

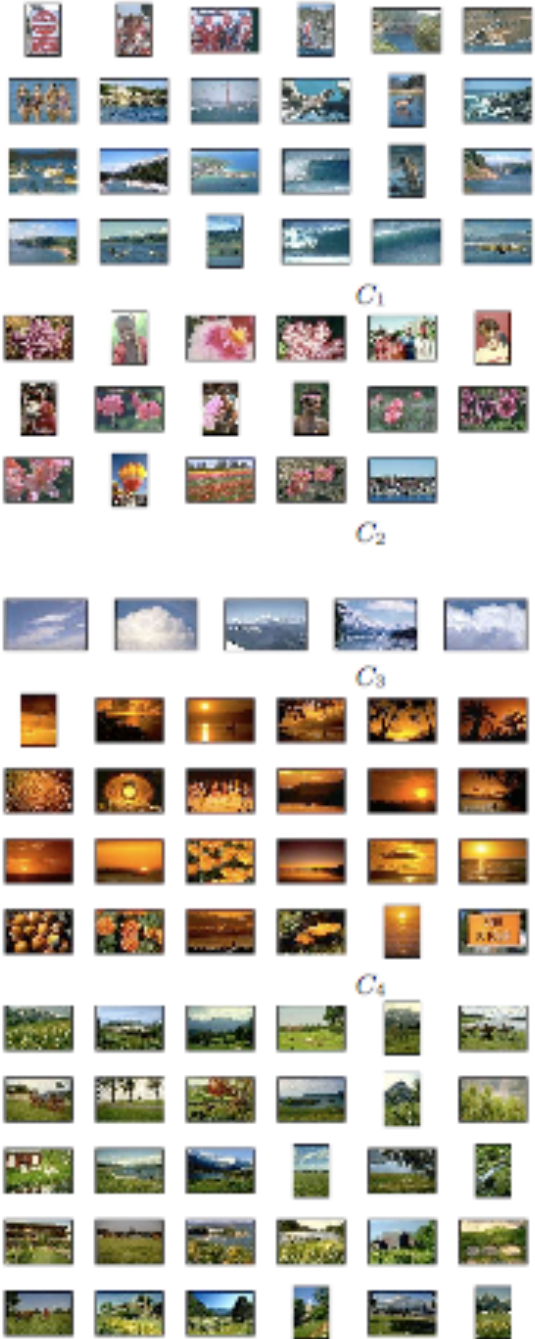
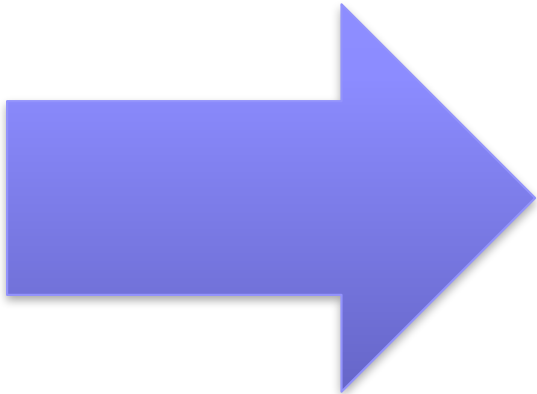
# Clustering

# K-means

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# Clustering images



[Goldberger et al.]

# Clustering web search results

web news images wikipedia blogs jobs more »

Clusty

race Search advanced preferences

clusters sources sites remix

All Results (238)




- Car (28)
- Race cars (7)
- Photos, Races Scheduled (5)
- Game (4)
- Track (3)
- Nascar (2)
- Equipment And Safety (2)
- Other Topics (7)
- Photos (22)
- Game (14)
- Definition (13)
- Team (18)
- Human (8)
  - Classification Of Human (2)
  - Statement, Evolved (2)
  - Other Topics (4)
- Weekend (8)
- Ethnicity And Race (7)
- Race for the Cure (8)
- Race Information (8)

[more](#) | [all clusters](#)




find in clusters:  Find

Cluster **Human** contains **8** documents.




Search Results

- [Race \(classification of human beings\) - Wikipedia, the free ...](#)   




The term **race** or racial group usually refers to the concept of dividing **humans** into populations or groups on the basis of various sets of characteristics. The most widely used **human** racial categories are based on visible traits (especially skin color, cranial or facial features and hair texture), and self-identification. Conceptions of **race**, as well as specific ways of grouping **races**, vary by culture and over time, and are often controversial for scientific as well as social and political reasons. History · Modern debates · Political and ...

[en.wikipedia.org/wiki/Race\\_\(classification\\_of\\_human\\_beings\)](http://en.wikipedia.org/wiki/Race_(classification_of_human_beings)) - [cache] - Live, Ask
- [Race - Wikipedia, the free encyclopedia](#)   




General. **Racing** competitions The **Race** (yachting **race**), or La course du millénaire, a no-rules round-the-world sailing event; **Race** (biology), classification of flora and fauna; **Race** (classification of human beings) **Race** and ethnicity in the United States Census, official definitions of "**race**" used by the US Census Bureau; **Race** and genetics, notion of racial classifications based on genetics. Historical definitions of **race**; **Race** (bearing), the inner and outer rings of a rolling-element bearing. **RACE** in molecular biology "Rapid ... General · Surnames · Television · Music · Literature · Video games

[en.wikipedia.org/wiki/Race](http://en.wikipedia.org/wiki/Race) - [cache] - Live, Ask
- [Publications | Human Rights Watch](#)   




The use of torture, unlawful rendition, secret prisons, unfair trials, ... Risks to Migrants, Refugees, and Asylum Seekers in Egypt and Israel ... In the run-up to the Beijing Olympics in August 2008, ...

[www.hrw.org/backgroundunder/usa/race](http://www.hrw.org/backgroundunder/usa/race) - [cache] - Ask
- [Amazon.com: Race: The Reality Of Human Differences: Vincent Sarich ...](#)   




Amazon.com: **Race: The Reality Of Human Differences: Vincent Sarich, Frank Miele: Books ...** From Publishers Weekly Sarich, a Berkeley emeritus anthropologist, and Miele, an editor ...

