

Boosting and Additive Models

Boosting

- 1988 Kearns and Valiant: “Can **weak learners** be combined to create a **strong learner**?”

Weak learner definition (informal):

An algorithm \mathcal{A} is a *weak learner* for a hypothesis class \mathcal{H} that maps \mathcal{X} to $\{-1, 1\}$ if for all input distributions over \mathcal{X} and $h \in \mathcal{H}$, we have that \mathcal{A} correctly classifies h with error at most $1/2 - \gamma$

- 1990 Robert Schapire: “Yup!”
- 1995 Schapire and Freund: “Practical for 0/1 loss” AdaBoost
- 2001 Friedman: “Practical for arbitrary losses”
- 2014 Tianqi Chen: “Scale it up!” XGBoost

Additive models

- **Given:** $\{(x_i, y_i)\}_{i=1}^n$ $x_i \in \mathbb{R}^d, y_i \in \{-1, 1\}$
- **Generate random functions:** $\phi_t : \mathbb{R}^d \rightarrow \mathbb{R}$ $t = 1, \dots, p$
- **Learn some weights:** $\hat{w} = \arg \min_w \sum_{i=1}^n \text{Loss} \left(y_i, \sum_{t=1}^p w_t \phi_t(x_i) \right)$
- **Classify new data:** $f(x) = \text{sign} \left(\sum_{t=1}^p \hat{w}_t \phi_t(x) \right)$

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An interpretation:

Each $\phi_t(x)$ is a classification rule that we are assigning some weight \hat{w}_t

$$\hat{w}, \hat{\phi}_1, \dots, \hat{\phi}_p = \arg \min_{w, \phi_1, \dots, \phi_p} \sum_{i=1}^n \text{Loss} \left(y_i, \sum_{t=1}^p w_t \phi_t(x_i) \right)$$

is in general computationally hard

Forward Stagewise Additive models

$b(x, \gamma)$ is a function with parameters γ

Examples: $b(x, \gamma) = \frac{1}{1 + e^{-\gamma^T x}}$

Algorithm 10.2 *Forward Stagewise Additive Modeling.*

1. Initialize $f_0(x) = 0$.

2. For $m = 1$ to M :

(a) Compute

$$(\beta_m, \gamma_m) = \arg \min_{\beta, \gamma} \sum_{i=1}^N L(y_i, f_{m-1}(x_i) + \beta b(x_i; \gamma)).$$

(b) Set $f_m(x) = f_{m-1}(x) + \beta_m b(x; \gamma_m)$.

Idea: greedily add one function at a time

$$b(x, \gamma) = \gamma_1 \mathbf{1}\{x_3 \leq \gamma_2\}$$

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AdaBoost: $b(x, \gamma)$: classifiers to $\{-1, 1\}$

$$L(y, f(x)) = \exp(-yf(x))$$

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Boosted Regression Trees:

$$L(y, f(x)) = (y - f(x))^2$$

$b(x, \gamma)$: regression trees

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Boosted Regression Trees: $L(y, f(x)) = (y - f(x))^2$

$$\begin{aligned} L(y_i, f_{m-1}(x_i) + \beta b(x_i; \gamma)) &= (y_i - f_{m-1}(x_i) - \beta b(x_i; \gamma))^2 \\ &= (r_{im} - \beta b(x_i; \gamma))^2, \quad r_{im} = y_i - f_{m-1}(x_i) \end{aligned}$$

Efficient: No harder than learning regression trees!

Additive models

- Boosting is popular at parties: Invented by theorists, heavily adopted by practitioners.
- Computationally efficient with “weak” learners. But can also use trees! Boosting can scale.
- Gradient boosting generalization with good software packages (e.g., *XGBoost*). Effective on Kaggle

