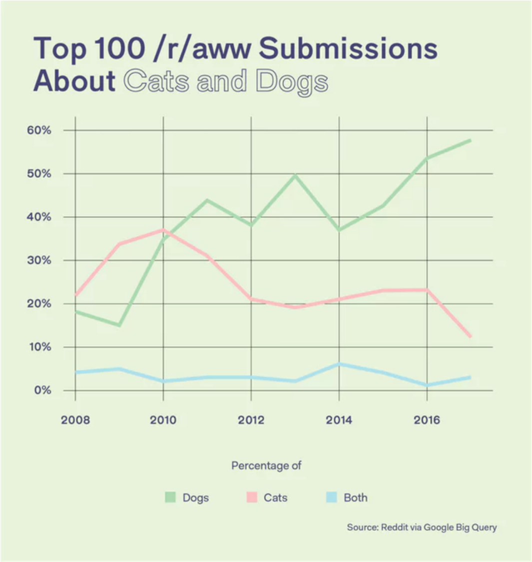
CSE 446: Machine Learning

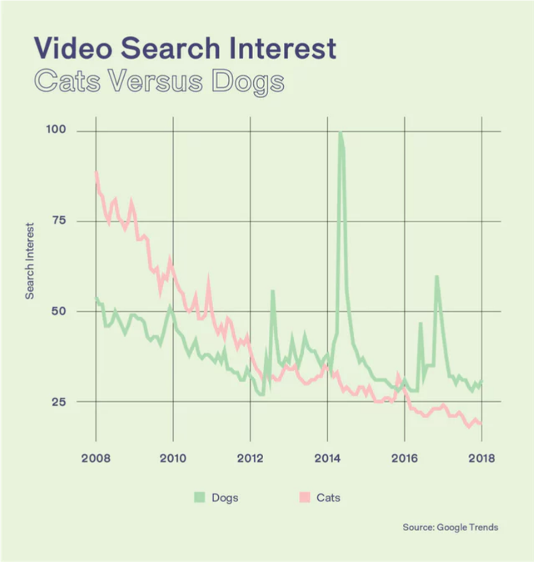
Jamie Morgenstern

Traditional Algorithms

Social media mentions of Cats vs. Dogs

Reddit Google Twitter?





Graphics courtesy of https://theoutline.com/post/3128/dogs-cats-internet-popularity?zd=1

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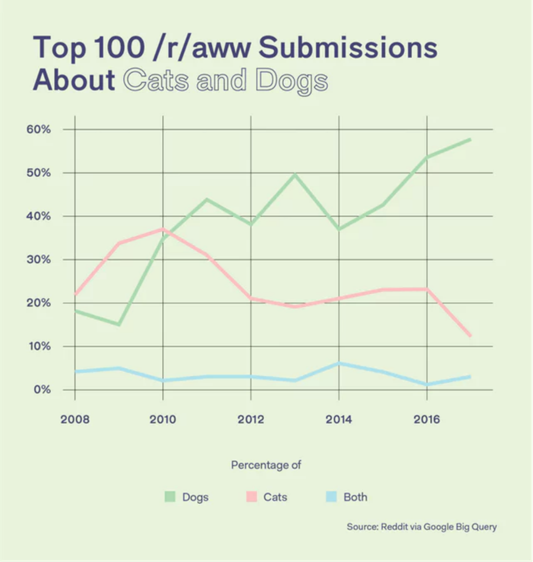

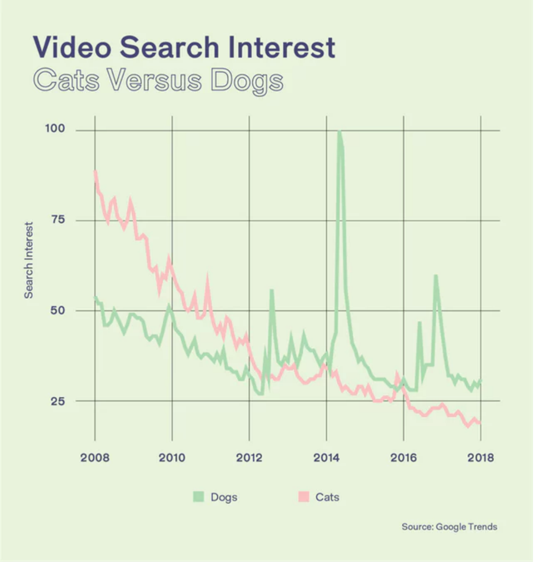

Write a program that sorts tweets into those containing "**cat**", "**dog**", or ***other***

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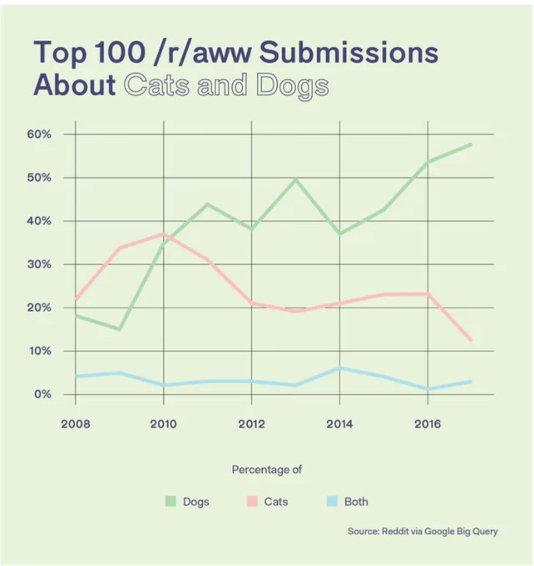
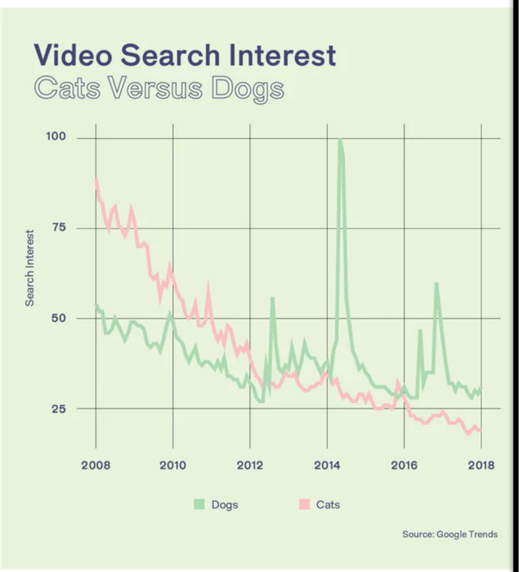


