

CSE 446/546: Machine Learning

Kevin Jamieson and Simon Du

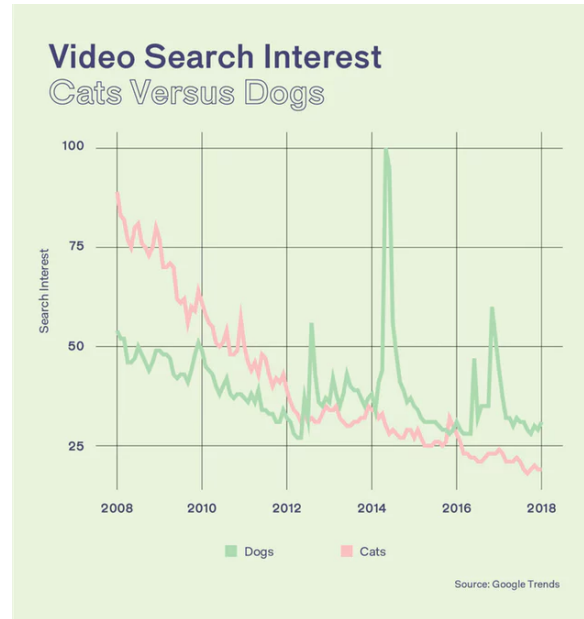
Traditional algorithms

Social media mentions of Cats vs. Dogs

Reddit

Google

Twitter?



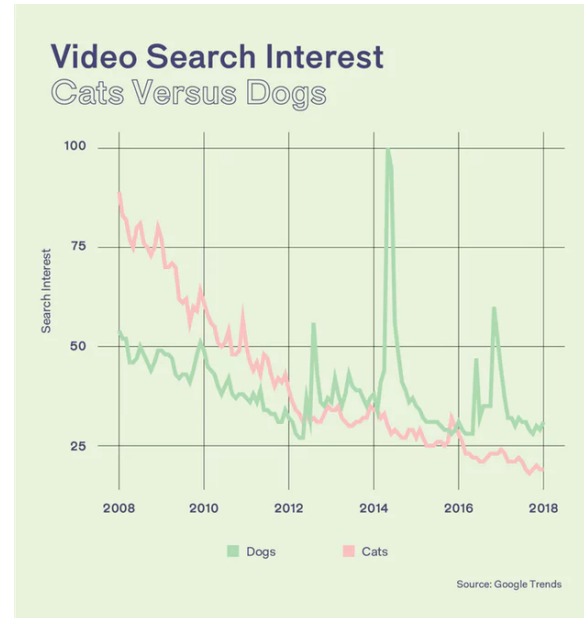
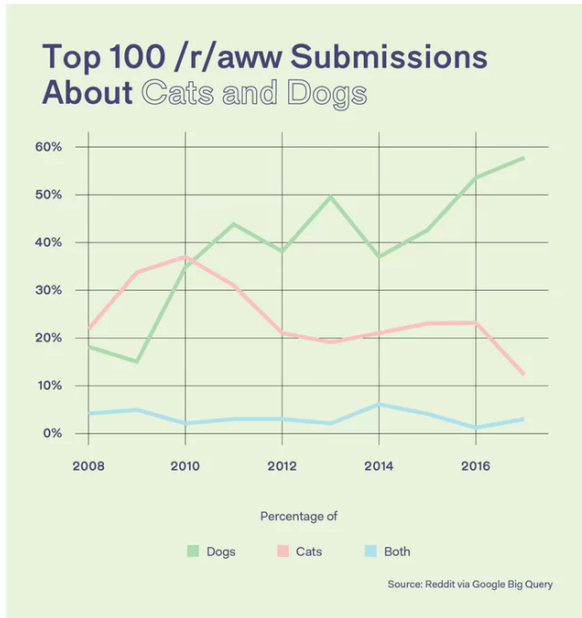
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Social media mentions of Cats vs. Dogs

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Twitter?



Write a program that sorts tweets into those containing “cat”, “dog”, or *other*

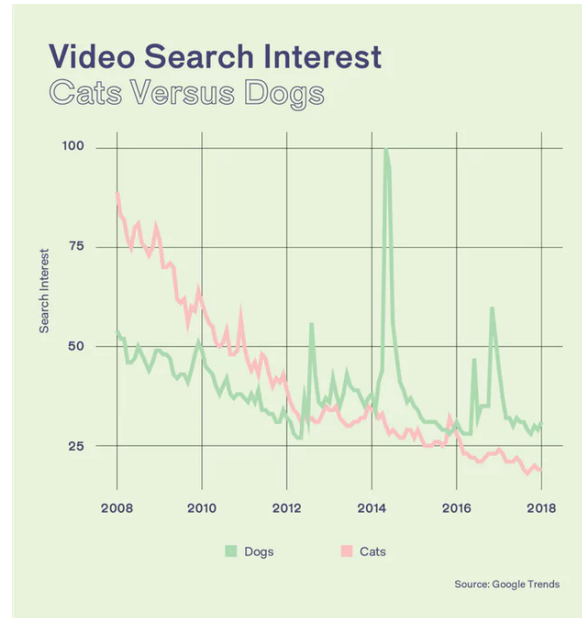
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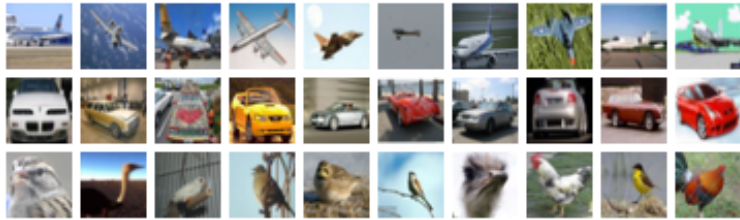
Twitter?

```
cats = []
dogs = []
other = []
for tweet in tweets:
    if "cat" in tweet:
        cats.append(tweet)
    elseif "dog" in tweet:
        dogs.append(tweet)
    else:
        other.append(tweet)
return cats, dogs, other
```