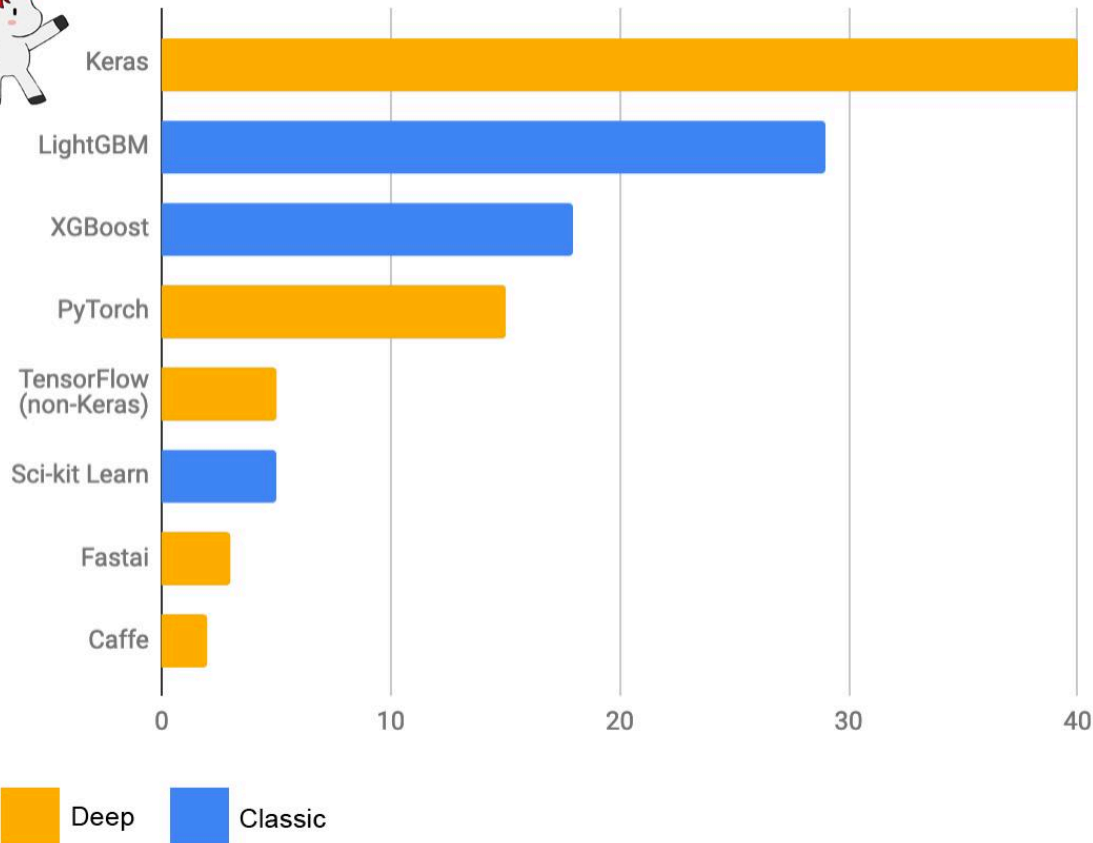
Additive models



François Chollet  @fchollet · Apr 3, 2019

What machine learning tools do Kaggle champions use? We ran a survey among teams that ranked in the *top 5* of a competition since 2016.

Primary ML software tool used by top-5 teams on Kaggle in each competition (n=120)



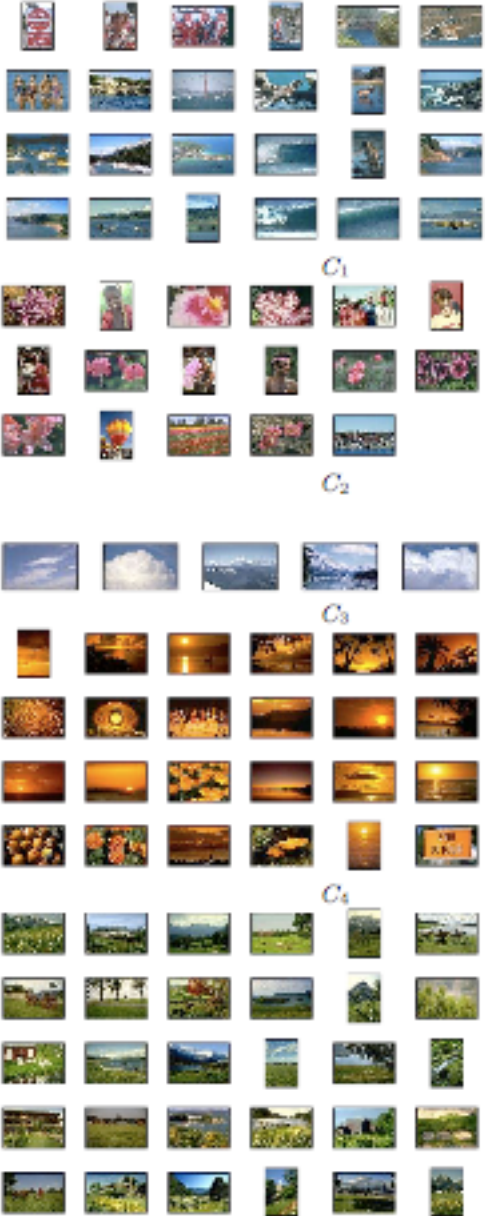
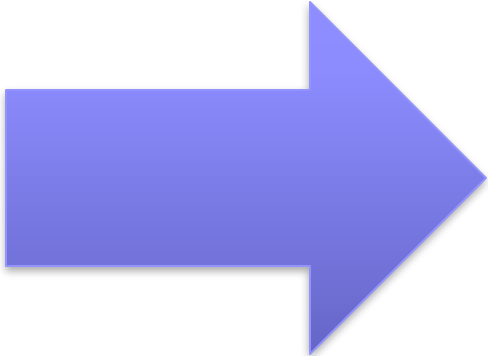
Bagging versus Boosting

- Bagging *averages* many **low-bias, lightly dependent** classifiers to reduce the variance
- Boosting *learns* linear combination of **high-bias, highly dependent** classifiers to reduce error

Clustering

K-means

Clustering images



[Goldberger et al.]