Google



Twitter?

```
cats = []
dogs = []
other = []
for tweet in tweets:
    if "cat" in tweet:
        cats.append(tweet)
    elseif "dog" in tweet:
        dogs.append(tweet)
    else:
        other.append(tweet)
return cats, dogs, other
```

Write a program that sorts tweets into those containing "**cat**", "**dog**", or ***other***

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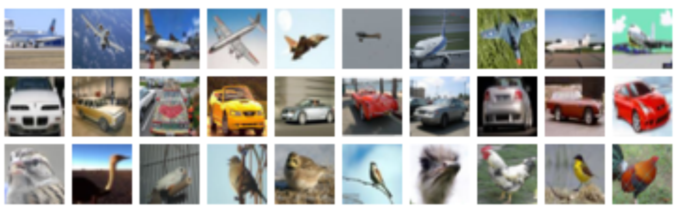
Machine Learning algorithms

Write a program that sorts images
into those containing "**birds**",
"**airplanes**", or ***other.***



airplane
other
bird

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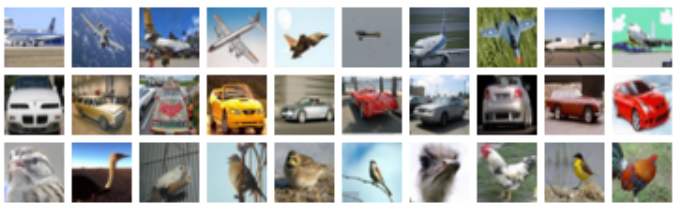