Write a program that sorts tweets into those containing **"cat"**, **"dog"**, or **other**

Machine learning algorithms

Write a program that sorts images into those containing “**birds**”, “**airplanes**”, or ***other***.



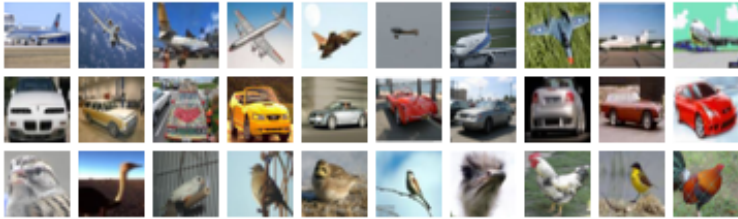
airplane

other

bird

Machine learning algorithms

Write a program that sorts images into those containing “**birds**”, “**airplanes**”, or **other**.



airplane

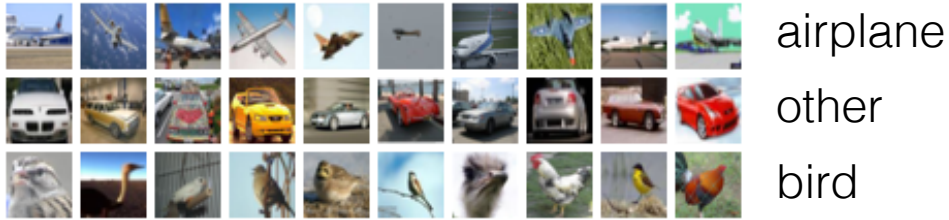
other

bird

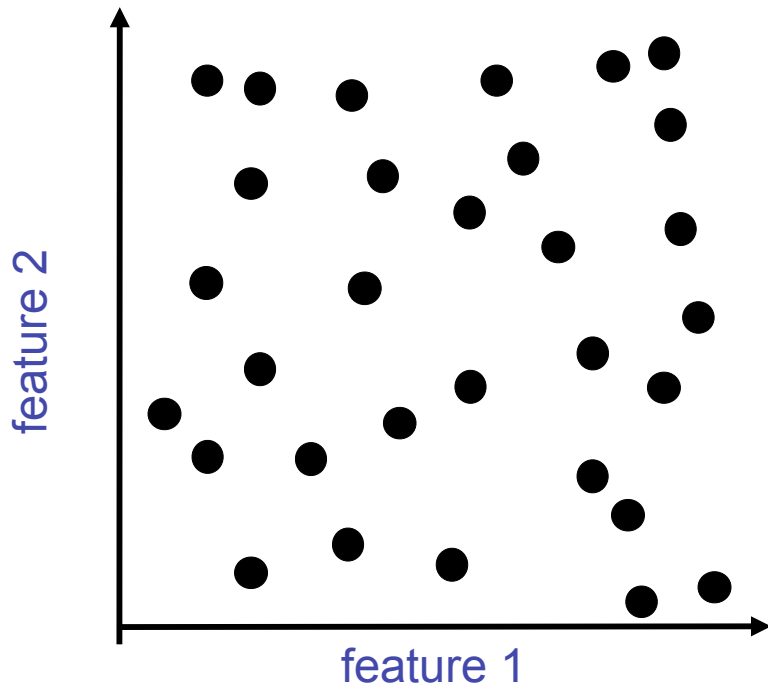
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return birds, planes, other
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Machine learning algorithms

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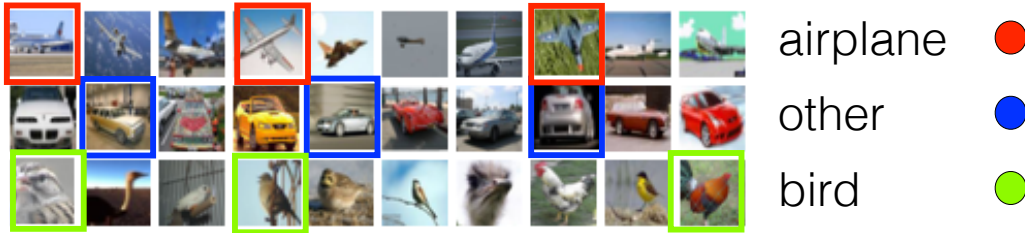