Clustering web search results

Clusty [web](#) [news](#) [images](#) [wikipedia](#) [blogs](#) [jobs](#) [more »](#)

race Search [advanced preferences](#)

clusters sources sites

All Results (238) remix

- Car (28)
 - Race cars (7)
 - Photos, Races Scheduled (5)
 - Game (4)
 - Track (3)
 - Nascar (2)
 - Equipment And Safety (2)
 - Other Topics (7)
- Photos (22)
- Game (14)
- Definition (13)
- Team (18)
- Human (8)**
 - Classification Of Human (2)
 - Statement, Evolved (2)
 - Other Topics (4)
- Weekend (8)
- Ethnicity And Race (7)
- Race for the Cure (8)
- Race Information (8)

[more](#) | [all clusters](#)

find in clusters:

Cluster **Human** contains 8 documents.

Search Results

- [Race \(classification of human beings\) - Wikipedia, the free ...](#)

The term **race** or racial group usually refers to the concept of dividing **humans** into populations or groups on the basis of various sets of characteristics. The most widely used **human** racial categories are based on visible traits (especially skin color, cranial or facial features and hair texture), and self-identification. Conceptions of **race**, as well as specific ways of grouping **races**, vary by culture and over time, and are often controversial for scientific as well as social and political reasons. History · Modern debates · Political and ...
[en.wikipedia.org/wiki/Race_\(classification_of_human_beings\)](#) - [cache] - Live, Ask
- [Race - Wikipedia, the free encyclopedia](#)

General. **Racing** competitions The **Race** (yachting **race**), or La course du millénaire, a no-rules round-the-world sailing event; **Race** (biology), classification of flora and fauna; **Race** (classification of **human** beings) **Race** and ethnicity in the United States Census, official definitions of "**race**" used by the US Census Bureau; **Race** and genetics, notion of racial classifications based on genetics. Historical definitions of **race**; **Race** (bearing), the inner and outer rings of a rolling-element bearing. **RACE** in molecular biology "Rapid ... General · Surnames · Television · Music · Literature · Video games
[en.wikipedia.org/wiki/Race](#) - [cache] - Live, Ask
- [Publications | Human Rights Watch](#)

The use of torture, unlawful rendition, secret prisons, unfair trials, ... Risks to Migrants, Refugees, and Asylum Seekers in Egypt and Israel ... In the run-up to the Beijing Olympics in August 2008, ...
[www.hrw.org/backgrounder/usa/race](#) - [cache] - Ask
- [Amazon.com: Race: The Reality Of Human Differences: Vincent Sarich ...](#)

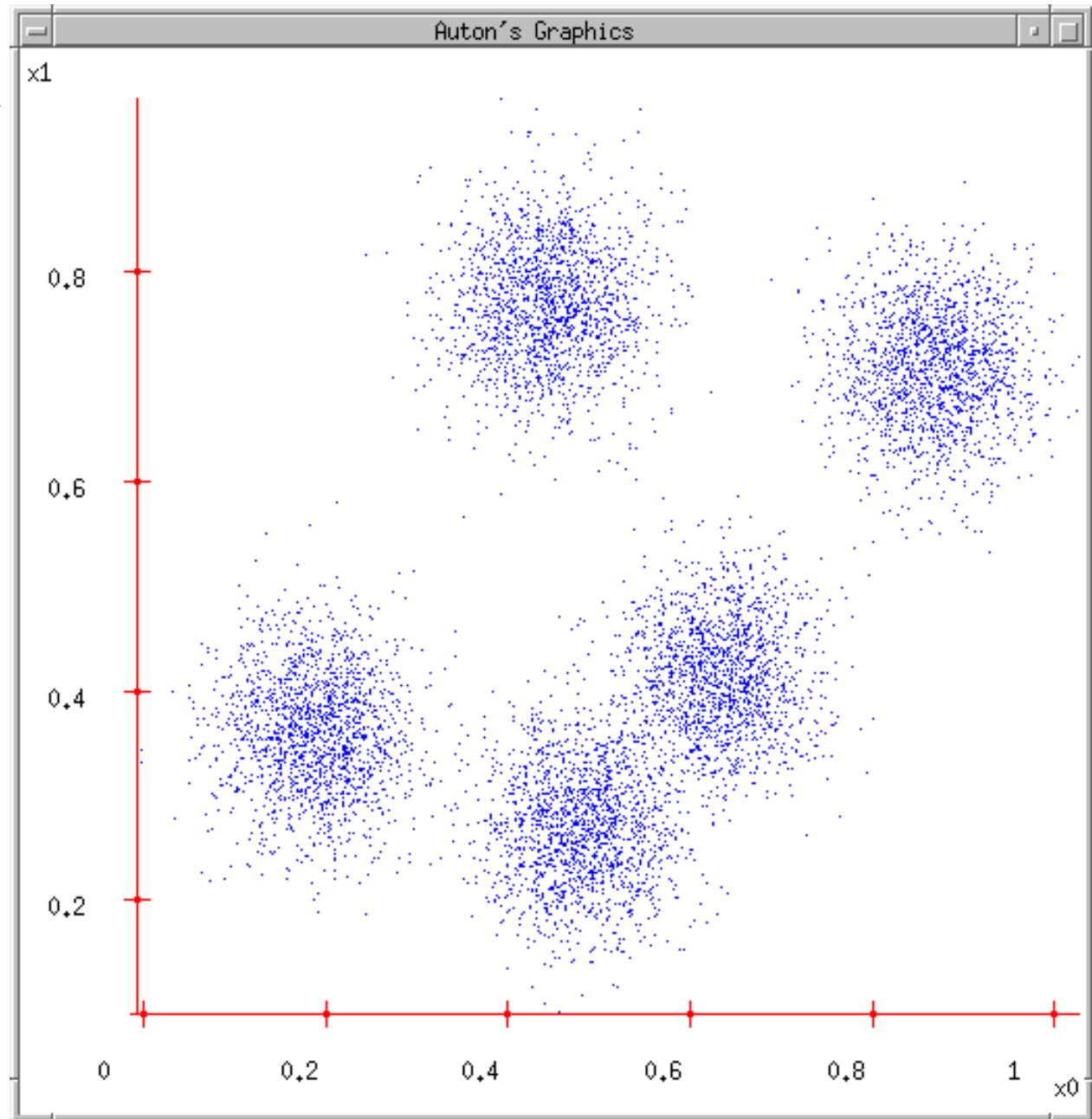
Amazon.com: **Race: The Reality Of Human Differences: Vincent Sarich, Frank Miele: Books ...** From Publishers Weekly Sarich, a Berkeley emeritus anthropologist, and Miele, an editor ...
[www.amazon.com/Race-Reality-Differences-Vincent-Sarich/dp/0813340861](#) - [cache] - Live
- [AAPA Statement on Biological Aspects of Race](#)