[www.amazon.com/Race-Reality-Differences-Vincent-Sarich/dp/0813340861](http://www.amazon.com/Race-Reality-Differences-Vincent-Sarich/dp/0813340861) - [cache] - Live
- [AAPA Statement on Biological Aspects of Race](#)   

AAPA Statement on Biological Aspects of **Race** ... Published in the American Journal of Physical Anthropology, vol. 101, pp 569-570, 1996 ... PREAMBLE As scientists who study **human** evolution and variation, ...

[www.physanth.org/positions/race.html](http://www.physanth.org/positions/race.html) - [cache] - Ask
- [race: Definition from Answers.com](#)   

**race** n. A local geographic or global **human** population distinguished as a more or less distinct group by genetically transmitted physical

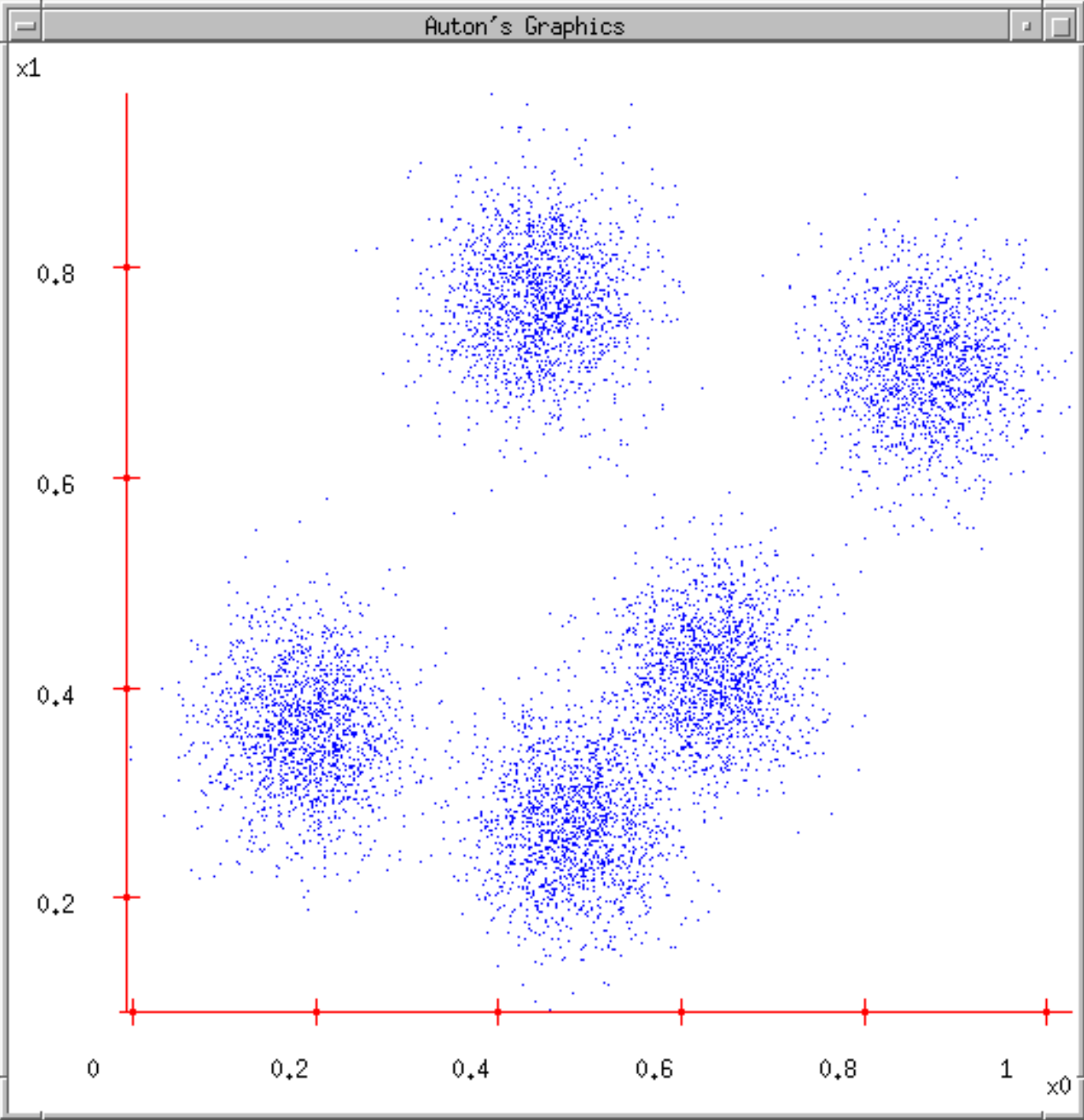
[www.answers.com/topic/race-1](http://www.answers.com/topic/race-1) - [cache] - Live
- [Dopefish.com](#)   

Site for newbies as well as experienced Dopefish followers, chronicling the birth of the Dopefish, its numerous appearances in several computer games, and its eventual take-over of the **human race**. Maintained by Mr. Dopefish himself, Joe Siegler of Apogee Software.

