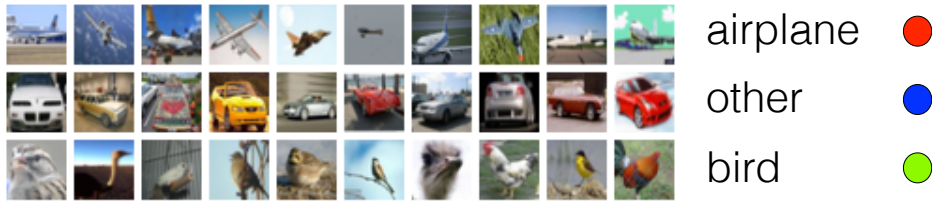


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# Machine Learning Algorithms

Write a program that sorts **images** into those containing “**birds**”, “**airplanes**”, or **other**.



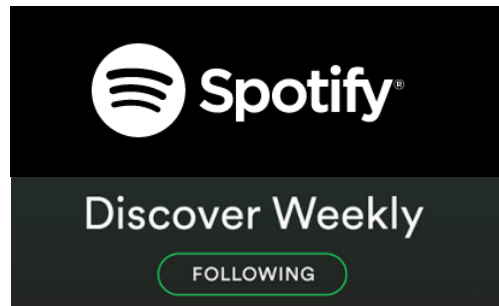
```
birds = []  
planes = []  
other = []  
for image in images:  
    if bird in image:  
        birds.append(image)  
    elif plane in image:  
        planes.append(image)  
    else: ...
```

The decision rule of  
*if "cat" in tweet:*  
is **hard coded by expert**.

The decision rule of  
*if bird in image:*  
is **LEARNED using DATA**

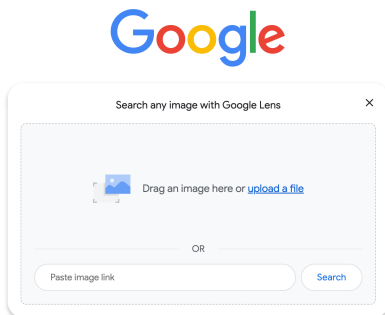
# Machine Learning Ingredients

- **Data:** past observations
- **Hypotheses/Models:** devised to capture the patterns in data
- **Prediction:** apply model to forecast future observations

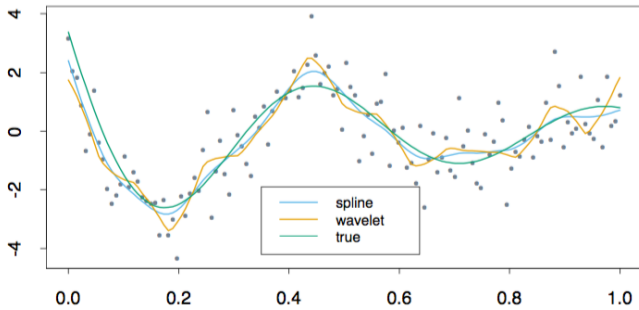


You may also like...

Even before ChatGPT, you were already interacting with ML algorithms every day

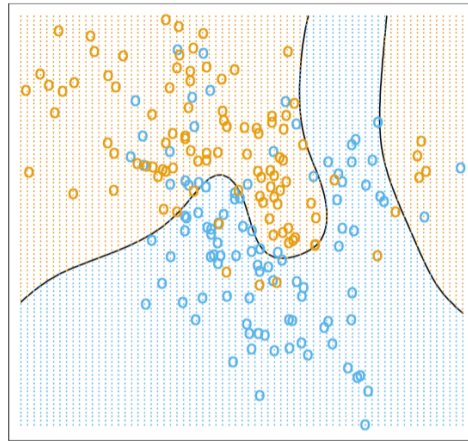


# Flavors of ML



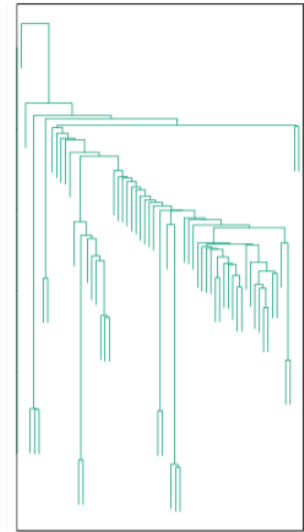
## Regression

Predict continuous value:  
ex: stock market, credit score,  
temperature, Netflix rating



## Classification

Predict categorical value:  
loan or not? spam or not? what  
disease is this?