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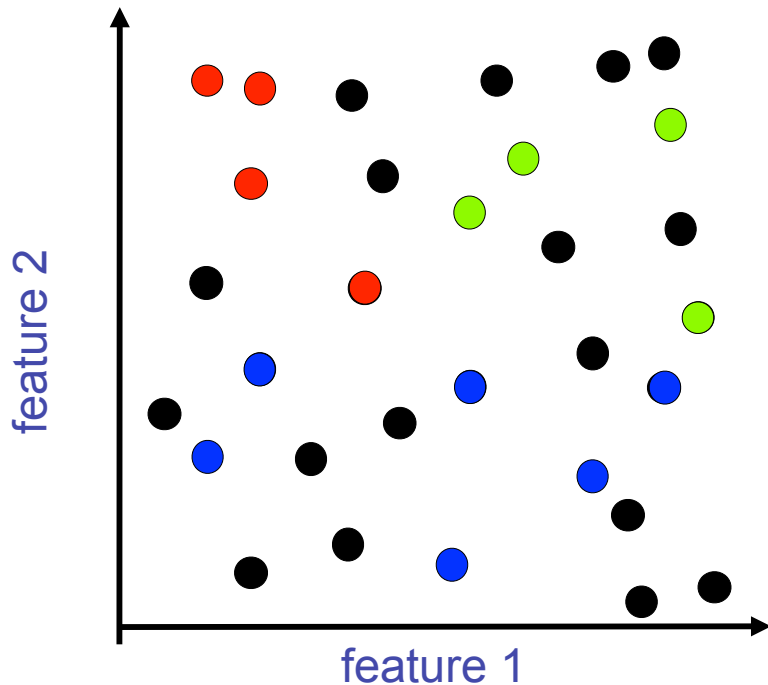


Machine learning algorithms

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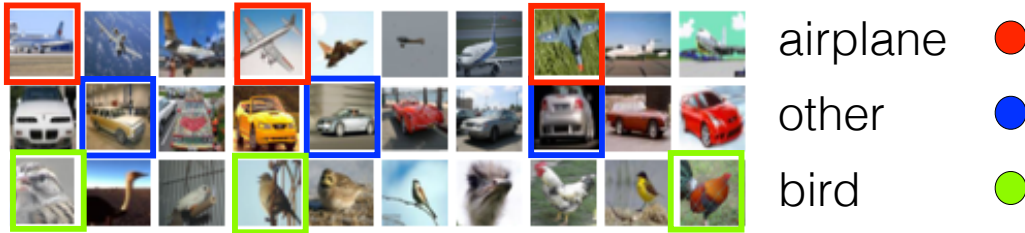


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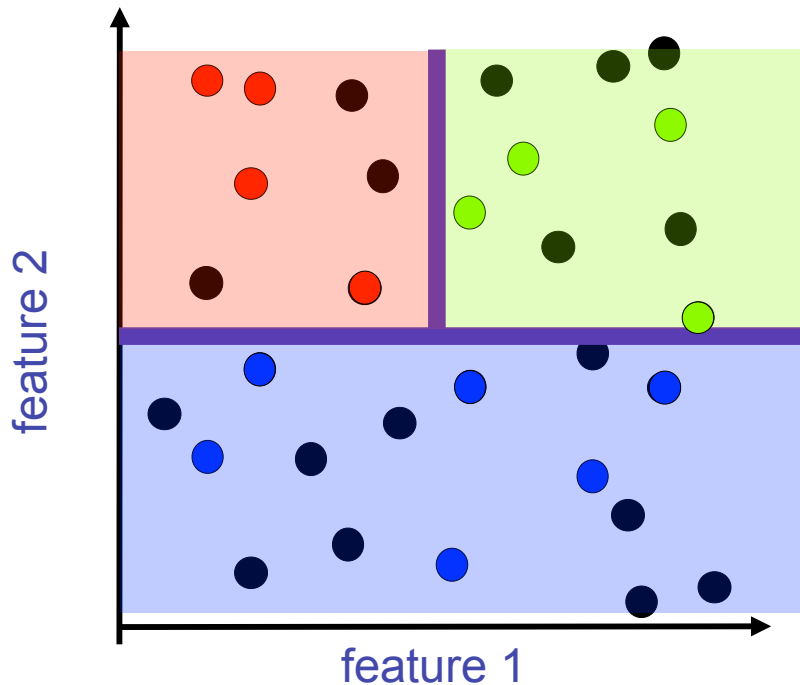


Machine learning algorithms

Write a program that sorts images into those containing “**birds**”, “**airplanes**”, or **other**.