[www.dopefish.com](http://www.dopefish.com) - [cache] - Open Directory

# Some Data

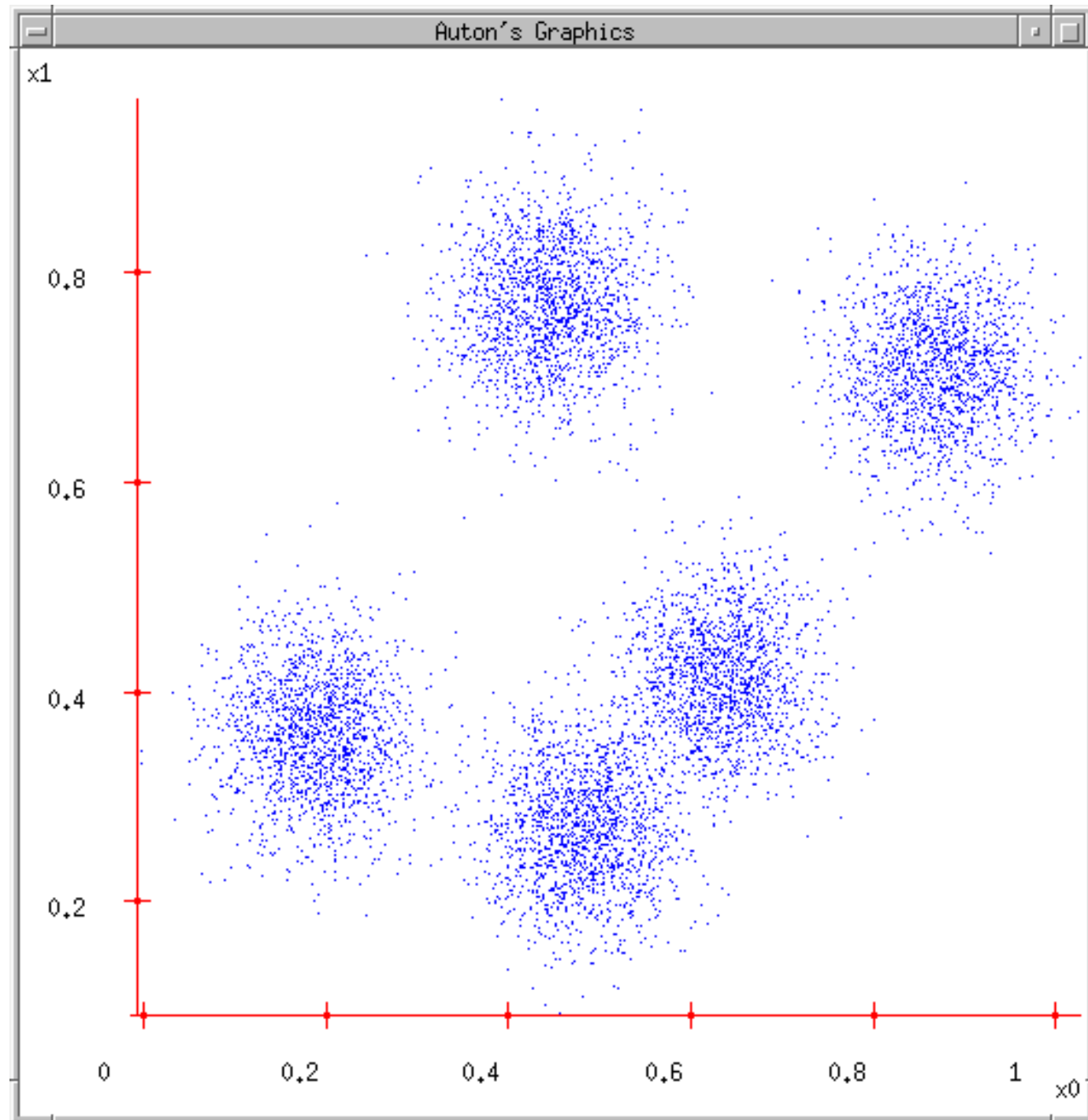
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# Clustering

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1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Pick clusters to minimize some objective fn.



# Clustering

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1. Fix a # of clusters (*e.g.*  $k=5$ )
2. Choose/Assign each point  $x_j$  to  $C(j) \in \{1, \dots, k\}$ 
  1. Sometimes, pick centers  $\mu_1, \dots, \mu_k$

To minimize

$$F(\mu, C)$$

**K-means refers to optimizing this objective:**

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$$F(\mu, C) = \sum_{j=1}^m \|\mu_{C(j)} - x_j\|^2$$

# K-means refers to optimizing this objective:

---

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How to minimize this quantity?



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How to minimize this quantity?

NP-Hard to minimize exactly. :(

# K-means refers to optimizing this objective:

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$$F(\mu, C) = \sum_{j=1}^m \|\mu_{C(j)} - x_j\|^2$$

How to minimize this quantity?