airplane
other
bird

```
birds = []
planes = []
other = []
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    elseif plane in image:
        planes.append(image)
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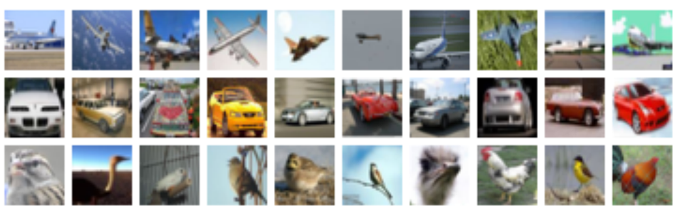

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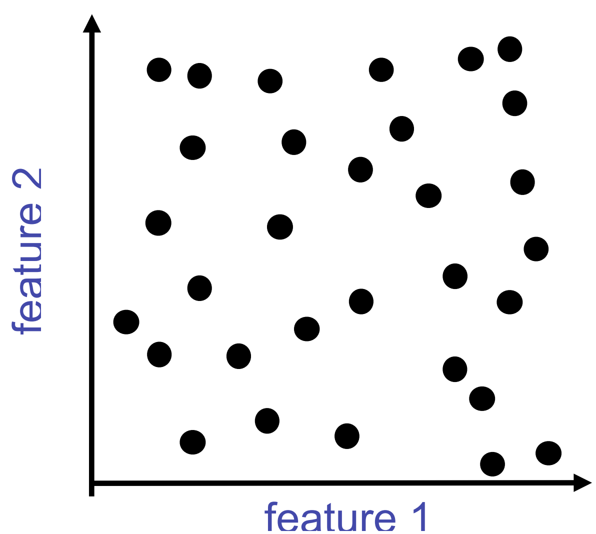


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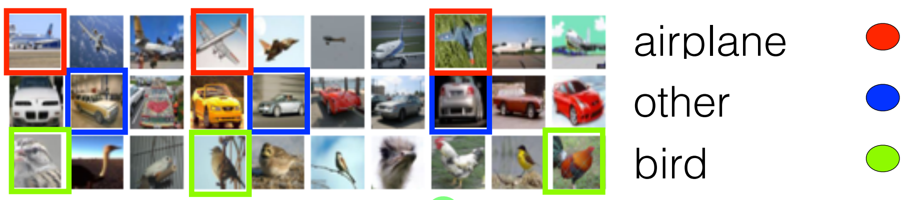

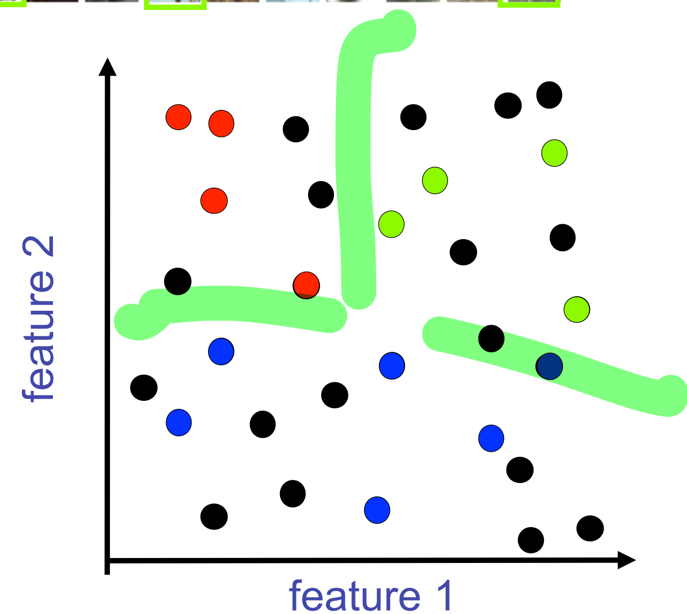


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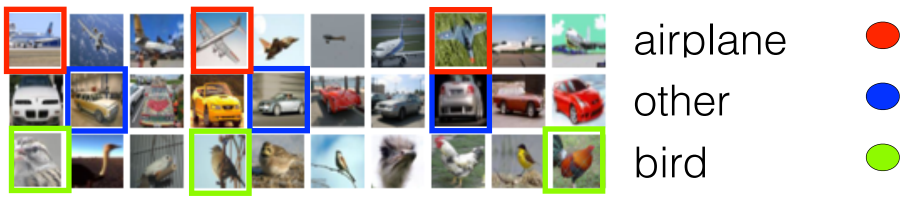
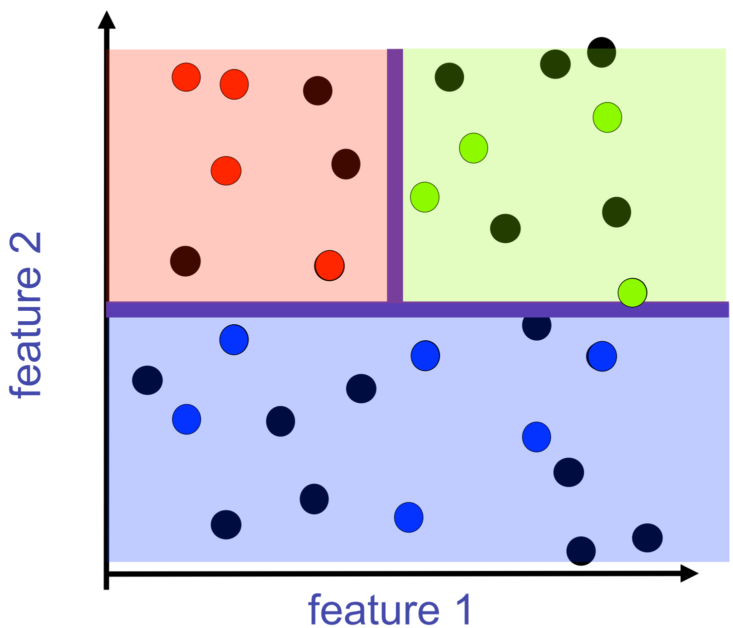




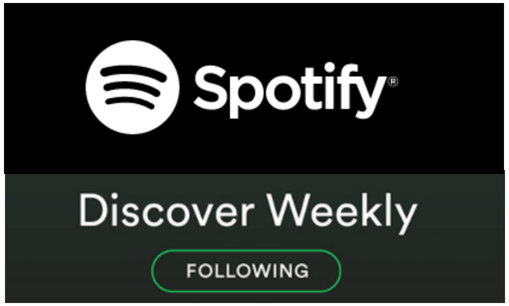
The decision rule of
`if "cat" in tweet:`
is **hard coded by expert.**

The decision rule of
`if bird in image:`
is **LEARNED using DATA**

Machine Learning Ingredients

- **Data**: past observations
- **Hypotheses/Models**: devised to capture the patterns in data
- **Prediction**: apply model to forecast future observations

ML uses past data to make personalized predictions



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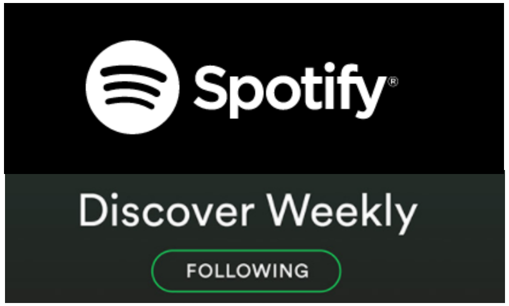
Tick Tock



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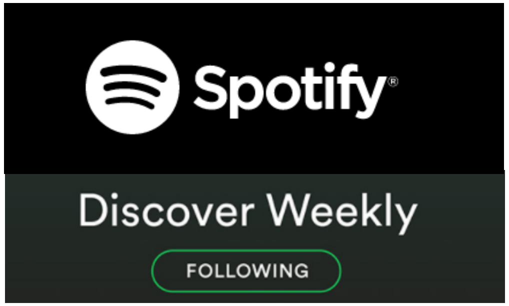

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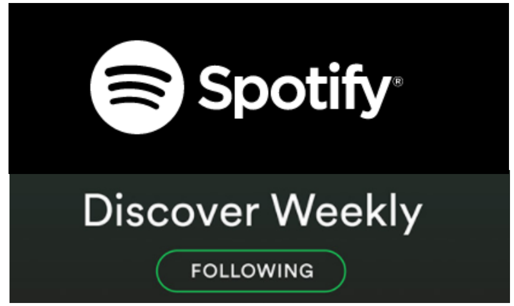

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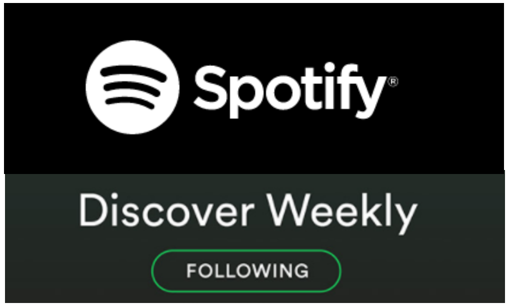

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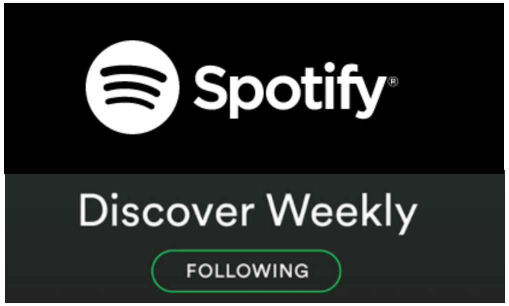

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Flavors of ML

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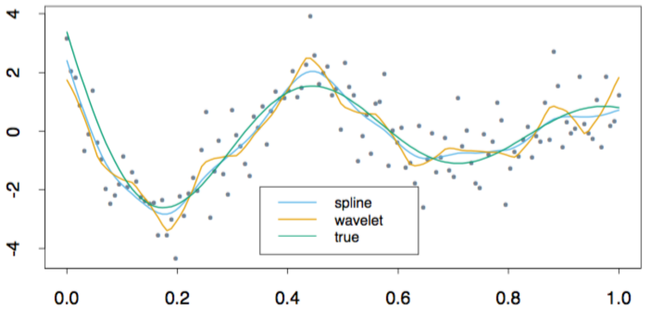

Regression

Predict continuous value:
ex: stock market, credit score, temperature, Netflix rating

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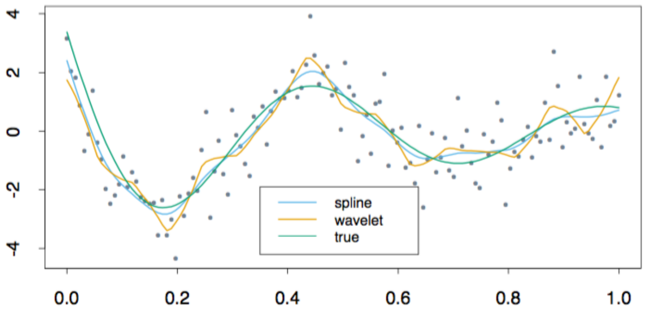

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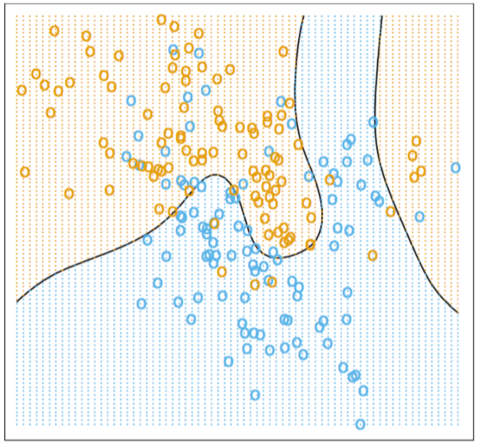

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Predict categorical value:
loan or not? spam or not? what disease is this?

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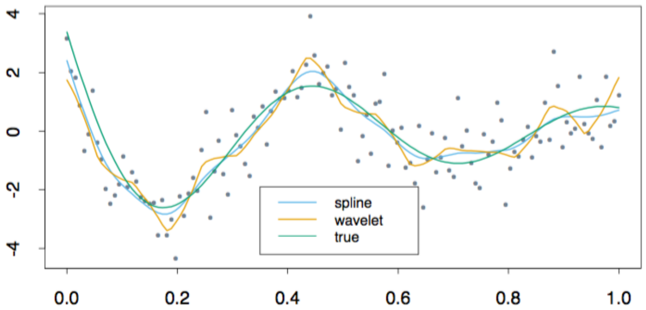

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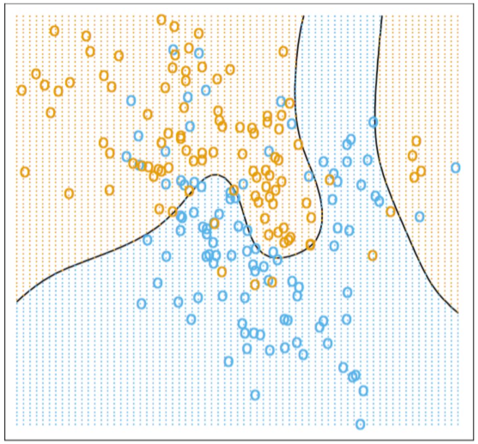

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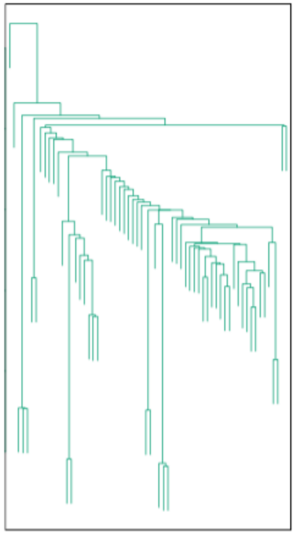
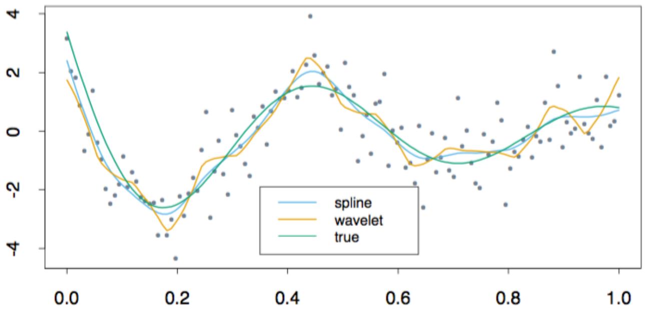


Unsupervised Learning

Predict structure:
tree of life from DNA, find similar images, community detection

Flavors of ML



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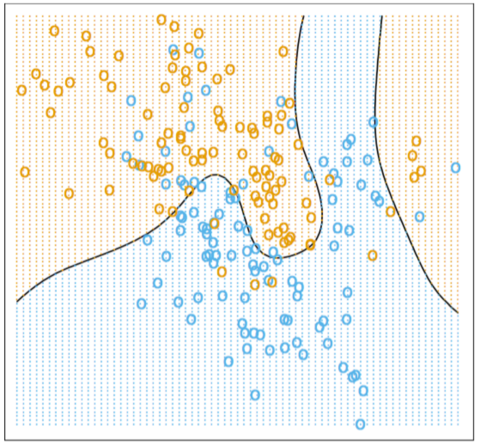

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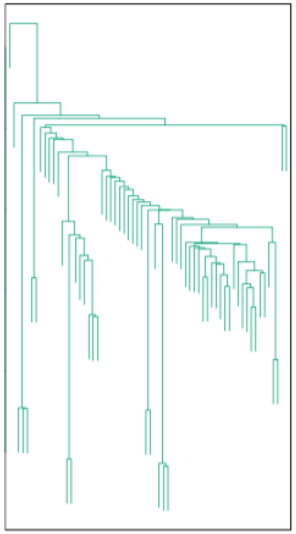

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Mix of statistics (theory) and algorithms (programming)

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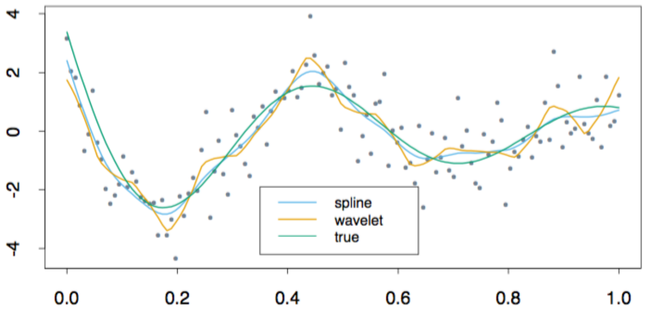

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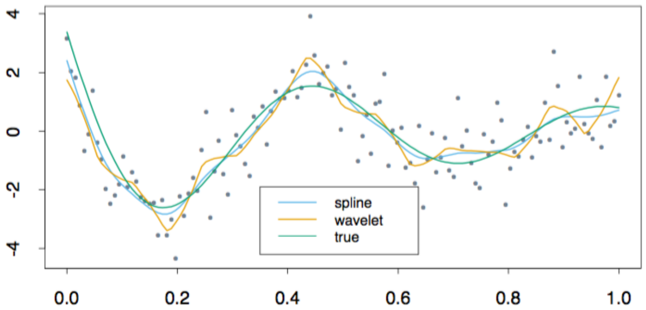

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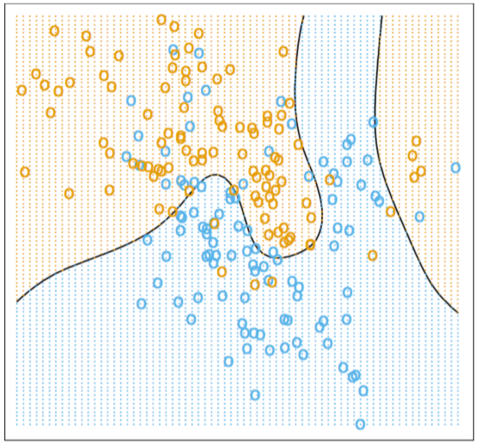

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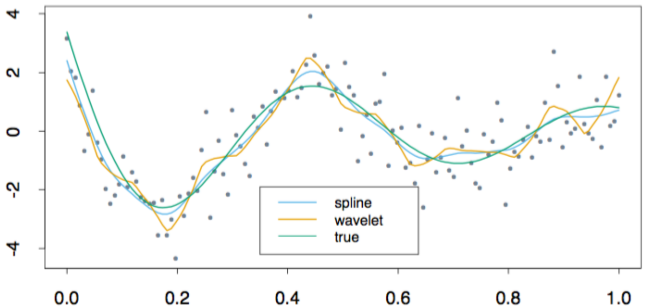

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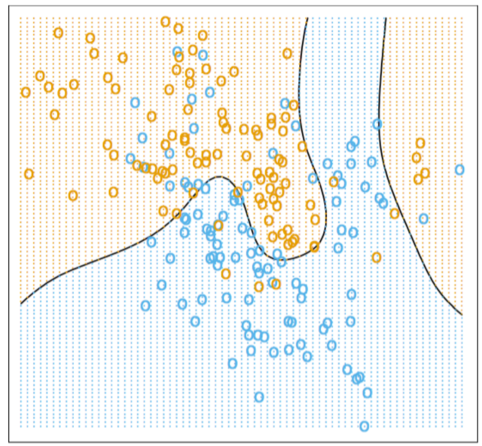

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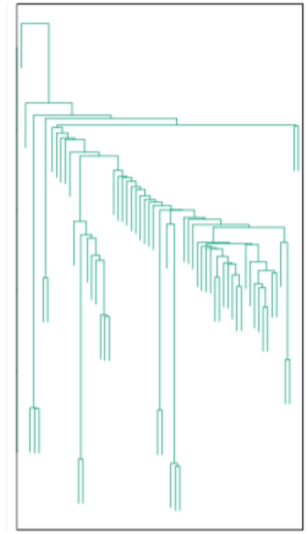

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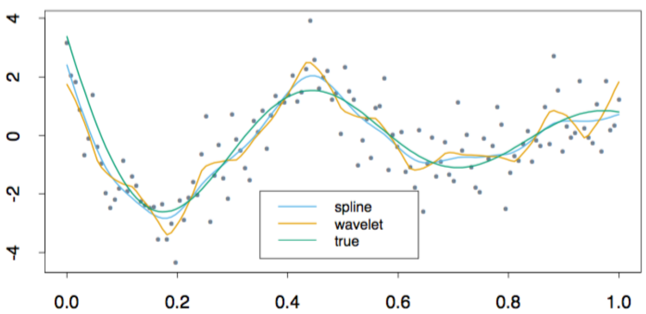

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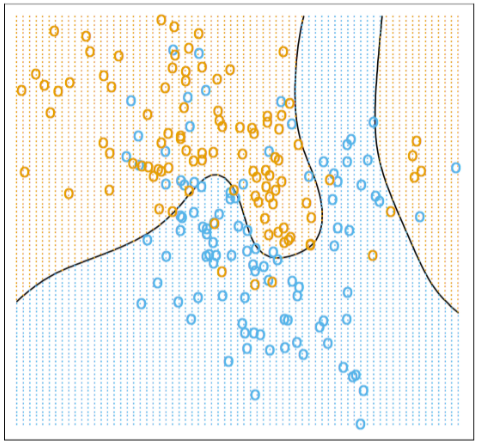

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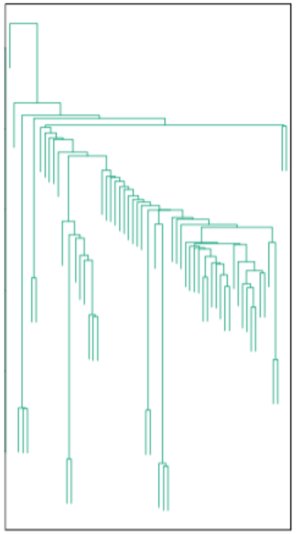

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Mix of statistics (theory) and algorithms (programming)

CSE446/546: Machine Learning

Instructor: Jamie Morgenstern

Contact: cse446-staff@cs.washington.edu

Course Website: https://courses.cs.washington.edu/courses/cse446/23wi

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What this class is:

- **Fundamentals of ML:** bias/variance tradeoff, overfitting, optimization and computational tradeoffs, supervised learning (e.g., linear, boosting, deep learning), unsupervised models (e.g. k-means, EM, PCA)
- **Preparation for further learning:** the field is fast-moving, you will be able to apply the basics and teach yourself the latest

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- **Preparation for further learning:** the field is fast-moving, you will be able to apply the basics and teach yourself the latest

What this class is not:

- **Survey course:** laundry list of algorithms, how to win Kaggle
- **An easy course:** familiarity with intro linear algebra and probability are assumed, homework will be time-consuming

Prerequisites

- Formally:
  - MATH 308, CSE 312, STAT 390 or equivalent
- Familiarity with:
  - Linear algebra
    - linear dependence, rank, linear equations, SVD