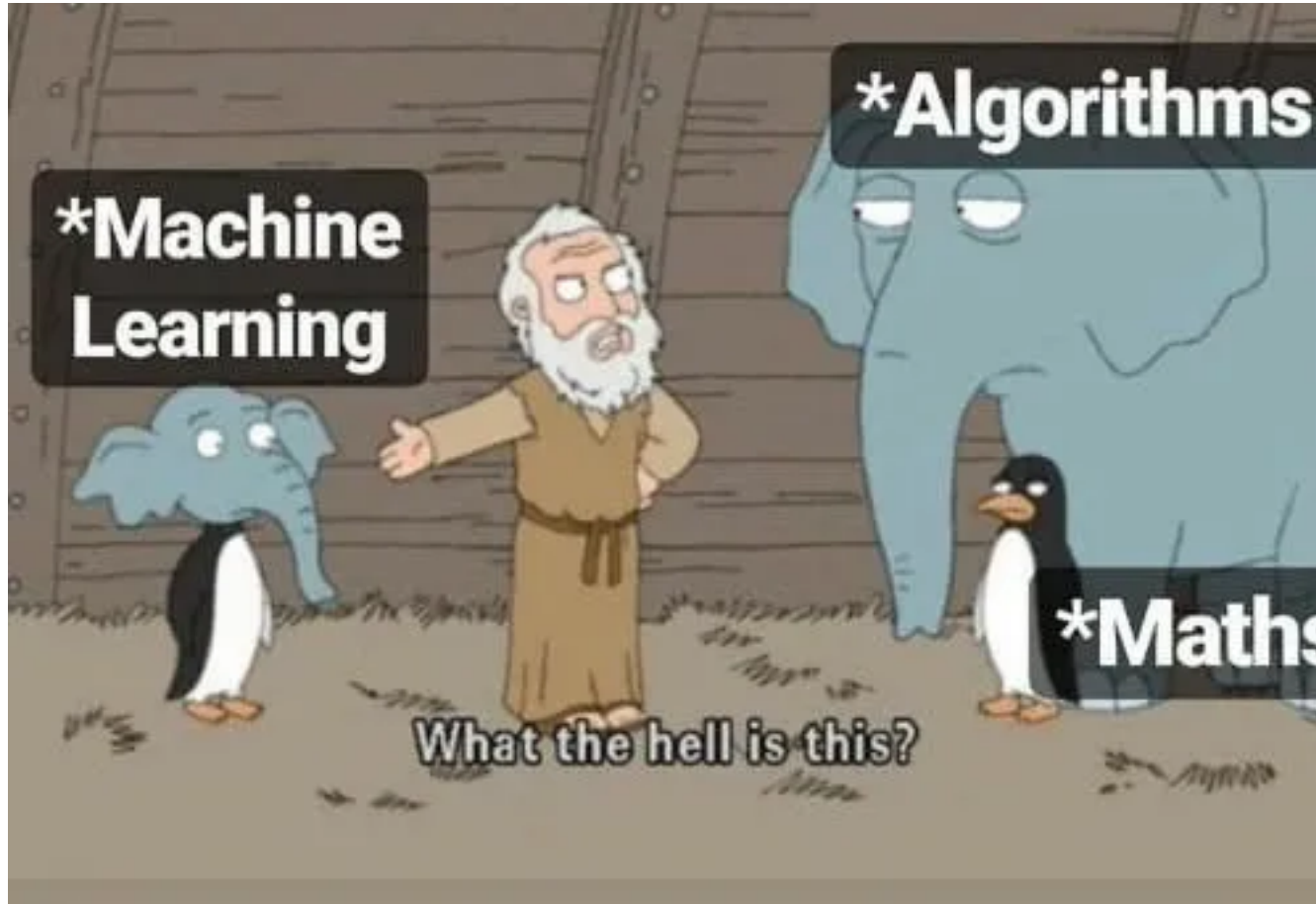
## Unsupervised Learning

Predict structure:  
tree of life from DNA, find  
similar images, community  
detection

Self-supervised:  
Model distribution of large-  
scale data, like text or  
images (large vision and  
language models)

**Mix of statistics (conceptual) and algorithms (programming)**

# Mix of statistics (conceptual) and algorithms (programming)



## What this class is:

- **Fundamentals of ML:** bias/variance tradeoff, overfitting, optimization and computational tradeoffs, supervised learning (e.g., linear, boosting, deep learning), unsupervised models (e.g. k-means, EM, PCA)
- **Preparation for further learning:** the field is fast-moving, you will learn the foundations of ML to understand the latest results

## What this class is not:

- **Survey course:** laundry list of algorithms, how to win Kaggle
- **An easy course:** familiarity with intro linear algebra and probability are assumed, homework will be time-consuming

# Prerequisites

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- Familiarity with:
  - Linear algebra
    - linear dependence, rank, linear equations
  - Multivariate calculus
  - Probability and statistics
    - Distributions, densities, marginalization, moments
  - Algorithms
    - Basic data structures, complexity
- Use HW0 as a refresher on the skills you will need
- **See assigned reading and website for additional review materials!**

# Course Registration

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- As of today, all enrollment restrictions have been dropped.
- If there are open spaces, any UW student may register for those spaces.
- All CSE course registration processes are managed centrally by CSE. **Do not email instructors, we cannot help.**

## Resources:

- <https://www.cs.washington.edu/academics/ugrad/advising/>
- <https://www.cs.washington.edu/academics/ugrad/courses/petition>
- <https://www.cs.washington.edu/students/ugrad/non-major-registration>

# Lectures

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- Will be broadcast on Zoom.
- Will be recorded and posted shortly after class.
  - Find Zoom links and videos in Canvas—>Zoom.
- **In-person attendance is strongly encouraged.**
  - Material can be tough, and I can teach better if I know where you get lost
  - **Please ask lots of questions!**
  - We can afford to slow down
- **For exams, in-person attendance is mandatory.**

# Grading

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- 5 homeworks (60%)
  - Each contain both theoretical questions and will have programming.
  - Collaboration okay. You must write, submit, and understand your answers and code (which we may run).
  - **READ COLLABORATION POLICY ON WEBSITE**
  - If you get someone (or ChatGPT) to do your homework, did you actually learn it?
    - This stuff is foundational, if you want to work in this area need to actually understand
  - HW0 (8%), HW1 (13%), HW2 (13%), HW3 (13%), HW4 (13%)
- Midterm (20%) and Final (20%)

# Homeworks

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- **HW 0** is out (**Due next Wednesday 4/9 @ 11:59pm**)
  - Should be review (but being rusty is expected)
  - Work individually, treat as guide for what to brush up on
- **HW 1,2,3,4.** They are not easy or short. Start early.
- **Submit** to Gradescope.
- **Regrade requests** on Gradescope.
- **Late days:** 5 days total over the quarter; no more than 2 per assignment. If an assignment is submitted late and this exceeds your 5 late days, that assignment will receive 0 credit.
  - **Do not need to email us about using a late day**
- **Assignments due at 11:59pm**, submit early and often (do not email us at 12:05).

- 1. All code must be written in Python**
- 2. All written work must be typeset (e.g., LaTeX)**

**See course website for tutorials and references.**

# Communication Channels

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- **Announcements, questions about class, homework help -> [EdStem](#)**
  - “I think there is a typo in the homework?”
  - “What does this notation mean?”
  - “Is this an accurate description of how this works?”
- **Personal concerns -> [cse446-staff@cs.washington.edu](mailto:cse446-staff@cs.washington.edu)**
  - “Was in hospital...”, “Laptop was stolen...”
  - Do not email to ask if you can use one of your allowed late days
  - Do not email professor directly
- **Office hours**
  - “How do I get started on problem 2?”
  - “Am I on the right track?”
  - “I have this problem at work—can you point me in the right direction?”
  - We will not be able to give detailed code debugging help
- **Regrade requests**
  - Directly submit on Gradescope
- **Anonymous feedback (<https://feedback.cs.washington.edu/>)**
  - “Your real-world example X lacked nuance. I would like you to...”