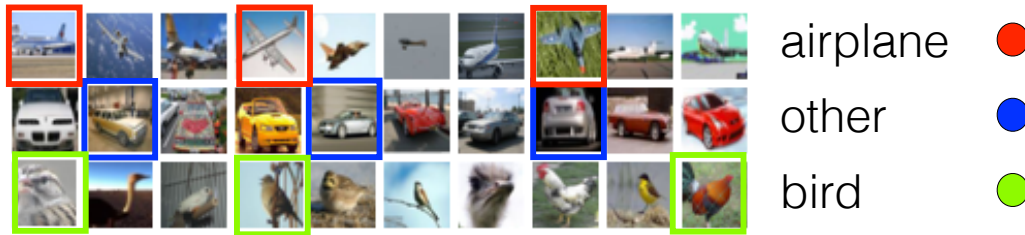


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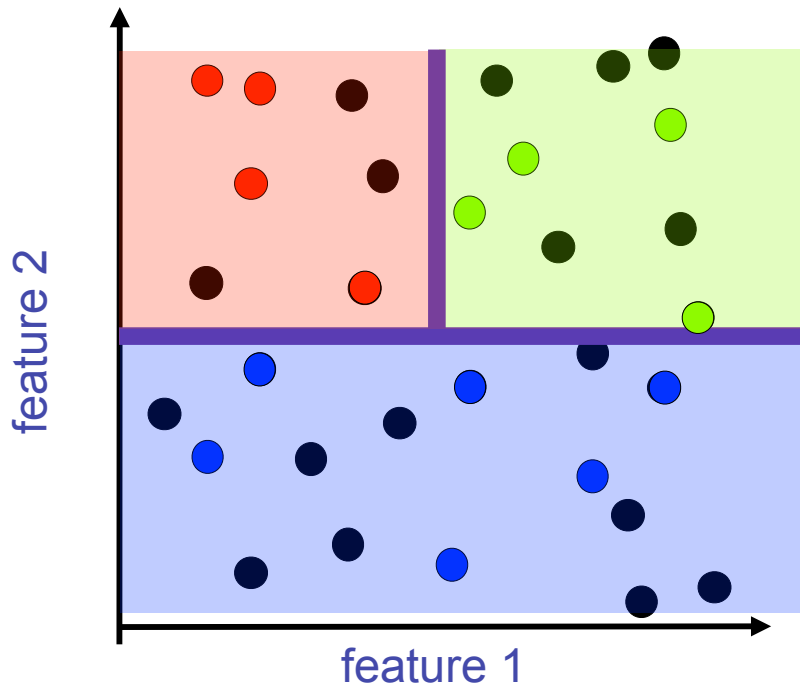


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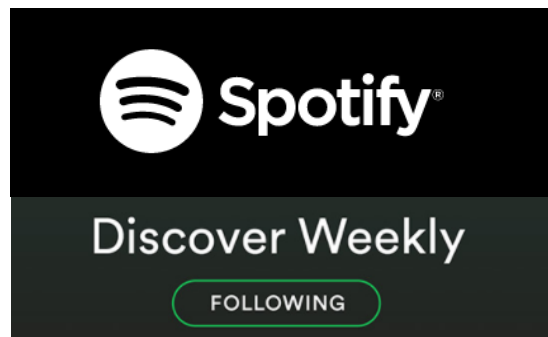


The decision rule of
if "cat" in tweet:
is **hard coded by expert.**

The decision rule of
if bird in image:
is **LEARNED using DATA**

Machine Learning Ingredients

- **Data:** past observations
- **Hypotheses/Models:** devised to capture the patterns in data
- **Prediction:** apply model to forecast future observations



You may also like...

ML uses past data to make personalized predictions



Mix of statistics (conceptual) and algorithms (programming)

CSEP546: Machine Learning

Instructor: [Kevin Jamieson](#) and [Simon Du](#)

Contact: cse446-staff@cs.washington.edu

Website: <https://courses.cs.washington.edu/courses/cse446/23sp/>

What this class is:

- **Fundamentals of ML:** bias/variance tradeoff, overfitting, optimization and computational tradeoffs, supervised learning (e.g., linear, boosting, deep learning), unsupervised models (e.g. k-means, EM, PCA)
- **Preparation for further learning:** the field is fast-moving, you will learn the foundations of ML to understand the latest results

What this class is not:

- **Survey course:** laundry list of algorithms, how to win Kaggle
- **An easy course:** familiarity with intro linear algebra and probability are assumed, homework will be time-consuming

Prerequisites

- Familiarity with:
 - Linear algebra
 - linear dependence, rank, linear equations
 - Multivariate calculus
 - Probability and statistics
 - Distributions, densities, marginalization, moments
 - Algorithms
 - Basic data structures, complexity
- Use HW0 to judge skills
- **See assigned reading and website for additional review materials!**

Grading

- 5 homeworks
 - Each contains both theoretical questions and will have programming
 - Collaboration okay. You must write, submit, and understand your answers and code (which we may run)
 - Do not Google for answers or ask chatGPT to do it.
- Midterm and Final

Grading: Your grade will be based on 5 homework assignments:

HW0 (8%), HW1 (13%), HW2 (13%), HW3 (13%), HW4 (13%).

There will be one midterm worth 20% and a final worth another 20%.