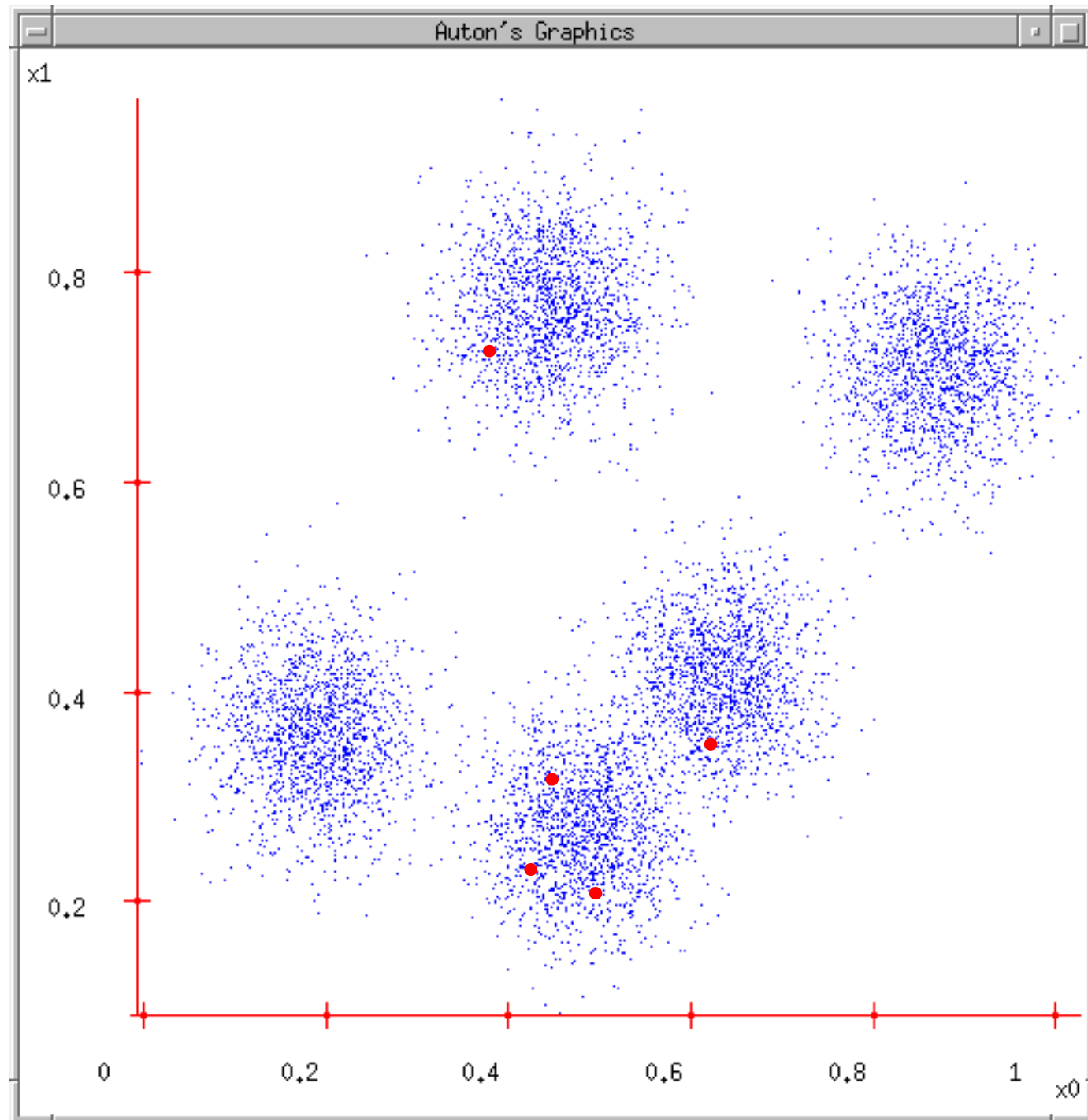
NP-Hard to minimize exactly. :(

But, several natural algorithms work well in practice!

# Lloyd's algorithm

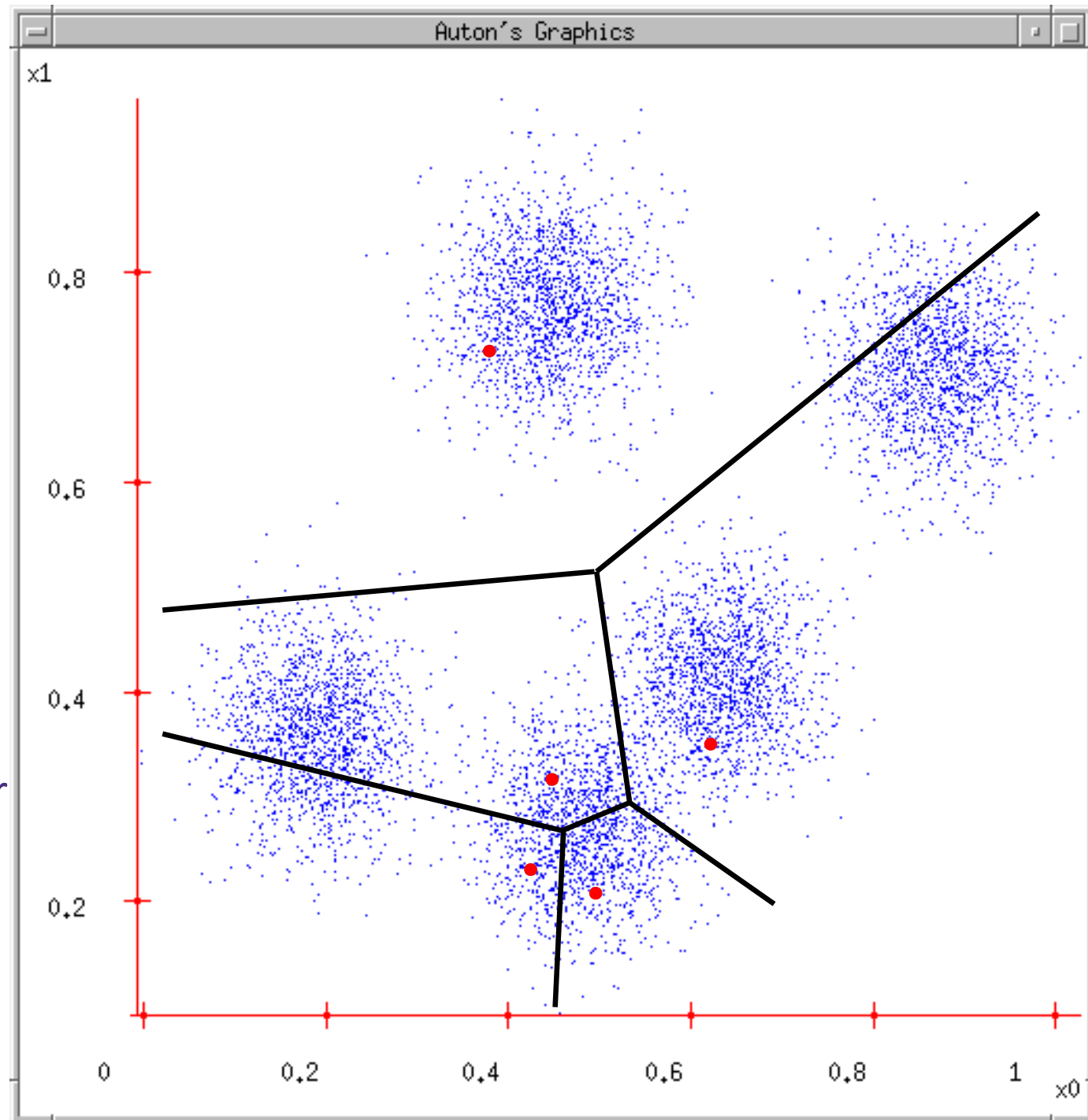
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1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Randomly guess  $k$  cluster Center locations