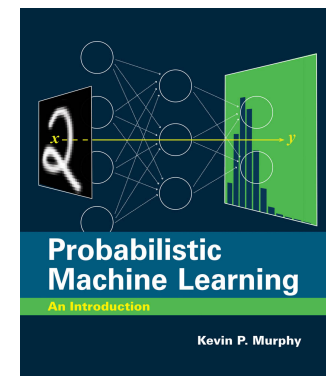
# Textbooks

- Free PDF Textbook we will assign most reading from:

## *Probabilistic Machine Learning: An Introduction*

Kevin Murphy

[PDF](#), also in print



- So many more resources on the website!

The textbook for the course is:

- [Murphy] [Probabilistic Machine Learning: An Introduction](#), Kevin Murphy, 2022 (Note, this is not the 2012 edition used in past years). Follow the link to obtain a free, PDF pre-print as well as options to purchase a hard copy.

For a gentler introduction to machine learning the following text is available for free online:

- [CIML] [A Course in Machine Learning](#) by Hal Daume III.

The following three texts are also excellent general machine learning texts and their PDFs are available for free online.

- [B] [Pattern Recognition and Machine Learning](#), Christopher Bishop.
- [HTF] [The Elements of Statistical Learning: Data Mining, Inference, and Prediction](#), Trevor Hastie, Robert Tibshirani, Jerome Friedman.
- [EH] [Computer Age Statistical Inference: Algorithms, Evidence and Data Science](#), Bradley Efron, Trevor Hastie.
- [ZLLS] [Dive into Deep Learning](#), Aston Zhang, Zach Lipton, Mu Li, Alex Smola.

You may also find these reference materials useful throughout the quarter.

#### Machine Learning (and related topics)

- [Understanding Machine Learning: From Theory to Algorithms](#), Shai Shalev-Shwartz, Shai Ben-David. An introduction to theoretical machine learning.
- [Fairness and Machine Learning](#), Solon Barocas, Moritz Hardt, Arvind Narayanan
- [Foundations of Data Science](#), by Avrim Blum, John Hopcroft and Ravi Kannan. This freely available pdf has nice chapters on machine learning (chapter 5), clustering (chapter 7) and SVD (chapter 3).
- [Crib sheet of math for ML](#) by Iain Murray

#### Linear Algebra and Matrix Analysis

- [These wonderful videos](#) by 3blue1brown provide a gentle and highly intuitive overview of linear algebra. (The same person created most of the videos on multivariable calculus on Khan Academy -- also excellent).
- [Linear Algebra Review and Reference](#) by Zico Kolter and Chuong Do (free). Light refresher for linear algebra and matrix calculus if you're a bit rusty.
- [Linear Algebra](#), David Cherney, Tom Denton, Rohit Thomas and Andrew Waldron (free). Introductory linear algebra text.
- [Matrix Analysis](#), Horn and Johnson. A great reference from elementary to advanced material.
- [The Matrix Cookbook](#), Pederson and Pederson. Contains many useful matrix manipulation identities, including derivatives

#### Probability and Statistics

- [20su offering of CSE312 materials and video lectures](#) by Alex Tsun. Slides, full notes, and short video lecture snippets on basic probability and statistics. This course was recently redesigned in part to better prepare students for CSE 446, making it an excellent resource for review.
- [Probability Review](#) by Arian Maleki and Tom Do. (From Andrew Ng's machine learning class.)
- Section notes from Anna Karlin's 18au offering of 312: [Counting, Combinatorics + intro probability, Conditional probability, Random variables & linearity of expectation, Variance and discrete r.v.s, Conditional expectation, Joint distributions, Continuous random variables, CLT, tail bounds and MLE.](#)
- [All of Statistics](#), Larry Wasserman. Chapters 1-5 are a great probability refresher and the book is a good reference for statistics.
- [A First Course in Probability](#), Sheldon Ross. Elementary concepts (previous editions are a couple bucks on Amazon)

#### Optimization

- [Numerical Optimization](#), Nocedal, Wright. Practical algorithms and advice for general optimization problems.
- [Convex Optimization: Algorithms and Complexity](#), Sébastien Bubeck. Elegant proofs for the most popular optimization procedures used in machine learning.

#### Python

- [www.learnpython.org](http://www.learnpython.org) "Whether you are an experienced programmer or not, this website is intended for everyone who wishes to learn the Python programming language."
- [NumPy for Matlab users](#)

#### Latex

- [Learn Latex in 30 minutes](#)
- [Overleaf](#). An online Latex editor.
- [Standalone Latex editor](#) on your local machine
- [Latex Math symbols](#)
- [Detexify](#) LaTeX handwritten symbol recognition

# Enjoy!

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- ML is:
  - Becoming ubiquitous in science, engineering and beyond
  - Transforming the world
- This class should give you a basic foundation for understanding and applying ML

# Probability review

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# Definitions

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- **Random Variable:** A variable that takes on different values determined randomly.
  - Example: The height of a person from the US.
- **Distribution:** The different values a random variable can take on along with the probability of that value.
- We talk about **sampling** from a distribution:
  - “Consider a sample of 100 different heights of people from the US drawn randomly from the distribution of all heights.”

# Independence

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Let  $X$  and  $Y$  be **random variables**

Ex.  $X$  is the outcome of the first roll of a 6-sided dice,  $Y$  is the outcome of the second roll of the dice

( $X$  and  $Y$  take values in  $\{1,2,3,4,5,6\}$  each with equal probability)

An **event** is statement about the world that holds or not:

Define events  $A = \{X \in \{3,4\}\},$

$B = \{X = 1\},$

$C = \{Y \in \{3,4\}\}$

Every event is assigned a **probability**:

$$P(A) = P(X \in \{3,4\}) =$$

# Independence

---

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Every event is assigned a **probability**:

$$P(A) = P(X \in \{3,4\}) = 1/3$$

For any events  $U, V$  we have  $P(U \cup V) = P(U) + P(V) - P(U \cap V)$

# Independence

Let  $X$  and  $Y$  be **random variables**

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Any events  $U, V$  are **independent** if  $P(U \cap V) = P(U)P(V)$

Are  $A, B$  independent?

$B, C$ ?

$A, C$ ?