  - Multivariate calculus
  - Probability and statistics
    - Distributions, marginalization, moments, conditional expectation
  - Algorithms
    - Basic data structures, complexity
- "Can I learn these topics concurrently?"
- Use HW0 to judge skills
- **See website for review materials!**

Grading

- *5 homework*
  - *Each contains both theoretical questions and will have programming*
  - *Collaboration okay but must write who you collaborated with.* ***You must write, submit, and understand your answers and code (which run on autograder)***
  - ***WHITEBOARD POLICY***
  - *Do not Google for answers.*
- *2 exams, a midterm and a final*

Homework

- HW 0 is out (**Due next Wednesday 10/6 Midnight**)
  - Short *review*
  - Work individually, treat as barometer for readiness
- HW 1,2,3,4
  - They are not easy or short. Start early.
- Submit to Gradescope
- Regrade requests on Gradescope
- **There is no credit for late work, 5 late days**

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1. All code must be written in Python
2. All written work must be typeset (e.g., LaTeX)

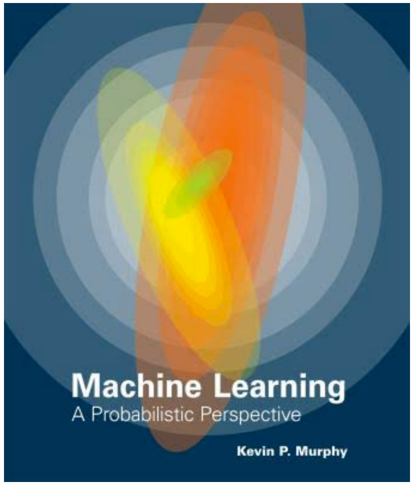
See course website for tutorials and references.

Communication Chanels

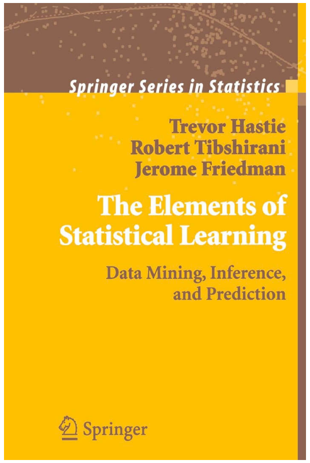
- **Announcements, questions about class, homework help**
  - EdStem (invitation sent, contact TAs if you need access)
  - Weekly Section
  - Office hours
- **Regrade requests**
  - Directly to Gradescope
- **Personal concerns**
  - Email: cse446-staff@cs.washington.edu
- **Anonymous feedback**
  - See website for link

Textbooks

- Required Textbook:
  - ***Machine Learning: a Probabilistic Perspective***; **Kevin Murphy**



- Optional Books (free PDF):
  - *The Elements of Statistical Learning: Data Mining, Inference, and Prediction;* Trevor Hastie, Robert Tibshirani, Jerome Friedman



Addcodes

- Email: Elle Brown (elleanb@cs.washington.edu) for addcodes

Enjoy!

- ML is becoming ubiquitous in science, engineering and beyond
- It's one of the hottest topics in industry today
- This class should give you the basic foundation for applying ML and developing new methods
- The fun begins…

Maximum Likelihood Estimation

Jamie Morgenstern

Your first consulting job

- *Billionaire*: I have a special coin, if I flip it, what's the probability it will be heads?
- *You*: Please flip it a few times:

$D = (HTHHT\ldots)$

$\underline{\qquad\qquad}$ n flips

k heads

- *You*: The probability is:

$\frac{k}{n}$