AAPA Statement on Biological Aspects of **Race** ... Published in the American Journal of Physical Anthropology, vol. 101, pp 569-570, 1996 ... PREAMBLE As scientists who study **human** evolution and variation, ...
[www.physanth.org/positions/race.html](#) - [cache] - Ask
- [race: Definition from Answers.com](#)

race n. A local geographic or global **human** population distinguished as a more or less distinct group by genetically transmitted physical
[www.answers.com/topic/race-1](#) - [cache] - Live
- [Dopefish.com](#)

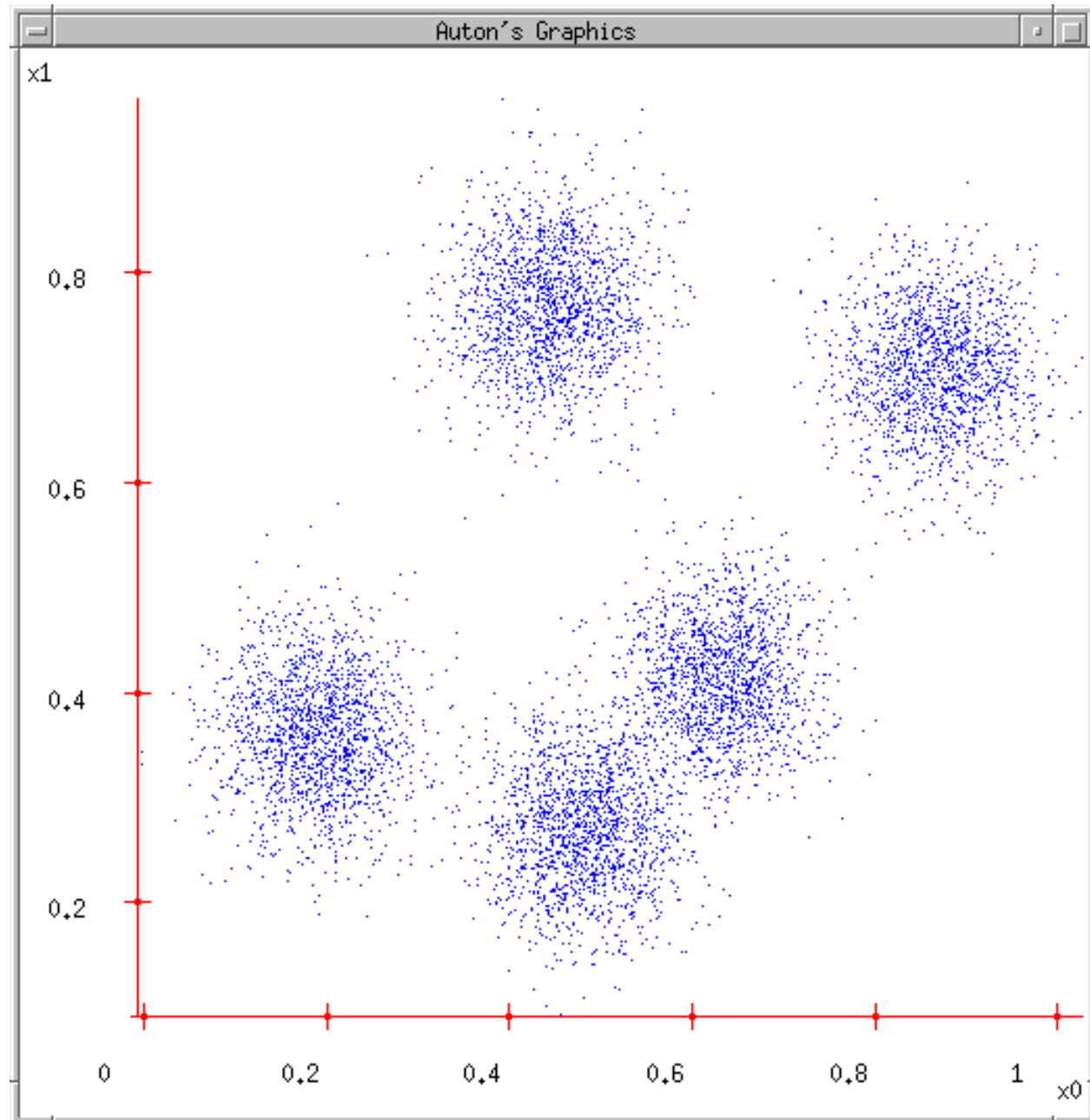
Site for newbies as well as experienced Dopefish followers, chronicling the birth of the Dopefish, its numerous appearances in several computer games, and its eventual take-over of the **human** **race**. Maintained by Mr. Dopefish himself, Joe Siegler of Apogee Software.
[www.dopefish.com](#) - [cache] - Open Directory