# Independence

Let  $X$  and  $Y$  be **random variables**

Ex.  $X$  is the outcome of the first roll of a 6-sided dice,  $Y$  is the outcome of the second roll of the dice

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Any events  $U, V$  are **independent** if  $P(U \cap V) = P(U)P(V)$

Are  $A, B$  independent? (no)

$B, C$ ? (yes)

$A, C$ ? (yes)

# Independence

Let  $X$  and  $Y$  be **random variables**

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An **event** is statement about the world that holds or not:

Define events  $A = \{X \in \{3,4\}\}$ ,

$B = \{X = 1\}$ ,

$C = \{Y \in \{3,4\}\}$

Any events  $U, V$  are **independent** if  $P(U \cap V) = P(U)P(V)$

We define the **conditional probability** of event  $U$  given  $V$  as

$$P(U | V) = \frac{P(U \cap V)}{P(V)}$$

What is  $P(X \leq 4 | X \geq 3)$ ?

# Independence

Let  $X$  and  $Y$  be **random variables**

Ex.  $X$  is the outcome of the first roll of a 6-sided dice,  $Y$  is the outcome of the second roll of the dice

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We define the **conditional probability** of event  $U$  given  $V$  as

$$P(U | V) = \frac{P(U \cap V)}{P(V)}$$

$$\text{What is } P(X \leq 4 | X \geq 3) = \frac{P(3 \leq X \leq 4)}{P(X \geq 3)} = \frac{1/3}{2/3} = 1/2$$

# Independence

Let  $X$  and  $Y$  be **random variables**

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Any events  $U, V$  are **independent** if  $P(U \cap V) = P(U)P(V)$

We define the **conditional probability** of event  $U$  given  $V$  as

$$P(U | V) = \frac{P(U \cap V)}{P(V)}$$

Observe: if  $U, V$  are independent then  $P(U | V) = P(U)$ .

In words: if independent,  $V$  tells you nothing about  $U$  (and vice versa)

# Mean, variance

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Mean  $\mathbb{E}[X], \mu$

The expected value of  $X$ , each value is weighted by the probability of seeing it.

$$\mathbb{E}[X] = \sum_x P(X = x)x$$

Variance  $\text{Var}(X), \sigma^2$

The expected squared deviation of  $X$  from its mean.

$$\mathbb{E}[(X - \mathbb{E}[X])^2]$$

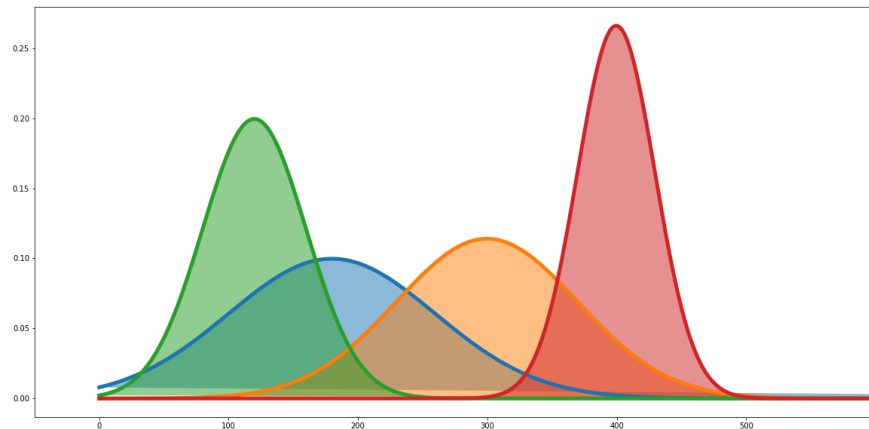
Median  $M$

The value of  $X$  that is separating the higher half of its range from the lower half.

$$P(X \leq M) = .5$$

# Mean, variance

The mean is a prediction of the value of the random variable. Answers the question "What do I expect the height of a random person to be?"



The variance captures the spread in your data. Also captures the error in the prediction using the mean. "How much do people's heights deviate?"

$$\mathbb{E}[(X - \mathbb{E}[X])^2]$$

# Maximum Likelihood Estimation

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# Your first consulting job

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- *Client*: I have special coin, if I flip it, what's the probability it will be heads?
- *You (a machine learner)*: I need to collect **data**.
  
- *You*: The probability is:

# Your first consulting job

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- *Client*: Uhhhh.... You sure about that? I just got a tails.
- *You (a machine learner)*: I need to collect **more data**.
  - \*flips coin 5 times, get HHTHT
  
- *You*: The probability is: 60% Heads, 40% Tails!

# Your first consulting job

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- *Client*: Uhhhh.... You sure about that? I just got a tails.
- *You (a machine learner)*: I need to collect **more data**.
  - \*flips coin 10000 times, it comes up Heads 70% of the time
- *You*: The probability is: 70% Heads, 30% Tails!
- *Client*: **Why should I believe you?**
- *You (a machine learner)*: Let's do some math!

# Coin – Bernoulli Distribution

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- **Data:** sequence  $D = (HHTHT\dots)$ , **k heads** out of **n flips**
- **Hypothesis:**  $P(\text{Heads}) = \theta$ ,  $P(\text{Tails}) = 1 - \theta$ 
  - Flips are i.i.d.:
    - Independent events
    - Identically distributed according to Bernoulli distribution
  
- $P(\mathcal{D}|\theta) =$

# Maximum Likelihood Estimation

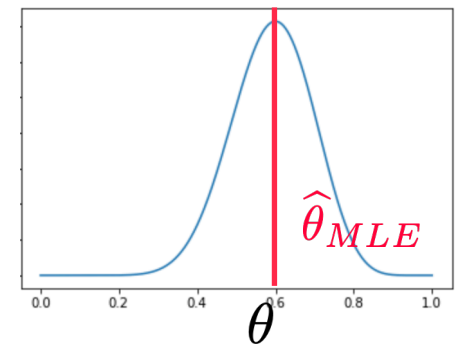
- **Data:** sequence  $D = (HHTHT\dots)$ , **k heads** out of **n flips**
- **Hypothesis:**  $P(\text{Heads}) = \theta$ ,  $P(\text{Tails}) = 1 - \theta$
- **Likelihood:**

$$P(\mathcal{D}|\theta) = \theta^k (1 - \theta)^{n-k}$$

- **Maximum likelihood estimation (MLE):** Choose  $\theta$  that maximizes the probability of observed data:

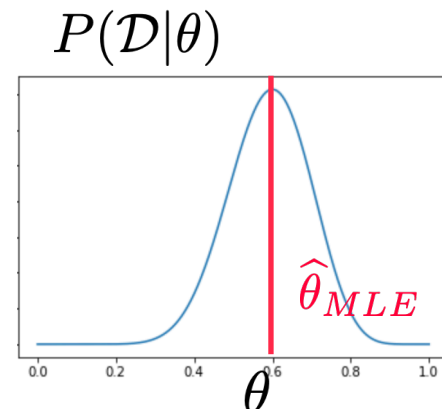
$$\hat{\theta}_{MLE} = \arg \max_{\theta} P(\mathcal{D}|\theta)$$

$$= \arg \max_{\theta} \log P(\mathcal{D}|\theta)$$



# MLE: Your first learning algorithm

$$\begin{aligned}\hat{\theta}_{MLE} &= \arg \max_{\theta} \log P(\mathcal{D}|\theta) \\ &= \arg \max_{\theta} \log \theta^k (1 - \theta)^{n-k}\end{aligned}$$



- How do we find  $\theta$  that maximizes likelihood?
- Use the fact that derivative is zero at maxima (also at minima)
- Set derivative to zero, and find  $\theta$  satisfying:

$$\frac{d}{d\theta} \log P(\mathcal{D}|\theta) = 0$$

# MLE: Your first learning algorithm

---

- First manipulate the log likelihood to make it easy to work with:

$$\begin{aligned}\hat{\theta}_{MLE} &= \arg \max_{\theta} \log P(\mathcal{D}|\theta) \\ &= \arg \max_{\theta} \log \theta^k (1 - \theta)^{n-k}\end{aligned}$$

- Then set derivative to 0, and find  $\theta$  satisfying:  $\frac{d}{d\theta} \log P(\mathcal{D}|\theta) = 0$

# How good is MLE? Well, it's unbiased

- We treat MLE  $\hat{\theta}_{\text{MLE}}$  as a random variable, where there is a ground truth parameter  $\theta^*$  that generates the data  $\mathcal{D} = (HHTTH\dots)$  of a fixed size  $n$

- What can we say about this random variable  $\hat{\theta}_{\text{MLE}}$ ?

- First good property of MLE for Binomial: **unbiased**

- Definition: **bias** of our MLE is

$$\text{Bias}(\hat{\theta}_{\text{MLE}}) := \mathbb{E}_{\mathcal{D} \sim P_{\theta^*}}[\hat{\theta}_{\text{MLE}}] - \theta^* =$$

- **Expectation** describes how the estimator behaves *on average*

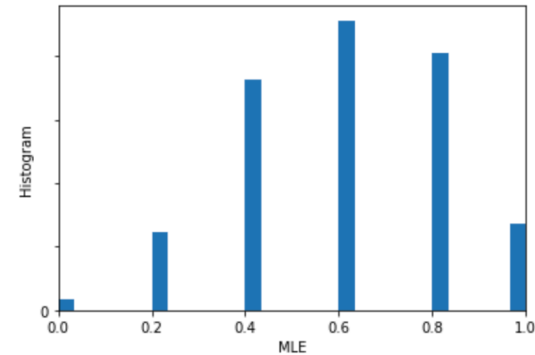
# How many flips do I need?

- Consider running many experiments with  $\theta^* = \frac{3}{5}$ , and observe many instances of the random variable

$$\hat{\theta}_{MLE} = \frac{k}{n}$$

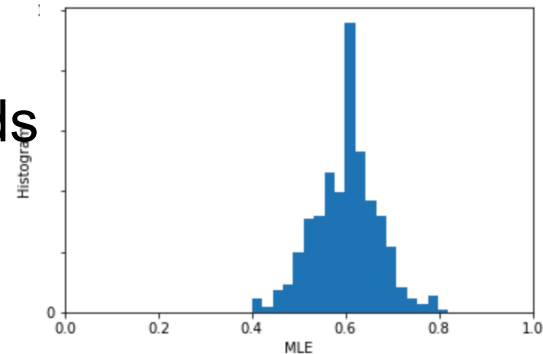
- Client:* I flipped the coin 5 times and got 2 heads.

$$\hat{\theta}_{MLE} =$$



- Client:* I flipped the coin 50 times and got 30 heads

$$\hat{\theta}_{MLE} =$$



- Client:* they are both unbiased, which one is right? Why?

- Variance goes down with larger n  $\sqrt{\text{Var}(\hat{\theta}_{MLE})} = \sqrt{\frac{\theta^*(1-\theta^*)}{n}}$

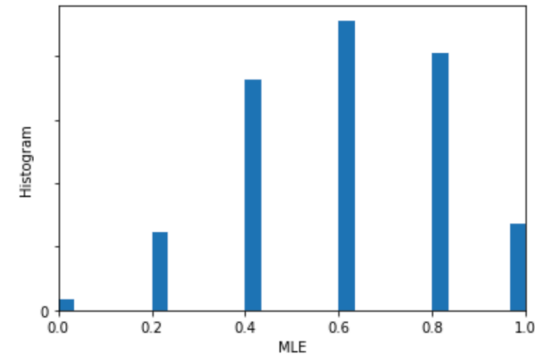
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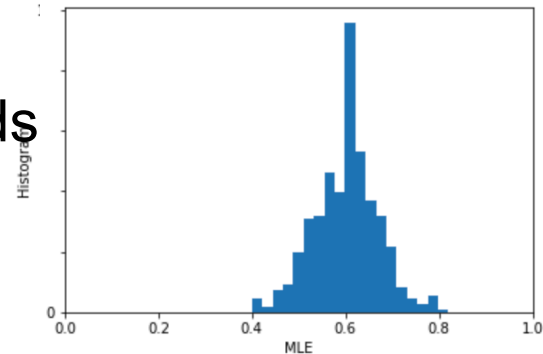
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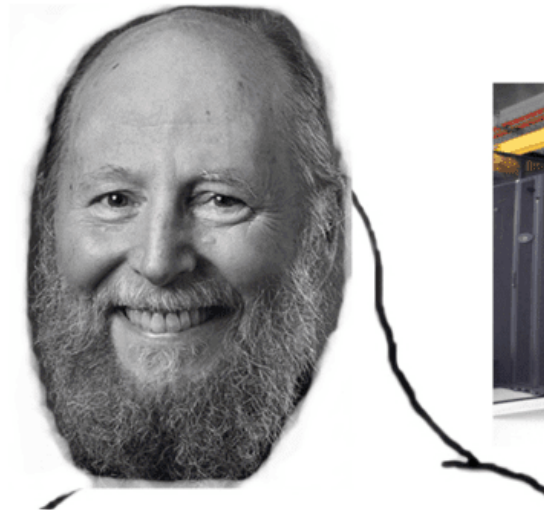
- Client:* they are both unbiased, which one is right? Why?