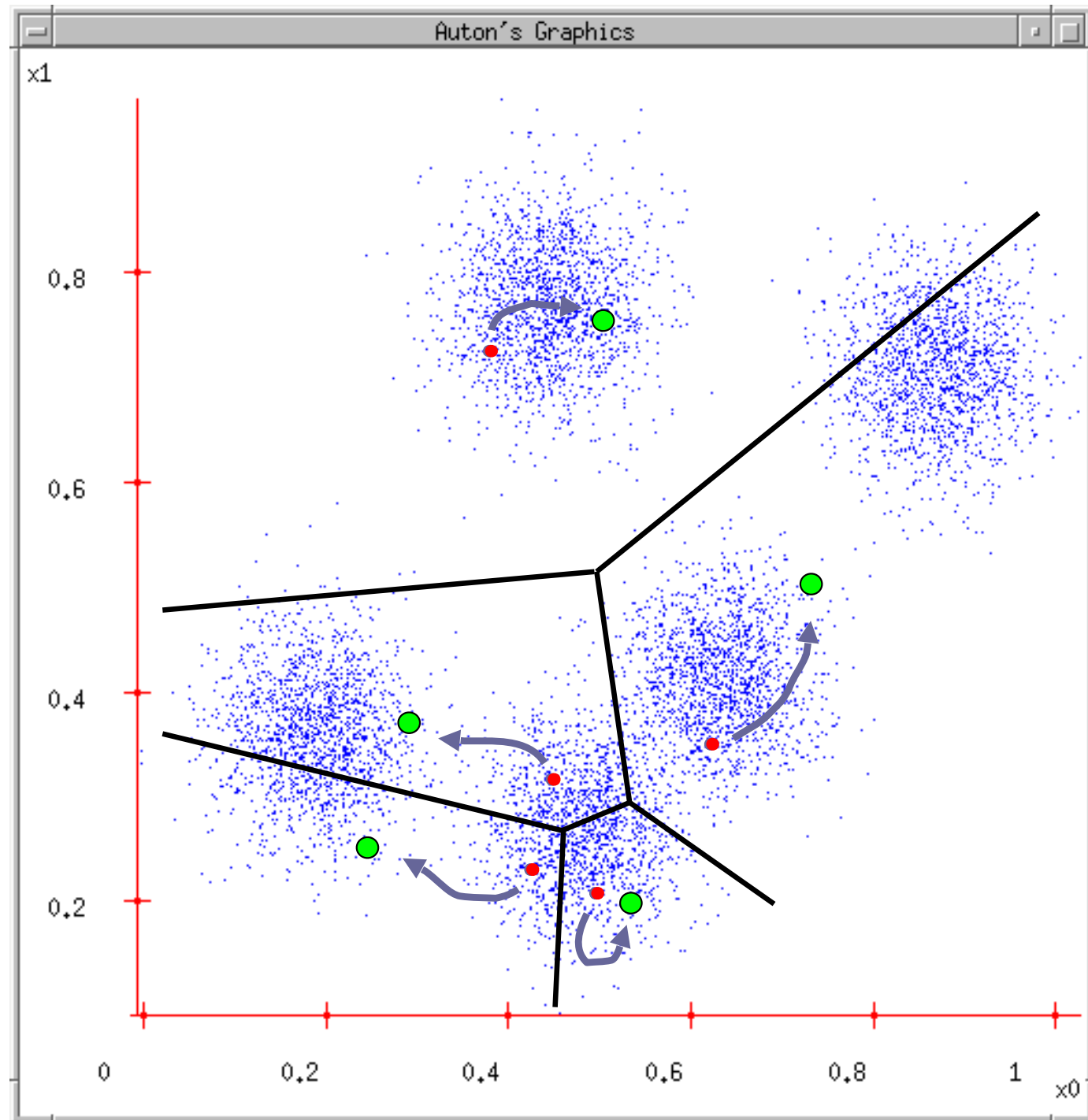
# Lloyd's algorithm

1. Ask user how many clusters they'd like.  
(e.g.  $k=5$ )
2. Randomly guess  $k$  cluster Center locations
3. Each datapoint finds out which Center it's closest to. (Each center "owns" a set of datapoints)



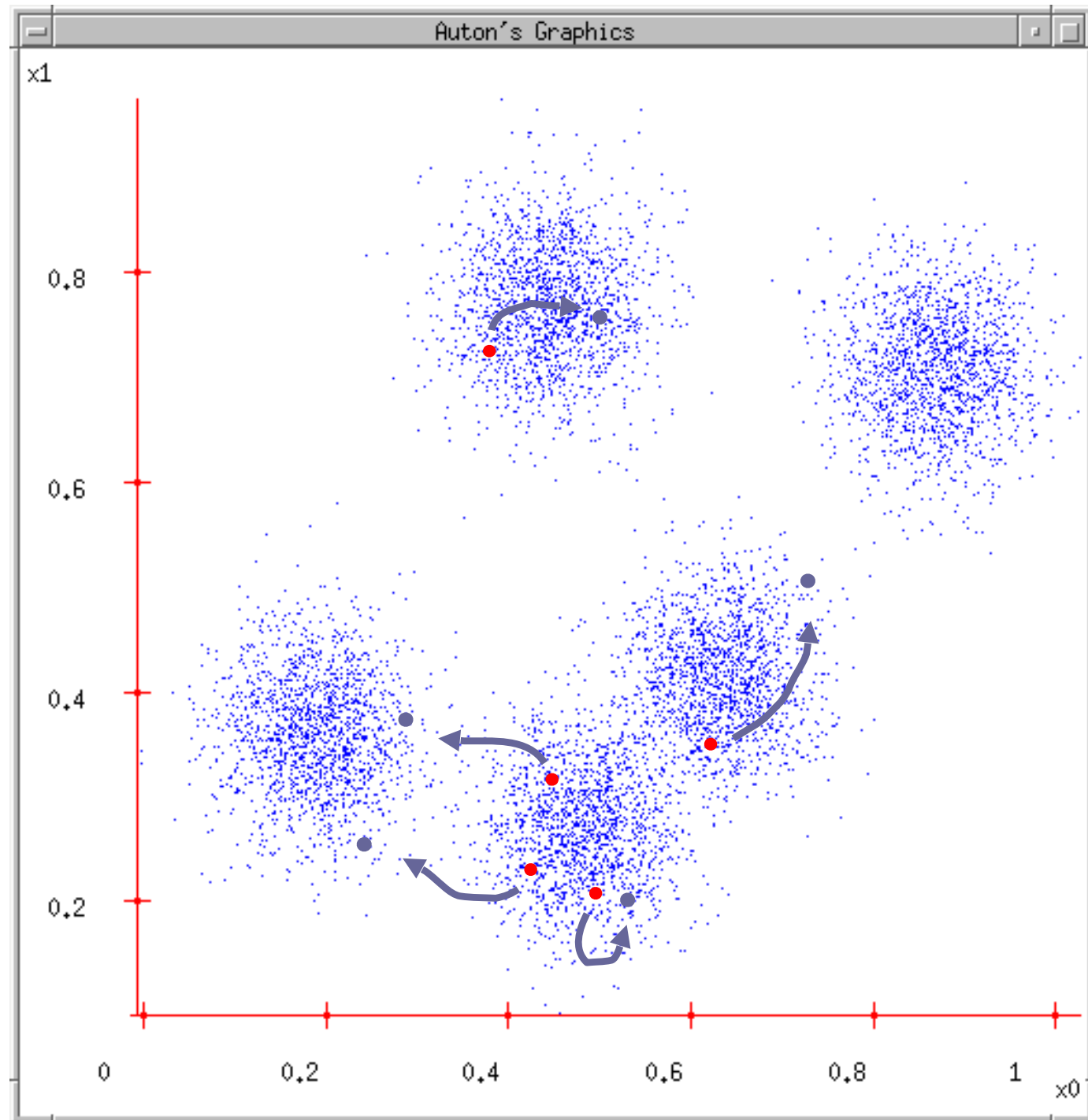
# Lloyd's algorithm

1. Ask user how many clusters they'd like.  
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4. Each Center finds the centroid of the points it owns



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3. Each datapoint finds out which Center it's closest to.
4. Each Center finds the centroid of the points it owns...
5. ...and jumps there
6. ...Repeat until terminated!