- *Billionaire:* Why?

Coin – Binomial Distribution

- **Data**: sequence *D= (HHTHT…)*, **k heads** out of **n flips**
- **Hypothesis:** P(Heads) = θ, P(Tails) = 1-θ
  - Flips are i.i.d.:
    - Independent events
    - Identically distributed according to Binomial distribution

- $P(\mathcal{D}|\theta) = \theta^{k}(1-\theta)^{n-k}$

Maximum Likelihood Estimation

- **Data**: sequence *D= (HHTHT…),* **k heads** out of **n flips**
- **Hypothesis:** P(Heads) = θ, P(Tails) = 1-θ

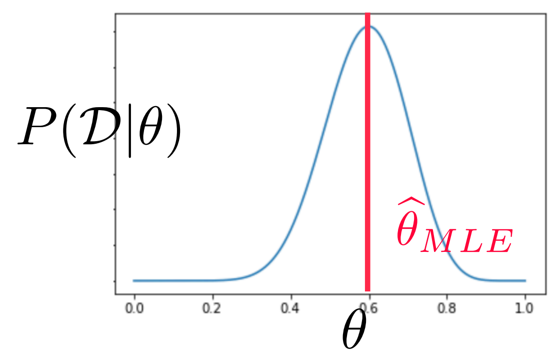
$$P(\mathcal{D}|\theta) = \theta^k (1-\theta)^{n-k}$$

When
x > y
log x > log y

- Maximum likelihood estimation (MLE): Choose θ that maximizes the probability of observed data:

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} \; P(\mathcal{D}|\theta)$$

$$= \arg\max_{\theta} \; \log P(\mathcal{D}|\theta)$$



Your first learning algorithm

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} \; \log P(\mathcal{D}|\theta)$$

$$= \arg\max_{\theta} \; \log \theta^k (1-\theta)^{n-k}$$

- Set derivative to zero: $\boxed{\dfrac{d}{d\theta} \log P(\mathcal{D}|\theta) = 0}$

$$\frac{d}{d\theta}\, k\log\theta + (n-k)\log(1-\theta)$$

$$= k \cdot \frac{1}{\theta} - (n-k)\frac{1}{1-\theta} = 0$$

$$\Leftrightarrow \frac{k}{\theta} = \frac{n-k}{(1-\theta)}$$

$$\frac{(1-\theta)}{\theta} = \frac{n-k}{k}$$

$$\frac{1}{\theta} - 1 = \frac{n}{k} - 1$$

$$\frac{1}{\theta} = \frac{n}{k} \Rightarrow \boxed{\frac{k}{n} = \theta}$$

How many flips do I need?

$$\widehat{\theta}_{MLE} = \frac{k}{n}$$

- *You*: flip the coin 5 times. *Billionaire*: I got 3 heads.

$$\widehat{\theta}_{MLE} = \frac{3}{5} = .6$$

- *You*: flip the coin 50 times. *Billionaire*: I got 20 heads.

$$\widehat{\theta}_{MLE} = \frac{2}{5} = .4$$

- *Billionaire:* Which one is right? Why?

Simple bound (based on Hoeffding's inequality)

- For **n flips** and **k heads** the MLE is **unbiased** for true θ^*:

$$\widehat{\theta}_{MLE} = \frac{k}{n} \qquad \mathbb{E}[\widehat{\theta}_{MLE}] = \theta^*$$

- Hoeffding's inequality says that for any $\varepsilon>0$:

$$P(|\widehat{\theta}_{MLE} - \theta^*| \geq \epsilon) \leq 2e^{-2n\epsilon^2}$$

PAC Learning

- PAC: Probably Approximate Correct
- *Billionaire*: I want to know the parameter θ^*, within $\varepsilon = 0.1$, with probability at least $1-\delta = 0.95$. How many flips?

$$P(|\widehat{\theta}_{MLE} - \theta^*| \geq \epsilon) \leq 2e^{-2n\epsilon^2}$$

What about continuous variables?

- *Billionaire*: What if I am measuring a **continuous variable**?