However, depending on whether you are enrolled in 446 or 546, the way the assignments are graded or curved varies (see below).

Communication Channels

- **Announcements, questions about class, homework help**
 - EdStem (<https://edstem.org/>)
 - “I think there is a typo in the homework?”
 - “What does this notation mean?”
 - “Is this an accurate description of how this works?”
- **Personal concerns** (cse446-staff@cs.washington.edu)
 - “Was in hospital...”, “laptop was stolen...”, etc.
- **Office hours**
 - “How do I get started on problem 2?”
 - “Am I on the right track?”
 - “I have this problem at work—can you point me in the right direction?”
- **Regrade requests**
 - Directly submit on Gradescope
- **Anonymous feedback** (<https://feedback.cs.washington.edu/>)
 - “Your real-world example X lacked nuance. I would like you to...”

Textbooks

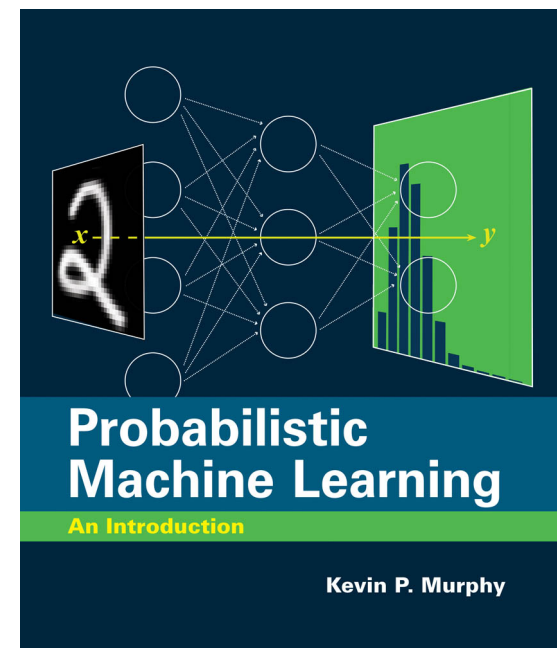
- Free PDF Textbook I will assign most reading from:

Probabilistic Machine Learning: An Introduction

Kevin Murphy

- PDF linked to on website, also in print

- So many more resources on the website!
- I may occasionally point you to other (free) readings



Homeworks

- HW 0 is out (**Due next Wednesday Midnight**)
 - Should be review (but being rusty is expected)
 - Work individually, treat as guide for what to brush up on
- HW 1,2,3,4
 - They are not easy or short. Start early.
- Submit to Gradescope
- Regrade requests on Gradescope
- **Late days: ~~2~~⁵ days total over the quarter**
- Assignments due at midnight, submit early and often (do not email me at 12:05)

- 1. All code must be written in Python**
- 2. All written work must be typeset (e.g., LaTeX)**

See course website for tutorials and references.

CSE 446 vs 546

Lecture	Lecture	Section	Homework	Grading
446	CSE2 G20 (Amazon Auditorium) MWF 9:30 -- 10:20am	Attend the section you are registered.	A problems only. No credit will be rewarded for completing B problems.	You will be graded (e.g., curved) against your peers in 446 only (on a 4.0 scale). For example, if you received a (curved) score of 0.9 on the A problems, then your full grade on your transcript will be $(4.0) \times (0.9) = 3.6$. Any attempt of the B problems will not influence your grade in any way.
546	CSE2 G20 (Amazon Auditorium) MWF 9:30 -- 10:20am	None	A and B problems.	You will be graded (e.g., curved) against your peers in 546 only. Your grade on the A and B problems will be curved separately, and then summed. For example, if you received a (curved) score of 0.9 on the A problems, and a (curved) score of 0.8 on the B problems, then your full grade on your transcript will be $(3.8) \times (0.9) + (0.2) \times (0.8) = 3.58$, rounded to 3.6. If only the A problems on the homework are attempted, the highest score attainable is a 3.8. If only the B problems are attempted, the highest score attainable is a 0.2.

Enjoy!

- ML is becoming ubiquitous in science, engineering and beyond
- This class should give you a basic foundation for understanding and applying ML