# Lloyd's algorithm

---

$$F(\mu, C) = \sum_{j=1}^m \|\mu_{C(j)} - x_j\|^2$$

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5. ...and jumps there
6. ...Repeat until terminated!

$$C^{(t)}(j) \leftarrow \arg \min_i \|\mu_i - x_j\|^2$$

$$\mu_i^{(t+1)} \leftarrow \arg \min_{\mu} \sum_{j:C(j)=i} \|\mu - x_j\|^2$$



# Does Lloyd's algorithm converge???

## Part 1

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$$\min_{\mu} \min_C F(\mu, C) = \min_{\mu} \min_C \sum_{i=1}^k \sum_{j:C(j)=i} \| \mu_i - x_j \|^2$$

First, fix  $\mu$

minimize w.r.t C

# Does Lloyd's algorithm converge??? Part 2

---

$$\min_{\mu} \min_C F(\mu, C) = \min_{\mu} \min_C \sum_{i=1}^k \sum_{j:C(j)=i} \| \mu_i - x_j \|^2$$

First, fix  $\mu$

minimize w.r.t  $C$

Then, fix  $C$

minimize w.r.t  $\mu$

$F(\mu, C)$  decreases each step  $\Rightarrow$  the algorithm doesn't cycle

Only  $\binom{n}{k} \approx n^k$  configurations  $\Rightarrow$  converges in finite # iterations

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# **A cool application of k-means clustering: compression**

# Vector Quantization, Fisher Vectors

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## Vector Quantization (for compression)

1. Represent image as grid of patches
2. Run k-means on the patches to build code book
3. Represent each patch as a code word.



**FIGURE 14.9.** *Sir Ronald A. Fisher (1890 – 1962) was one of the founders of modern day statistics, to whom we owe maximum-likelihood, sufficiency, and many other fundamental concepts. The image on the left is a  $1024 \times 1024$  grayscale image at 8 bits per pixel. The center image is the result of  $2 \times 2$  block VQ, using 200 code vectors, with a compression rate of 1.9 bits/pixel. The right image uses only four code vectors, with a compression rate of 0.50 bits/pixel*