- Variance goes down with larger n  $\sqrt{\text{Var}(\hat{\theta}_{MLE})} = \sqrt{\frac{\theta^*(1-\theta^*)}{n}}$

# Fundamental machine learning truth

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- More data -> better performance
  - “The Bitter Lesson”
  - [https://www.cs.utexas.edu/~eunsol/courses/data/bitter\\_lesson.pdf](https://www.cs.utexas.edu/~eunsol/courses/data/bitter_lesson.pdf)



haha gpus go bitterrr

# Maximum Likelihood Estimation

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**Observe**  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

**Likelihood function**  $L_n(\theta) = \prod_{i=1}^n f(X_i; \theta)$

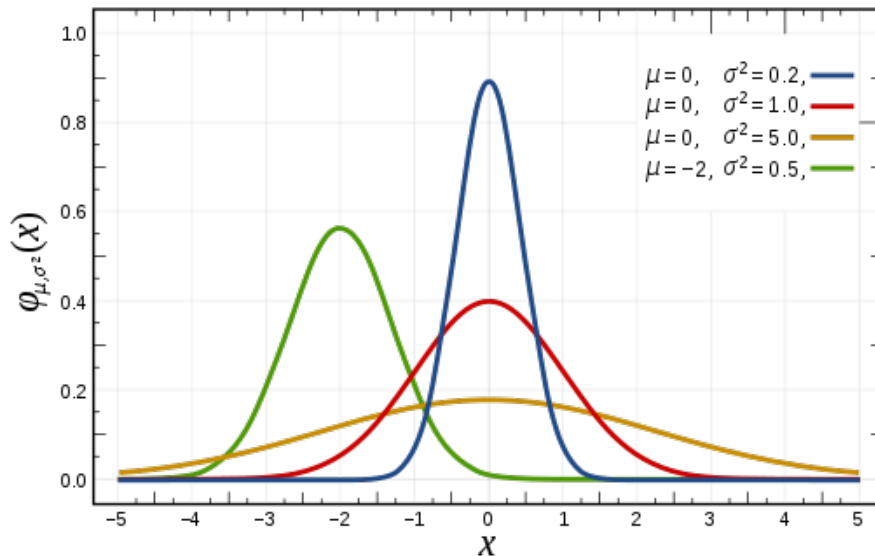
**Log-Likelihood function**  $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^n \log(f(X_i; \theta))$

**Maximum Likelihood Estimator (MLE)**  $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

# What about continuous variables?

- *Client*: What if I am measuring a **continuous variable**?
- *You*: Let me tell you about **Gaussians**...

$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



# Some properties of Gaussians

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- Affine transformation (multiplying by scalar and adding a constant)
  - $X \sim N(\mu, \sigma^2)$
  - $Y = aX + b \rightarrow Y \sim N(a\mu + b, a^2\sigma^2)$
- Sum of Gaussians
  - $X \sim N(\mu_X, \sigma_X^2)$
  - $Y \sim N(\mu_Y, \sigma_Y^2)$
  - $Z = X + Y \rightarrow Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$

# MLE for Gaussian

- Prob. of i.i.d. samples  $D=\{x_1, \dots, x_n\}$  (e.g., temperature):

$$\begin{aligned} P(\mathcal{D}|\mu, \sigma) &= P(x_1, \dots, x_n|\mu, \sigma) \\ &= \left( \frac{1}{\sigma\sqrt{2\pi}} \right)^n \prod_{i=1}^n e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} \end{aligned}$$

- Log-likelihood of data:

$$\log P(\mathcal{D}|\mu, \sigma) = -n \log(\sigma\sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2}$$

- What is  $\hat{\theta}_{MLE}$  for  $\theta = (\mu, \sigma^2)$ ? Draw a picture!