Probability review

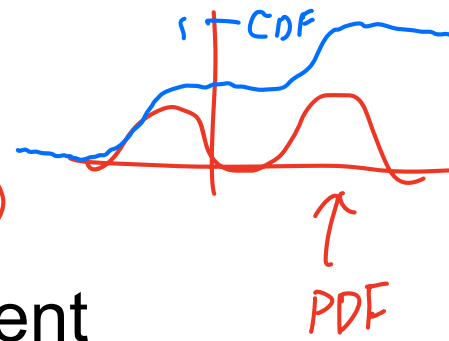
R.V. X

Definitions

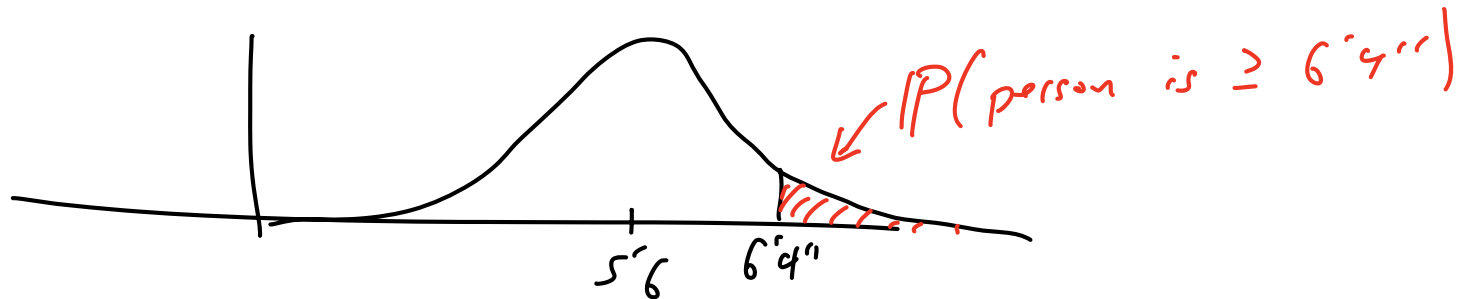
CDF $F(x) = IP(X \leq x)$

$U \sim \text{uniform}([0,1])$

$X \sim F^{-1}(U)$



- **Random Variable:** A variable that takes on different values determined randomly.
 - Example: The height of a person from the US.
- **Distribution:** The different values a random variable can take on along with the probability of that value.



- We talk about **sampling** from a distribution:
 - “Consider a sample of 100 different heights of people from the US drawn randomly from the distribution of all heights.”

Independence

Let X and Y be **random variables**

Ex. X is the outcome of the first roll of a 6-sided dice, Y is the outcome of the second roll of the dice

(X and Y take values in $\{1,2,3,4,5,6\}$ each with equal probability)

An **event** is statement about the world that holds or not:

Define events $A = \{X \in \{3,4\}\}$,

$B = \{X = 1\}$,

$C = \{Y \in \{3,4\}\}$

Every event is assigned a **probability**:

$$P(A) = P(X \in \{3,4\}) = \frac{1}{3}$$

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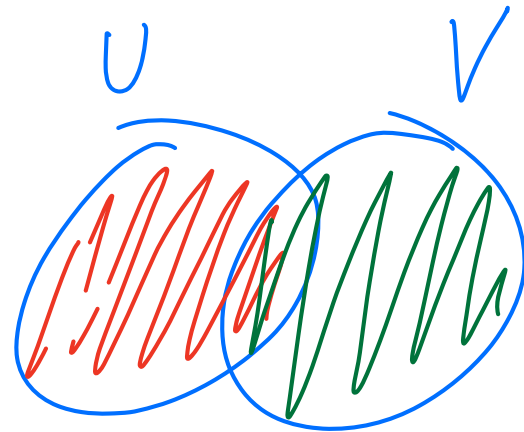
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For any events U, V we have $P(U \cup V) = \underbrace{P(U)} + \underbrace{P(V)} - P(U \cap V)$

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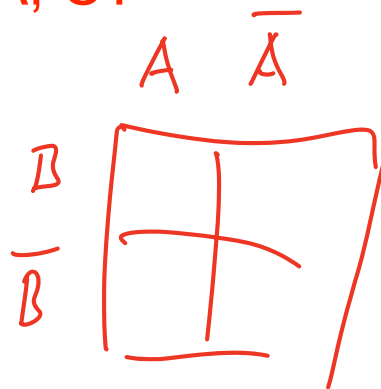
$C = \{Y \in \{3,4\}\}$

For say events U, V are **independent** if $P(U \cap V) = P(U)P(V)$

Are A, B independent? B, C ? A, C ?

$$A \cap B = \emptyset$$
$$P(A \cap B) = 0$$

$$P(A) = 1/3, \quad P(B) = 1/6$$



Independence

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Are A, B independent (no)? B, C (yes)? A, C (yes)?

Independence

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We define the **conditional probability** of event U given V as

$$P(U|V) = \frac{P(U \cap V)}{P(V)}$$

$$\text{What is } \underline{P(X \leq 4 | X \geq 3)}? = \frac{P(X \leq 4, X \geq 3)}{P(X \geq 3)} = \frac{P(X \in \{3,4\})}{2/3} = \frac{1/3}{2/3} = 1/2$$

Independence

Let X and Y be **random variables**

Ex. X is the outcome of the first roll of a 6-sided dice, Y is the outcome of the second roll of the dice

(X and Y take values in $\{1,2,3,4,5,6\}$ each with equal probability)

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We define the **conditional probability** of event U given V as

$$P(U|V) = \frac{P(U \cap V)}{P(V)}$$

Observe: if U, V are independent then $P(U|V) = P(U)$.