Some Data



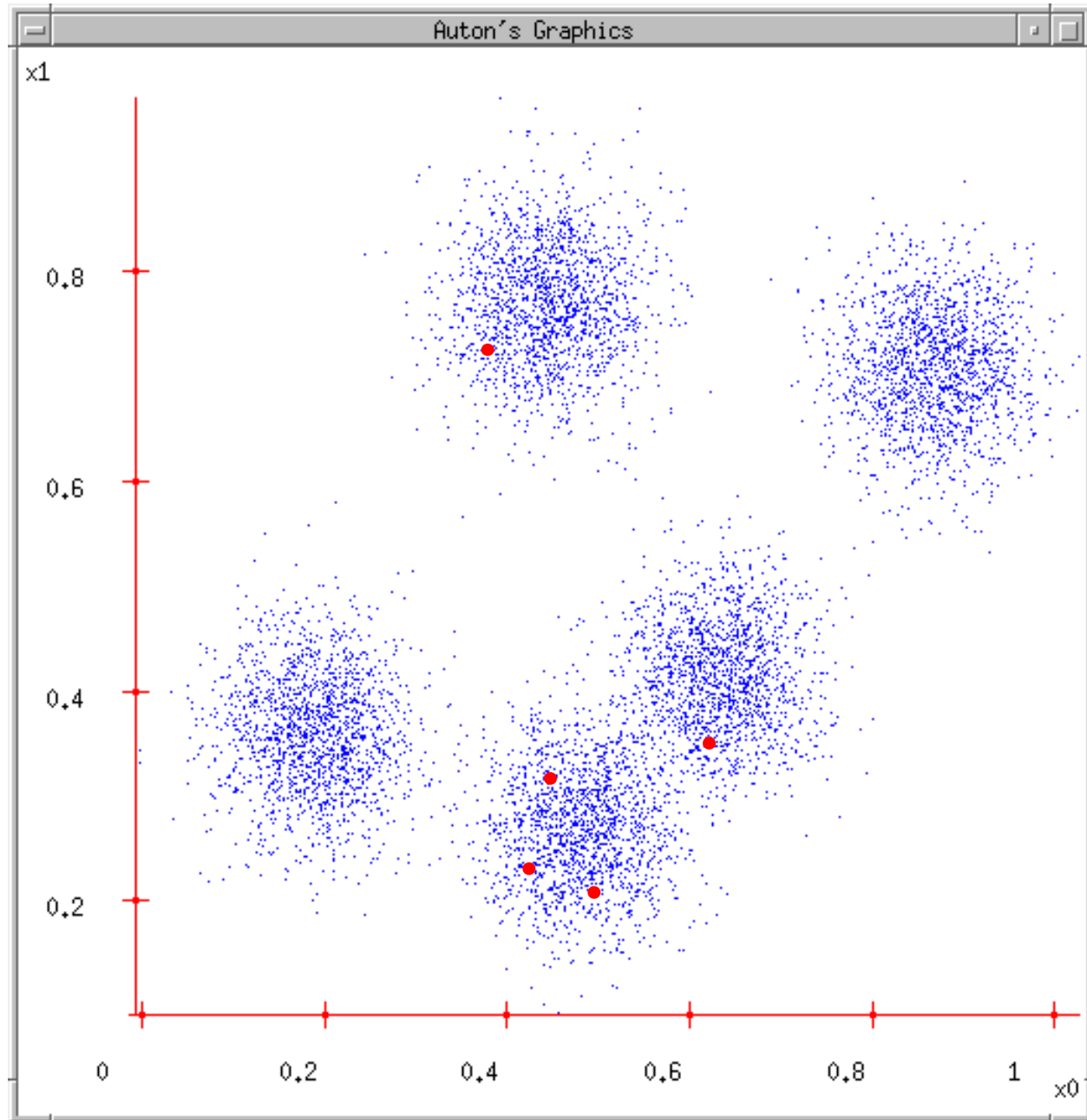
K-means

1. Ask user how many clusters they'd like.
(e.g. $k=5$)



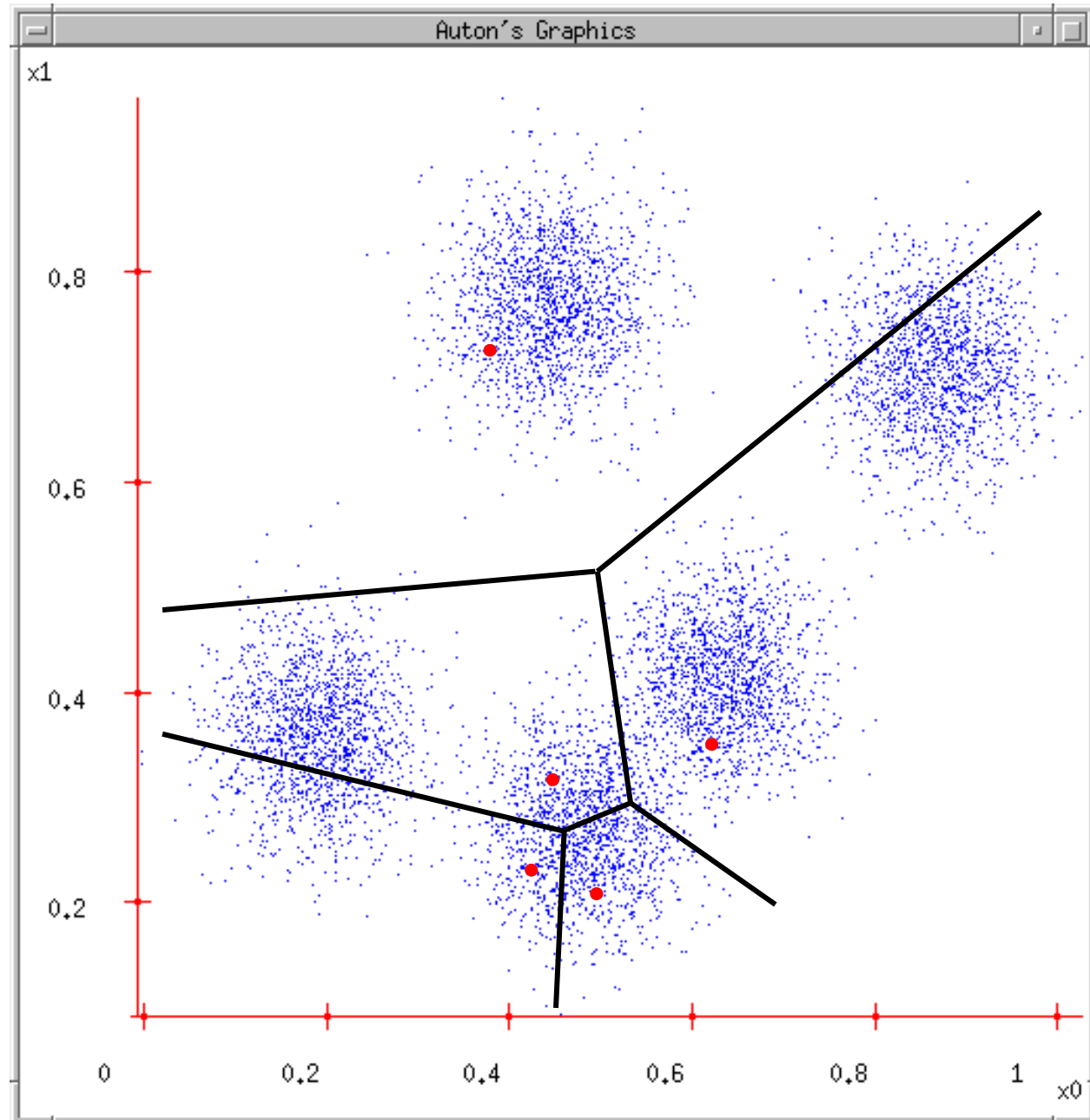
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2. Randomly guess k cluster Center locations