# MLE for Gaussian

Generate  $\mathcal{D} = \{x_1, \dots, x_n\}$ , where

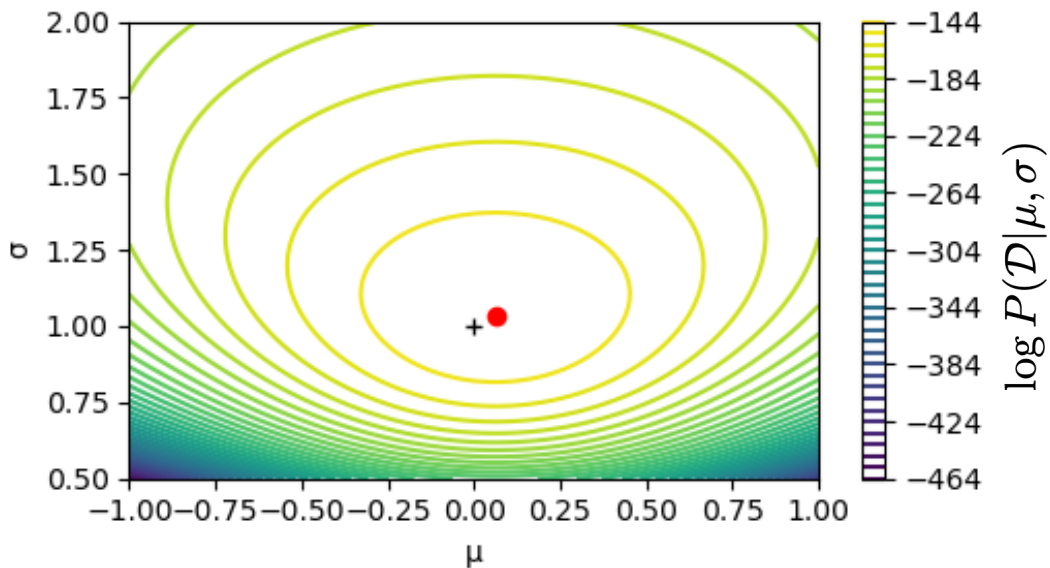
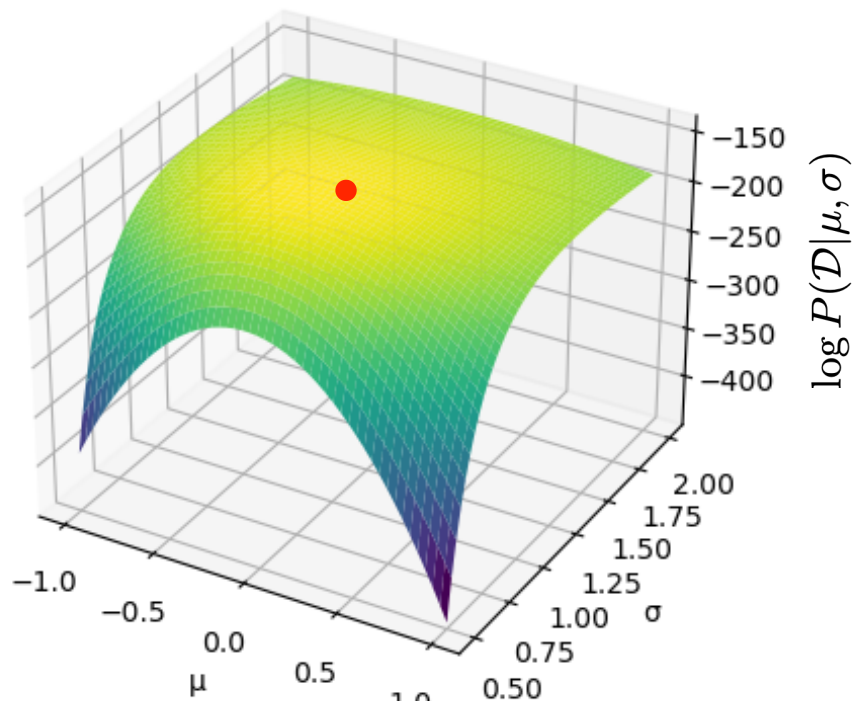
$$n = 100$$

$$x_i \sim \mathcal{N}(\mu, \sigma^2)$$

$$\mu = 0$$

$$\sigma^2 = 1$$

$$\log P(\mathcal{D}|\mu, \sigma) = -n \log(\sigma\sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2}$$



$$+ (\mu_{True}, \sigma_{True})$$

$$\bullet (\hat{\mu}_{MLE}, \hat{\sigma}_{MLE})$$

# Your second learning algorithm: MLE for mean of a Gaussian

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- What's MLE for mean? Set **partial derivative** to zero.

$$\frac{\partial}{\partial \mu} \log P(\mathcal{D} \mid \mu, \sigma) = \frac{\partial}{\partial \mu} \left[ -n \log(\sigma \sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

# MLE for variance

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- Again, set partial derivative to zero:

$$\frac{\partial}{\partial \sigma} \log P(\mathcal{D} \mid \mu, \sigma) = \frac{\partial}{\partial \sigma} \left[ -n \log(\sigma \sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

# Learning Gaussian parameters

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- MLE:

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{\mu}_{MLE})^2$$

- MLE for the variance of a Gaussian is **biased**

$$\mathbb{E}[\hat{\sigma}^2_{MLE}] \neq \sigma^2$$

- Unbiased variance estimator:

$$\hat{\sigma}^2_{unbiased} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu}_{MLE})^2$$

# Maximum Likelihood Estimation

Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

Likelihood function  $L_n(\theta) = \prod_{i=1}^n f(X_i; \theta)$

Log-Likelihood function  $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^n \log(f(X_i; \theta))$

Maximum Likelihood Estimator (MLE)  $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

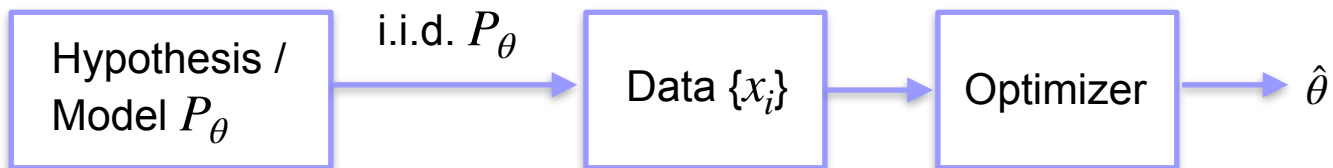
Under benign assumptions, as the number of observations  $n \rightarrow \infty$  we have  $\hat{\theta}_{MLE} \rightarrow \theta_*$

The MLE is a “recipe” that begins with a *model* for data  $f(x; \theta)$

# Recap

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- Learning is...
  - Collect some data
    - E.g., coin flips
  - Choose a hypothesis class or model
    - E.g., Bernoulli
  - Choose a loss function
    - E.g., data likelihood
  - Choose an optimization procedure
    - E.g., set derivative to zero to obtain MLE



# Applications preview

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# Maximum Likelihood Estimation

Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

**Likelihood function**  $L_n(\theta) = \prod_{i=1}^n f(X_i; \theta)$

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**Maximum Likelihood Estimator (MLE)**  $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

Under benign assumptions, as the number of observations  $n \rightarrow \infty$  we have  $\hat{\theta}_{MLE} \rightarrow \theta_*$

Why is it useful to recover the “true” parameters  $\theta_*$  of a probabilistic model?

- **Estimation** of the parameters  $\theta_*$  is the goal
- Help **interpret** or summarize large datasets
- Make **predictions** about future data
- **Generate** new data  $X \sim f(\cdot; \theta_{MLE})$

# Estimation

Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

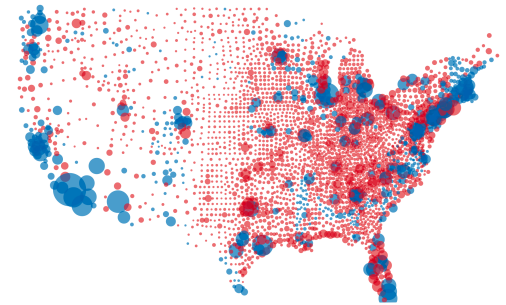
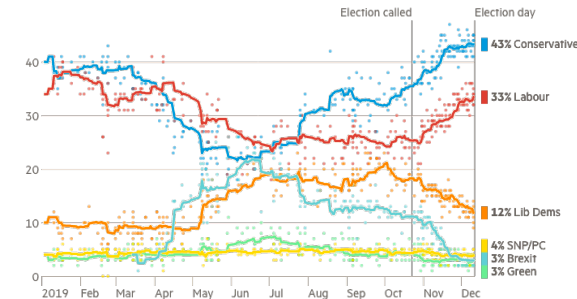
## Opinion polls

How does the greater population feel about an issue?  
Correct for over-sampling?