- ***You*: Let me tell you about Gaussians…**

$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Some properties of Gaussians

- affine transformation (multiplying by scalar and adding a constant)
  - $X \sim N(\mu,\sigma^2)$
  - $Y = aX + b \quad \rightarrow \quad Y \sim N(a\mu+b,a^2\sigma^2)$

- Sum of Gaussians
  - $X \sim N(\mu_X,\sigma^2_X)$
  - $Y \sim N(\mu_Y,\sigma^2_Y)$
  - $Z = X+Y \quad \rightarrow \quad Z \sim N(\mu_X+\mu_Y, \sigma^2_X+\sigma^2_Y)$

MLE for Gaussian

$\in \mathbb{R}_{\geq 0}$

- Prob. of i.i.d. samples $D=\{x_1,\ldots,x_N\}$ (e.g., exam scores):

$$P(\mathcal{D}|\mu,\sigma) = P(x_1,\ldots,x_n|\mu,\sigma)$$
$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \prod_{i=1}^n e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$\frac{d}{d\mu} = 0$

- Log-likelihood of data:

$\frac{d}{d\mu}$ $-2\frac{(x_i-\mu)}{2\sigma^2}$

$$\log P(\mathcal{D}|\mu,\sigma) = \boxed{-n\log(\sigma\sqrt{2\pi})} \boxed{-\sum_{i=1}^n \frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\frac{d}{d\mu}\log P(D|\mu,\sigma) = +\sum_i \frac{(x_i-\mu)}{\sigma^2}$$

Your second learning algorithm: MLE for mean of a Gaussian

- What's MLE for mean?

$$\frac{d}{d\mu} \log P(\mathcal{D}|\mu, \sigma) = \frac{d}{d\mu}\left[-n \log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

$$\sum \frac{x_i - \mu)}{\cancel{\sigma^2}} = 0$$

$$\sum_i (x_i - \hat{\mu}) = 0$$

$$\sum x_i = n\hat{\mu}$$

$$\hat{\mu} = \frac{\sum x_i}{n}$$

MLE for variance

- Again, set derivative to zero:

$$\frac{d}{d\sigma}\log P(\mathcal{D}|\mu,\sigma) = \frac{d}{d\sigma}\left[-n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

Learning Gaussian parameters

$\mathbb{E}_{\mathcal{D}}[\hat{\mu}_{MLE}] = \mu$

- MLE:

$$\widehat{\mu}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\widehat{\sigma^2}_{MLE} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \widehat{\mu}_{MLE})^2$$

- MLE for the variance of a Gaussian is **biased**

$$\mathbb{E}[\widehat{\sigma^2}_{MLE}] \neq \sigma^2$$

$\sigma^2_{MLE} = \frac{1}{n}\sum (x_i - \hat{\mu}_{MLE})^2$

- Unbiased variance estimator:

$$\widehat{\sigma^2}_{unbiased} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \widehat{\mu}_{MLE})^2$$

Maximum Likelihood Estimation

Observe $X_1, X_2, \ldots, X_n$ drawn IID from $f(x;\theta)$ for some "true" $\theta = \theta_*$

Likelihood function $L_n(\theta) = \prod_{i=1}^{n} f(X_i;\theta)$

Log-Likelihood function $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^{n} \log(f(X_i;\theta))$