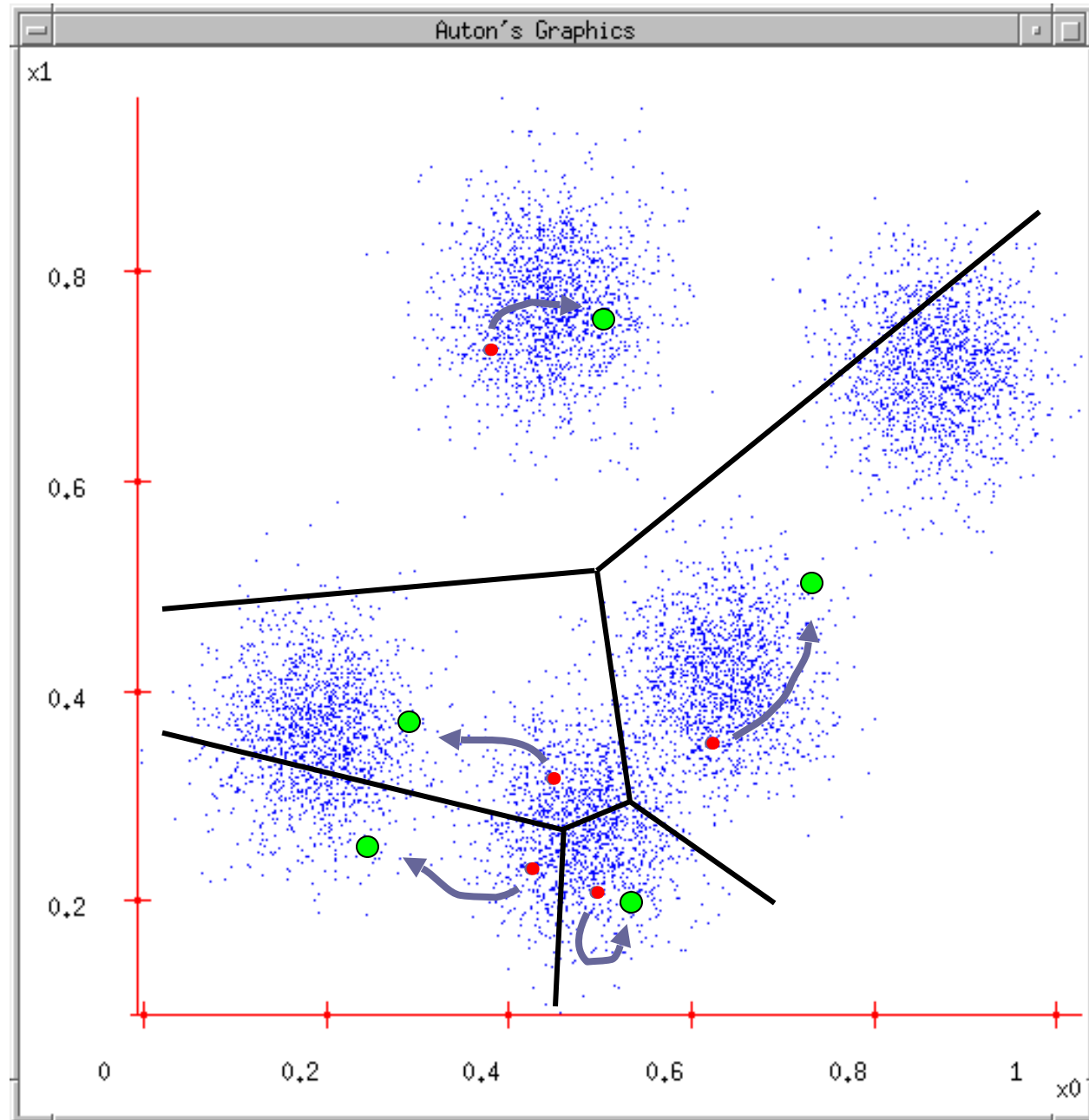
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3. Each datapoint finds out which Center it's closest to. (Thus each Center "owns" a set of datapoints)