- $\theta_*$  is “true” average opinion
- $X_1, X_2, \dots$  are sample calls

UK poll tracker

Lines represent weighted averages, points represent polls (%)



## A/B testing

How do we figure out which ad results in more click-through?

- $\theta_*$  are the “true” average rates
- $X_1, X_2, \dots$  are binary “clicks”

Save on prescription drugs - over \$3,637\* a year!

Last year, Humana's Medicare Advantage plan members saved, on average, \$3,637\* on prescription drugs! Choose your Humana Medicare Advantage plan and you could enjoy savings on prescription drugs, plus:

- Hospital, doctor AND drug coverage combined into one easy-to-use plan
- Extra benefits not offered by Original Medicare
- Affordable or no monthly plan premiums

Shop 2014 Medicare Plans

Control

Explore Humana's Medicare plans

Let us help you determine the Humana plan that's best for your needs.

Get started now

Treatment

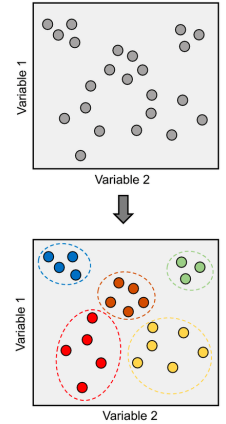
# Interpret

Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

## Customer segmentation / clustering

Can we identify distinct groups of customers by their behavior?

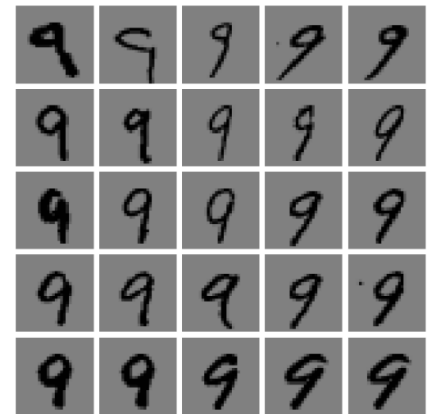
- $\theta_*$  describes “center” of distinct groups
- $X_1, X_2, \dots$  are individual customers



## Data exploration

What are the degrees of freedom of the dataset?

- $\theta_*$  describes the principle directions of variation
- $X_1, X_2, \dots$  are the individual images



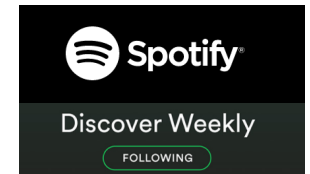
# Predict

Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

## Content recommendation

Can we predict how much someone will like a movie based on past ratings?

- $\theta_*$  describes user’s preferences
- $X_1, X_2, \dots$  are (movie, rating) pairs



## Object recognition / classification

Identify a flower given just its picture?

- $\theta_*$  describes the characteristics of each kind of flower
- $X_1, X_2, \dots$  are the (image, label) pairs



(a)



(b)



(c)

Figure 1.1: Three types of Iris flowers: Setosa, Versicolor and Virginica. Used with kind permission of Dennis Krumb and SIGNA.

index	sl	sw	pl	pw	label
0	5.1	3.5	1.4	0.2	Setosa
1	4.9	3.0	1.4	0.2	Setosa
...					
50	7.0	3.2	4.7	1.4	Versicolor
...					
149	5.9	3.0	5.1	1.8	Virginica

# Generate

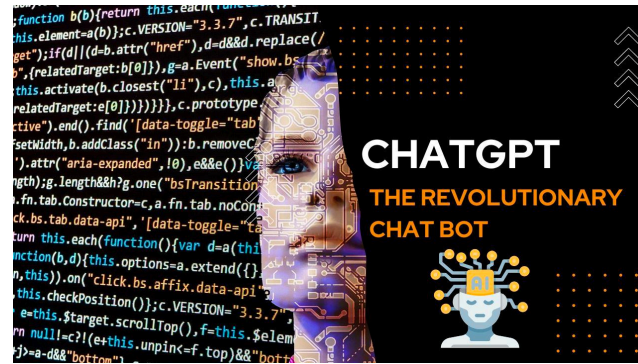
Observe  $X_1, X_2, \dots, X_n$  drawn IID from  $f(x; \theta)$  for some “true”  $\theta = \theta_*$

## Text generation

Can AI generate text that could have been written like a human?

- $\theta_*$  describes language structure
- $X_1, X_2, \dots$  are text snippets found online

“Kaia the dog wasn't a natural pick to go to mars. No one could have predicted she would...”



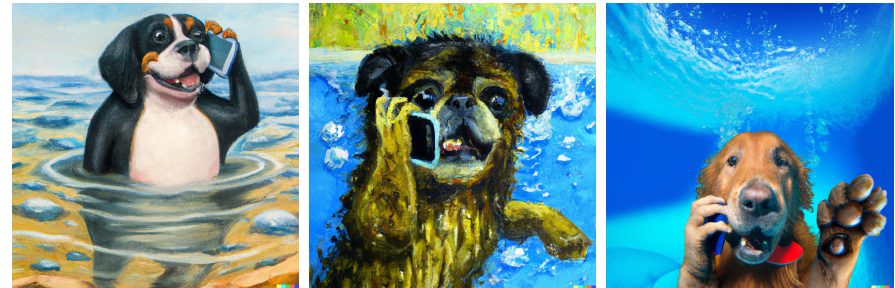
<https://chat.openai.com/chat>

## Image to text generation

Can AI generate an image from a prompt?

- $\theta_*$  describes the coupled structure of images and text
- $X_1, X_2, \dots$  are the (image, caption) pairs found online

“dog talking on cell phone under water, oil painting”



<https://labs.openai.com/>