In words: if independent, V tells you nothing about U (and vice versa)

Mean, variance

Mean $\mathbb{E}[X], \mu$

The expected value of X , each value is weighted by the probability of seeing it.

$$\mathbb{E}[X] = \sum_x P(X = x)x$$

Variance $\text{Var}(X), \sigma^2$

The expected squared deviation of X from its mean.

$$\mathbb{E}[(X - \mathbb{E}[X])^2]$$

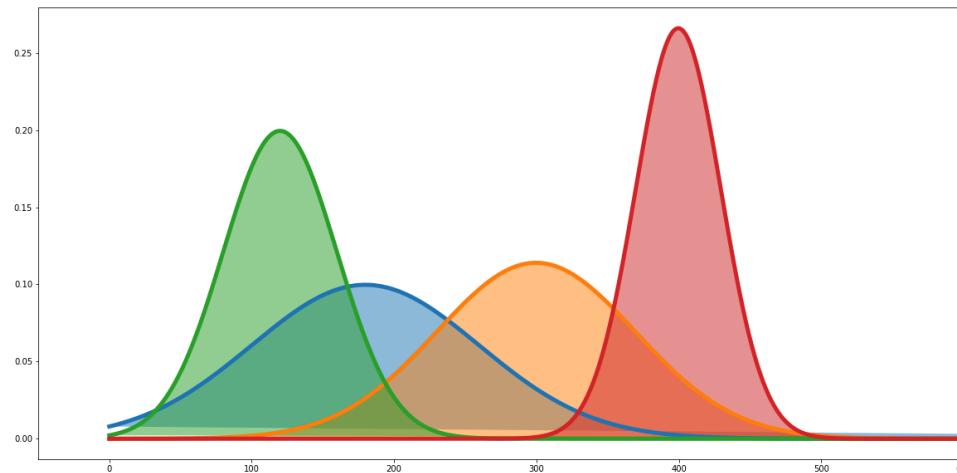
Median M

The value of X that is separating the higher half of its range from the lower half.

$$P(X \leq M) = .5$$

Mean, variance

The mean is a prediction of the value of the random variable. Answers the question “What do I expect the height of a random person to be?”



The variance captures the spread in your data. Also captures the error in the prediction using the mean. “How much do people’s heights deviate?”

$$\mathbb{E}[(X - \mathbb{E}[X])^2]$$

Maximum Likelihood Estimation

Your first consulting job

- *Billionaire*: I have special coin, if I flip it, what's the probability it will be heads?
- *You*: Please flip it a few times:

HHTHH

- *You*: The probability is: $\frac{3}{5}$
- *Billionaire*: Why?

Coin – Binomial Distribution

- **Data:** sequence $D = (HHTHT\dots)$, **k heads** out of **n flips**
- **Hypothesis:** $P(\text{Heads}) = \theta$, $P(\text{Tails}) = 1 - \theta$ $\theta \in [0, 1]$
 - Flips are i.i.d.:
 - Independent events
 - Identically distributed according to Binomial distribution

- $$P(D|\theta) = P(HHTHT|\theta)$$
$$= P(H|HTHT, \theta) P(HTHT|\theta)$$
$$= P(H|\theta) P(HTHT|\theta) = P(H|\theta)^k P(T|\theta)^{n-k}$$
$$= \theta^k (1 - \theta)^{n-k}$$

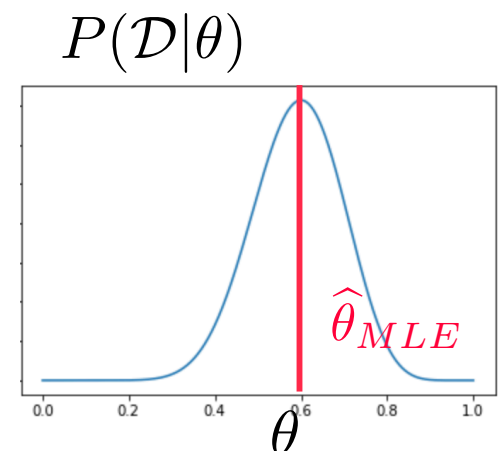
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- **Hypothesis:** $P(\text{Heads}) = \theta$, $P(\text{Tails}) = 1 - \theta$

$$\underline{P(\mathcal{D}|\theta) = \theta^k (1 - \theta)^{n-k}}$$

- Maximum likelihood estimation (MLE): Choose θ that maximizes the probability of observed data:

$$\begin{aligned}\hat{\theta}_{MLE} &= \arg \max_{\theta} P(\mathcal{D}|\theta) \\ &= \arg \max_{\theta} \log P(\mathcal{D}|\theta)\end{aligned}$$



Your first learning algorithm

$$\begin{aligned}\hat{\theta}_{MLE} &= \arg \max_{\theta} \log P(\mathcal{D}|\theta) \\ &= \arg \max_{\theta} \log \theta^k (1 - \theta)^{n-k}\end{aligned}$$

- Set derivative to zero:

$$\frac{d}{d\theta} \log P(\mathcal{D}|\theta) = 0$$

Maximum Likelihood Estimation

Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

Likelihood function $L_n(\theta) = \prod_{i=1}^n f(X_i; \theta)$

Log-Likelihood function $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^n \log(f(X_i; \theta))$

Maximum Likelihood Estimator (MLE) $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