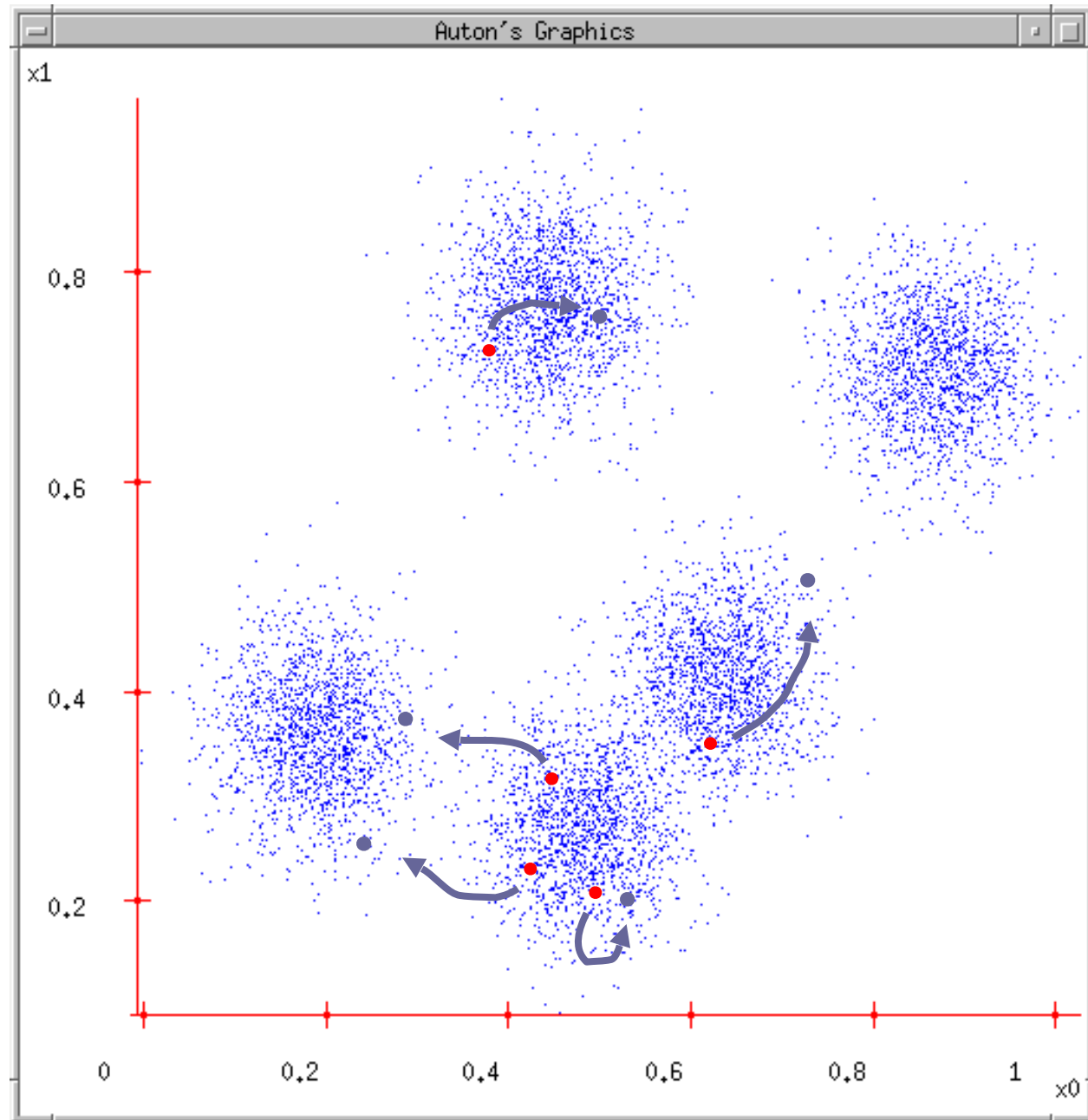
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4. Each Center finds the centroid of the points it owns...
5. ...and jumps there
6. ...Repeat until terminated!