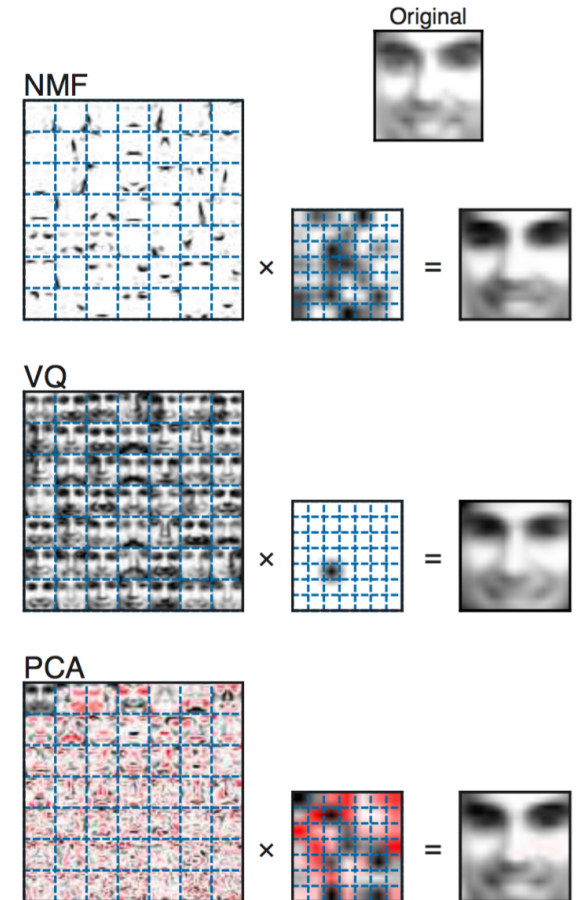
# Vector Quantization, Fisher Vectors

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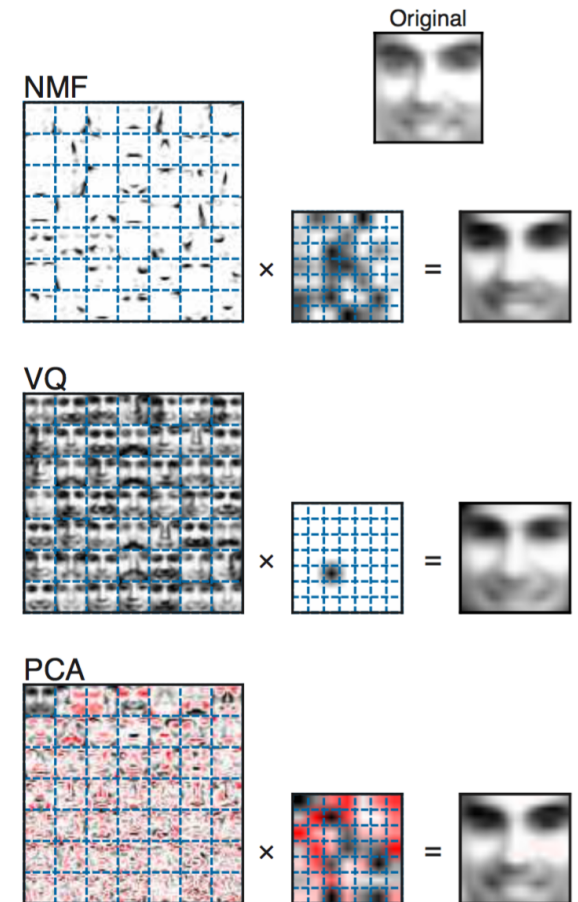
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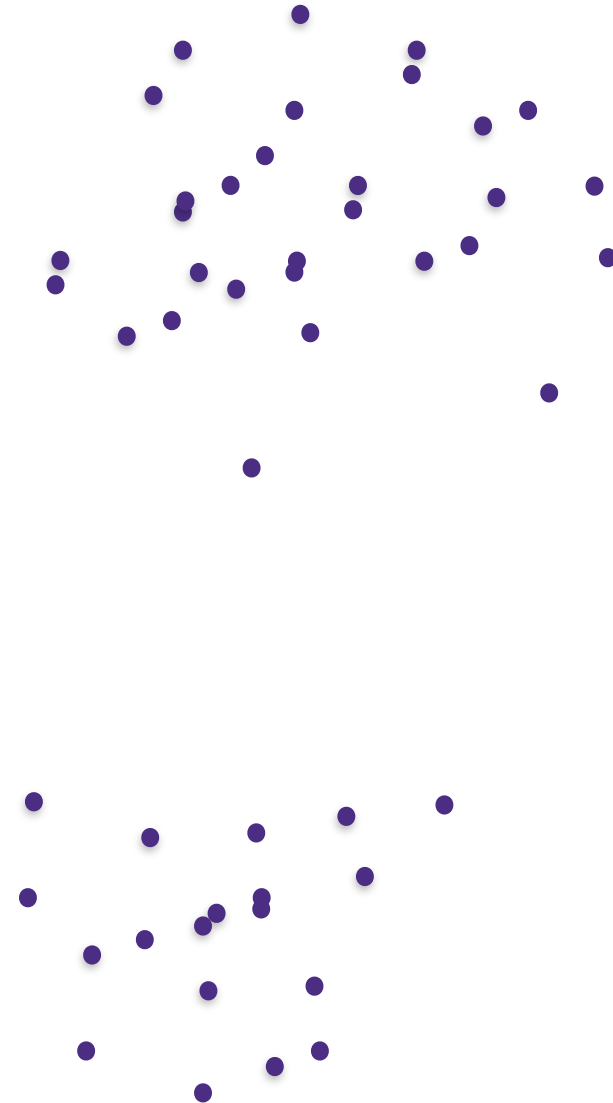
# When to use K-means, or something else?

What sort of groupings are desired?

Nonoverlapping, similar diameter clusters: k-means may work well

Otherwise, might want another objective function (spectral, k-median, k-mode, ...)

# One bad case for k-means





# K-means summary

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A clustering objective

minimize average L2 distance to centers of clusters

Lloyd's algorithm: a greedy heuristic for minimizing it

Will converge in finite time, may not find global minimum

Good for finding similar width, nonoverlapping clusters

Sensitive to initial center selection, and random may not be the best a priori

See k-means++, *The Advantages of Careful Seeding*, Arthur and Vassilvitskii

# Principal Component Analysis

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Figure credit:

Karlin  
Roughgarden + Valiant  
Benedetto  
Novembre et al  
Alex Williams  
Sandipan Dey  
Victor Lavenko



# PCA

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Data dependent dimensionality reduction

Useful for

Visualization

Interpretation

Compression

Understanding “intrinsic dimension”

	kale	taco bell	sashimi	pop tarts
Alice	10	1	2	7
Bob	7	2	1	10
Carolyn	2	9	7	3
Dave	3	6	10	2

Figure credit:

Karlin

Roughgarden + Valiant

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Victor Lavenko

# PCA

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Claim:

Each row can be expressed approximately as

$$x_i \approx \bar{x} + a_{i1}\vec{v}_1 + a_{i2}\vec{v}_2$$

$$\vec{v}_1 = [3 \quad -3 \quad -3 \quad 3]$$

$$\vec{v}_2 = [1 \quad -1 \quad 1 \quad -1]$$

	kale	taco bell	sashimi	pop tarts
Alice	10	1	2	7
Bob	7	2	1	10
Carolyn	2	9	7	3
Dave	3	6	10	2

$$\bar{x} = [5.5 \quad 4.5 \quad 5 \quad 5.5]$$

# PCA in one dimension

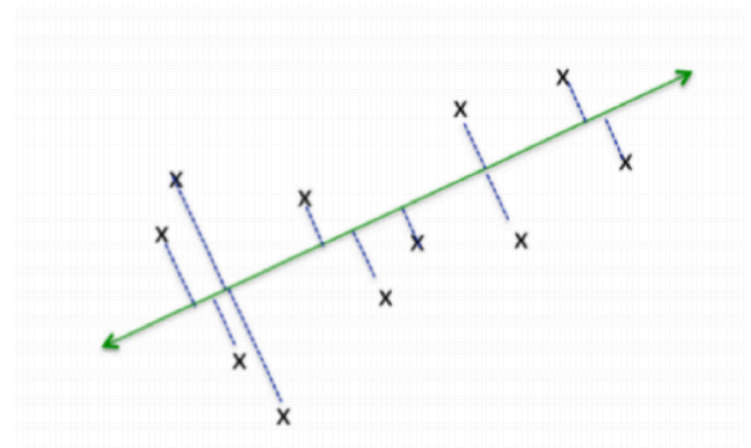
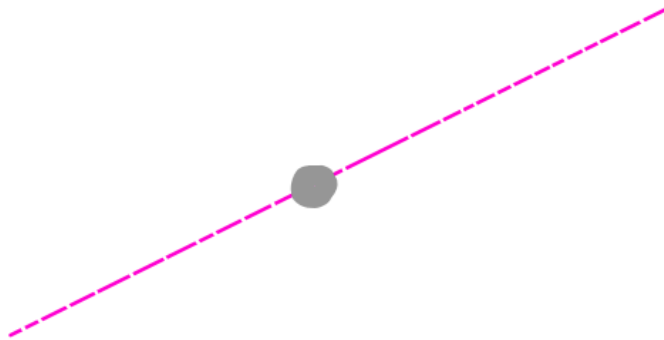
Goal: find a  $k < d$ -dimensional representation of  $X$

For  $k = 1$ :