Maximum Likelihood Estimator (MLE) $\widehat{\theta}_{MLE} = \arg\max_{\theta} L_n(\theta)$

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Maximum Likelihood Estimator (MLE) $\widehat{\theta}_{MLE} = \arg\max_{\theta} L_n(\theta)$

Properties (under benign regularity conditions—smoothness, identifiability, etc.):

- Asymptotically consistent and normal: $\frac{\widehat{\theta}_{MLE} - \theta_*}{\widehat{se}} \sim \mathcal{N}(0,1)$
- Asymptotic Optimality, minimum variance (see Cramer-Rao lower bound)

Recap

- Learning is…
  - Collect some data
    - E.g., coin flips
  - Choose a hypothesis class or model
    - E.g., binomial
  - Choose a loss function
    - E.g., data likelihood
  - Choose an optimization procedure
    - E.g., set derivative to zero to obtain MLE
  - Justifying the accuracy of the estimate
    - E.g., Hoeffding's inequality

Maximum Likelihood Estimation, cont.

Machine Learning – CSE446

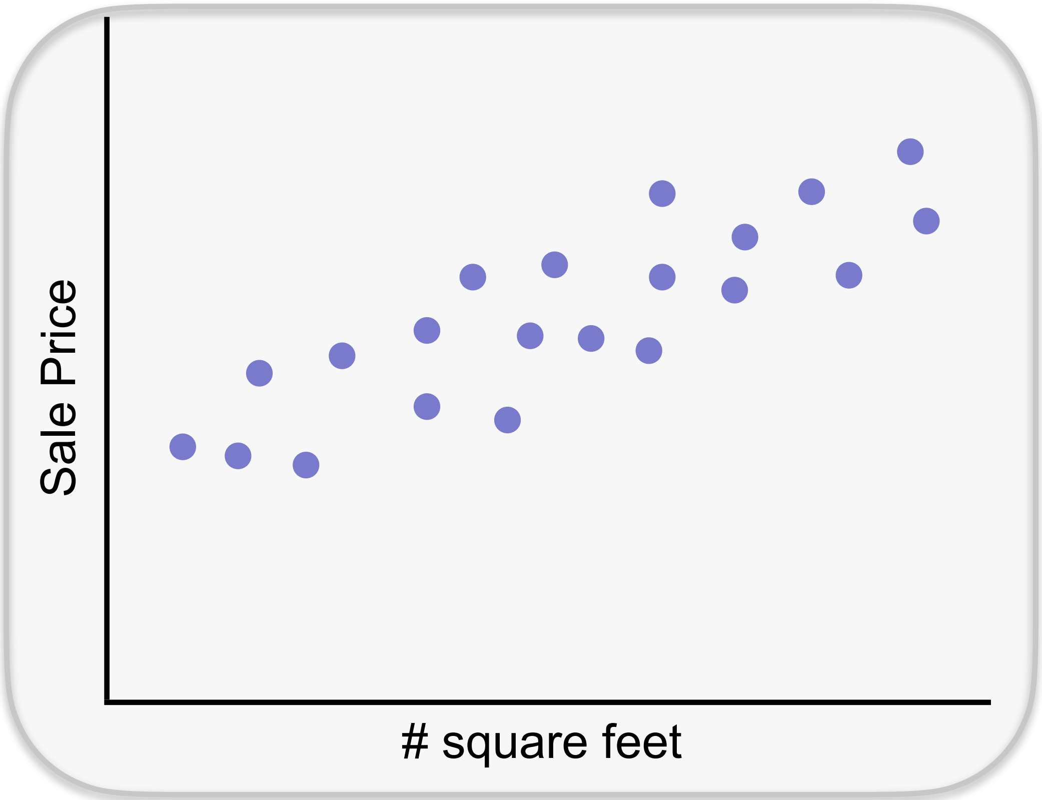
Jamie Morgenstern
University of Washington

The 1d regression problem

Given past sales data on zillow.com, predict:

y = **House sale price** *from*

x = **# sq. ft.**



Training Data:

$$\{(x_i, y_i)\}_{i=1}^n$$

Model:

$$y_i = x_i w + b + \epsilon_i$$

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

Linear model

Noise model

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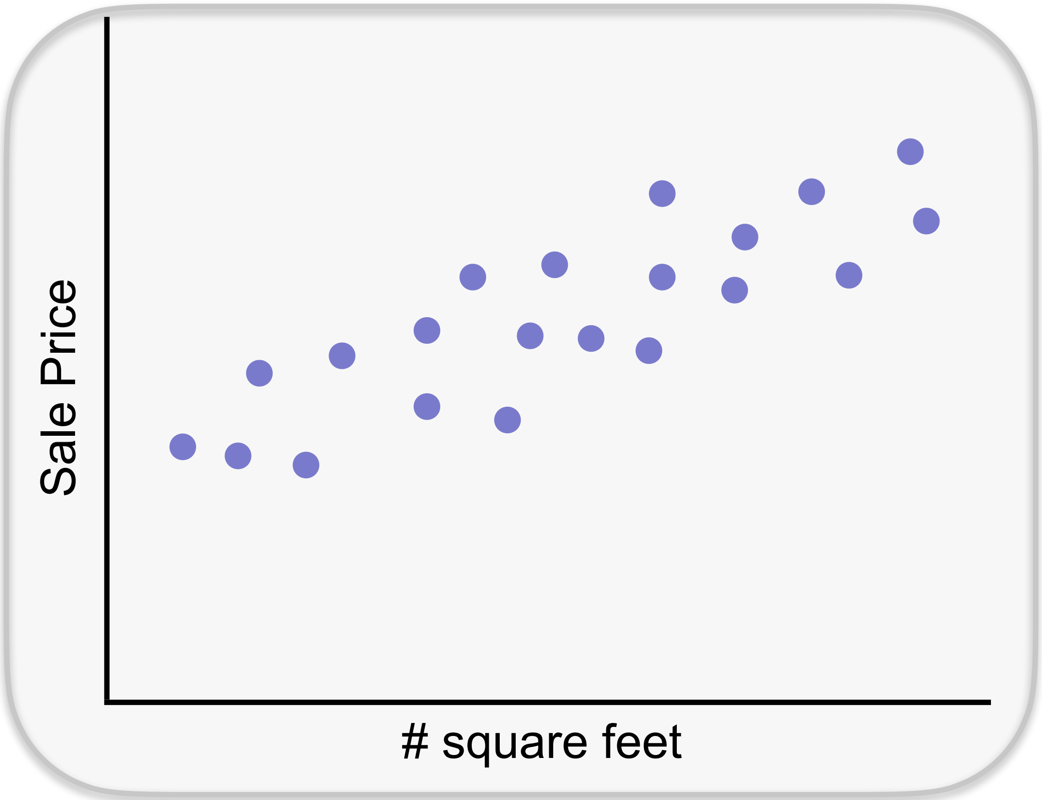

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Loss function:

$$\sum_{i=1}^n -\log(p(y_i | x_i, w, b))$$

The 1d regression problem

Loss function:

$$\sum_{i=1}^{n} -\log(p(y_i|x_i, w, b))$$

$$= \sum_{i=1}^{n} -\log(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(y_i - (wx_i + b))^2}{2\sigma^2}\right\})$$

Model:

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

$$y_i = x_i w + b + \epsilon_i$$

The 1d regression problem

Loss function:

$$\sum_{i=1}^{n} -\log(p(y_i|x_i, w, b))$$

$$= \sum_{i=1}^{n} -\log\left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(y_i - (wx_i + b))^2}{2\sigma^2}\right\}\right)$$

Model:

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

$$y_i = x_i w + b + \epsilon_i$$

$$\arg\min_{w,b} \sum_{i=1}^{n} -\log(p(y_i|x_i, w, b)) \equiv \arg\min_{w,b} \sum_{i=1}^{n} (y_i - (wx_i + b))^2$$

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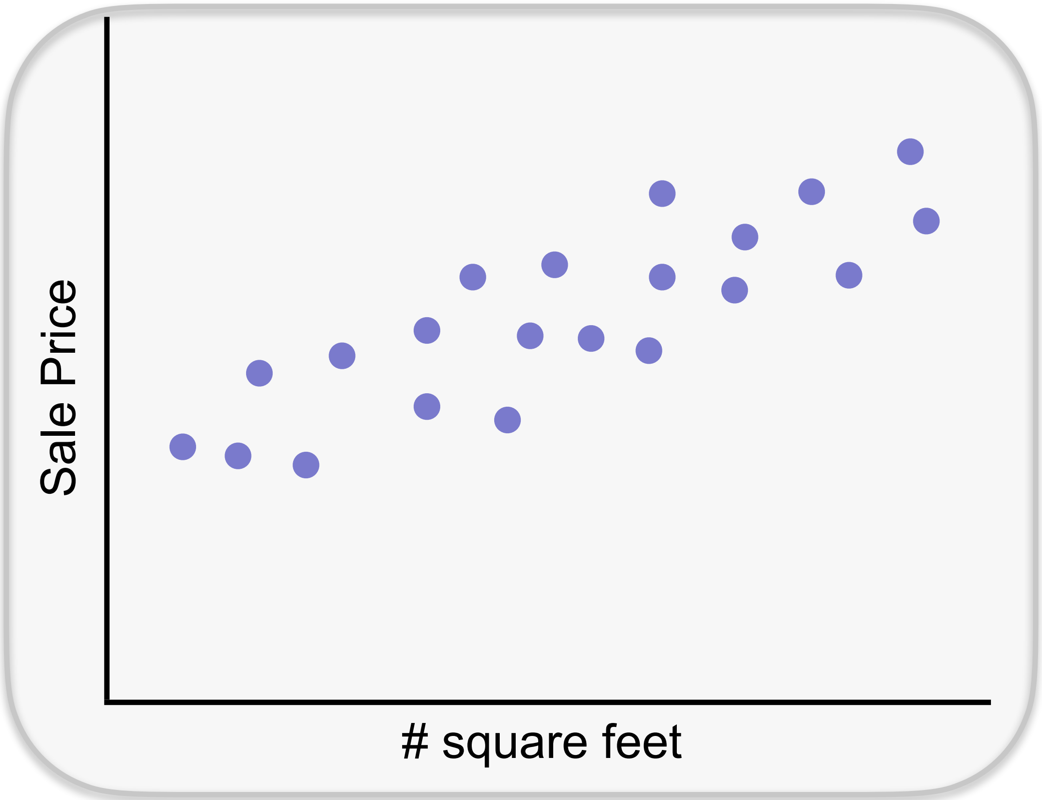

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Loss function:

$$\sum_{i=1}^n (y_i - (wx_i + b))^2$$