K-means

- Randomly initialize k centers
 - $\mu^{(0)} = \mu_1^{(0)}, \dots, \mu_k^{(0)}$
- Classify: Assign each point $j \in \{1, \dots, N\}$ to nearest center:
 - $C^{(t)}(j) \leftarrow \arg \min_i \|\mu_i - x_j\|^2$
- Recenter: μ_i becomes centroid of its point:
 - $\mu_i^{(t+1)} \leftarrow \arg \min_{\mu} \sum_{j: C(j)=i} \|\mu - x_j\|^2$

Does K-means converge???

Part 1

- Optimize potential function:

$$\min_{\mu} \min_C F(\mu, C) = \min_{\mu} \min_C \sum_{i=1}^k \sum_{j:C(j)=i} \|\mu_i - x_j\|^2$$

- Fix μ , optimize C

Does K-means converge???

Part 2

- Optimize potential function:

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Vector Quantization, Fisher Vectors

Vector Quantization (for compression)

1. Represent image as grid of patches
2. Run k-means on the patches to build code book
3. Represent each patch as a code word.



FIGURE 14.9. *Sir Ronald A. Fisher (1890 – 1962) was one of the founders of modern day statistics, to whom we owe maximum-likelihood, sufficiency, and many other fundamental concepts. The image on the left is a 1024×1024 grayscale image at 8 bits per pixel. The center image is the result of 2×2 block VQ, using 200 code vectors, with a compression rate of 1.9 bits/pixel. The right image uses only four code vectors, with a compression rate of 0.50 bits/pixel*

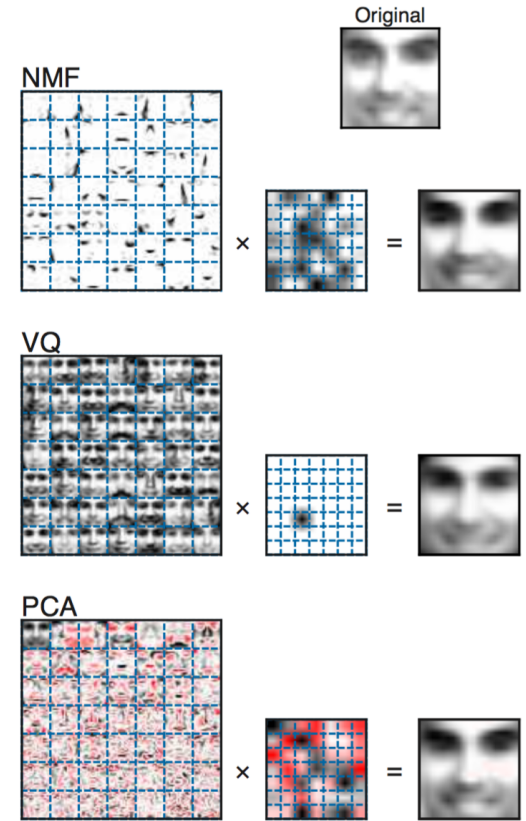
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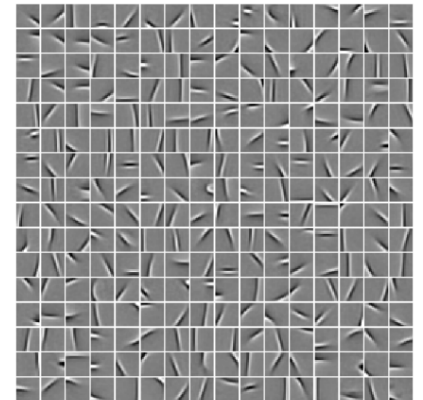
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Typical output of
on patches



Similar reduced representation can be used as a feature vector

Coates, Ng, *Learning Feature Representations with K-means*, 2012