Choose  $\vec{v} \in \mathbb{R}^d, \|\vec{v}\| = 1$

to minimize

$$\frac{1}{n} \sum_{i=1}^n \text{dist}(x_i, \text{line defined by } \vec{v})$$



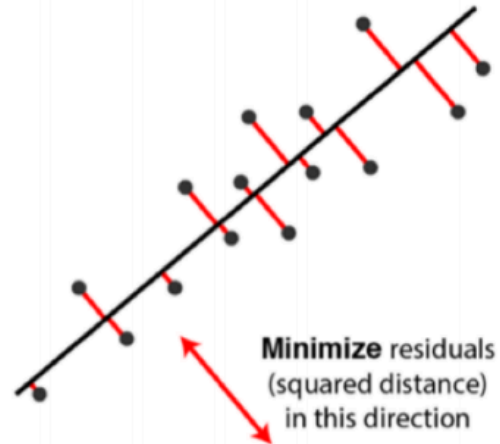
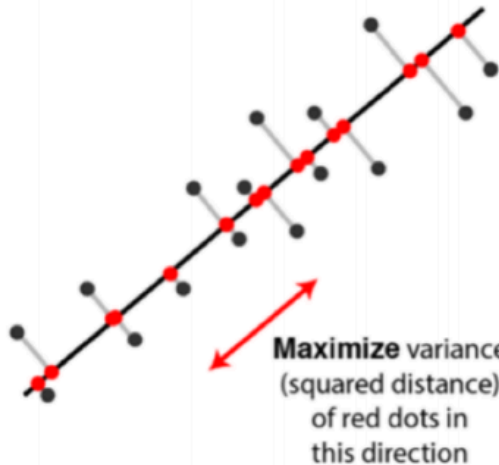
# PCA in one dimension, 2 equivalent views

Goal: find a  $k < d$ -dimensional representation of  $X$

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$$\frac{1}{n} \sum_{i=1}^n \text{dist}(x_i, \text{line defined by } \vec{v})$$



Two equivalent views of principal component analysis.

# PCA: finding a linear projection

---

Given  $x_1, \dots, x_n \in \mathbb{R}^d$ , for  $q \ll d$  find a compressed representation with  $\lambda_1, \dots, \lambda_n \in \mathbb{R}^q$  such that  $x_i \approx \mu + \mathbf{V}_q \lambda_i$  and  $\mathbf{V}_q^T \mathbf{V}_q = I$

$$\min_{\mu, \mathbf{V}_q, \{\lambda_i\}_i} \sum_{i=1}^n \|x_i - \mu - \mathbf{V}_q \lambda_i\|_2^2$$

# PCA: finding a linear projection

Given  $x_1, \dots, x_n \in \mathbb{R}^d$ , for  $q \ll d$  find a compressed representation with  $\lambda_1, \dots, \lambda_n \in \mathbb{R}^q$  such that  $x_i \approx \mu + \mathbf{V}_q \lambda_i$  and  $\mathbf{V}_q^T \mathbf{V}_q = \mathbf{I}$

$$\min_{\mu, \mathbf{V}_q, \{\lambda_i\}_i} \sum_{i=1}^n \|x_i - \mu - \mathbf{V}_q \lambda_i\|_2^2$$

Fix  $\mathbf{V}_q$  and solve for  $\mu, \lambda_i$ :

$$\mu = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$
$$\lambda_i = \mathbf{V}_q^T (x_i - \bar{x})$$

Which gives us:

$$\min_{\mathbf{V}_q} \sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|^2.$$

$\mathbf{V}_q \mathbf{V}_q^T$  is a *projection matrix* that minimizes error in basis of size  $q$



# Linear projections

---

$$\sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|_2^2$$

$$\Sigma := \sum_{i=1}^N (x_i - \bar{x})(x_i - \bar{x})^T$$
$$\mathbf{V}_q^T \mathbf{V}_q = I_q$$

# Linear projections

$$\sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|_2^2$$

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$$\min_{\mathbf{V}_q} \sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|_2^2 = \min_{\mathbf{V}_q} \text{Tr}(\Sigma) - \text{Tr}(\mathbf{V}_q^T \Sigma \mathbf{V}_q)$$

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# Linear projections

$$\sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|_2^2$$

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Eigenvalue decomposition

$\mathbf{V}_q$  are the first  $q$  eigenvectors of  $\Sigma$

Minimize reconstruction error and capture the most variance in your data.

# Linear projections

Given  $x_i \in \mathbb{R}^d$  and some  $q < d$  consider

$$\min_{\mathbf{V}_q} \sum_{i=1}^N \|(x_i - \bar{x}) - \mathbf{V}_q \mathbf{V}_q^T (x_i - \bar{x})\|^2.$$

where  $\mathbf{V}_q = [v_1, v_2, \dots, v_q]$  is orthonormal:

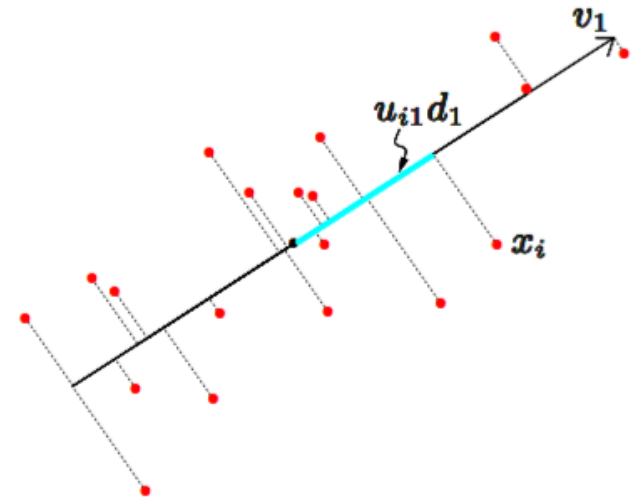
$$\mathbf{V}_q^T \mathbf{V}_q = I_q$$

$\mathbf{V}_q$  are the first  $q$  eigenvectors of  $\Sigma$

$\mathbf{V}_q$  are the first  $q$  principal components

Principal Component Analysis (PCA) projects  $(\mathbf{X} - \mathbf{1}\bar{x}^T)$  down onto  $\mathbf{V}_q$

$$(\mathbf{X} - \mathbf{1}\bar{x}^T) \mathbf{V}_q = \mathbf{U}_q \text{diag}(d_1, \dots, d_q) \quad \mathbf{U}_q^T \mathbf{U}_q = I_q$$



$$\Sigma := \sum_{i=1}^N (x_i - \bar{x})(x_i - \bar{x})^T$$