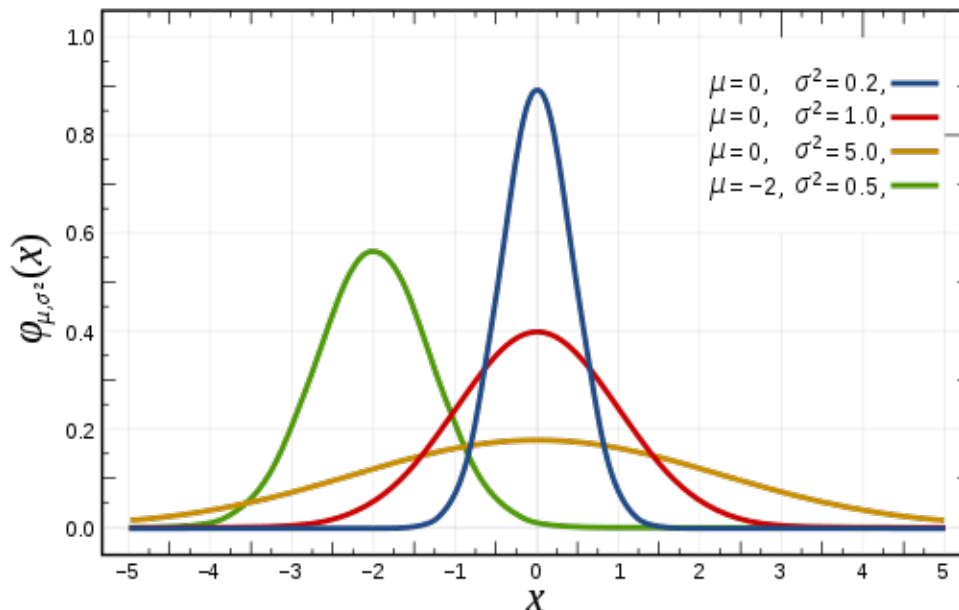
Recap

- Learning is...
 - Collect some data
 - E.g., coin flips
 - Choose a hypothesis class or model
 - E.g., binomial
 - Choose a loss function
 - E.g., data likelihood
 - Choose an optimization procedure
 - E.g., set derivative to zero to obtain MLE

What about continuous variables?

- *Billionaire*: What if I am measuring a **continuous variable**?
- *You*: Let me tell you about **Gaussians**...

$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Some properties of Gaussians

- affine transformation (multiplying by scalar and adding a constant)
 - $X \sim N(\mu, \sigma^2)$
 - $Y = aX + b \rightarrow Y \sim N(a\mu + b, a^2\sigma^2)$
- Sum of Gaussians
 - $X \sim N(\mu_X, \sigma_X^2)$
 - $Y \sim N(\mu_Y, \sigma_Y^2)$
 - $Z = X + Y \rightarrow Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$

MLE for Gaussian

- Prob. of i.i.d. samples $D=\{x_1, \dots, x_n\}$ (e.g., temperature):

$$\begin{aligned} P(\mathcal{D}|\mu, \sigma) &= P(x_1, \dots, x_n|\mu, \sigma) \\ &= \left(\frac{1}{\sigma\sqrt{2\pi}} \right)^n \prod_{i=1}^n e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} \end{aligned}$$

- Log-likelihood of data:

$$\log P(\mathcal{D}|\mu, \sigma) = -n \log(\sigma\sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2}$$

- What is $\hat{\theta}_{MLE}$ for $\theta = (\mu, \sigma^2)$? Draw a picture!

Your second learning algorithm: MLE for mean of a Gaussian

- What's MLE for mean?

$$\frac{d}{d\mu} \log P(\mathcal{D}|\mu, \sigma) = \frac{d}{d\mu} \left[-n \log(\sigma \sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

MLE for variance

- Again, set derivative to zero:

$$\frac{d}{d\sigma} \log P(\mathcal{D}|\mu, \sigma) = \frac{d}{d\sigma} \left[-n \log(\sigma \sqrt{2\pi}) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2} \right]$$

Learning Gaussian parameters

- MLE:

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{\mu}_{MLE})^2$$

- MLE for the variance of a Gaussian is **biased**

$$\mathbb{E}[\hat{\sigma}^2_{MLE}] \neq \sigma^2$$

- Unbiased variance estimator:

$$\hat{\sigma}^2_{unbiased} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu}_{MLE})^2$$

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Maximum Likelihood Estimator (MLE) $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

Under benign assumptions, as the number of observations $n \rightarrow \infty$ we have $\hat{\theta}_{MLE} \rightarrow \theta_*$

The MLE is a “recipe” that begins with a *model* for data $f(x; \theta)$

Applications preview



Maximum Likelihood Estimation

Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

Likelihood function $L_n(\theta) = \prod_{i=1}^n f(X_i; \theta)$

Log-Likelihood function $l_n(\theta) = \log(L_n(\theta)) = \sum_{i=1}^n \log(f(X_i; \theta))$

Maximum Likelihood Estimator (MLE) $\hat{\theta}_{MLE} = \arg \max_{\theta} L_n(\theta)$

Under benign assumptions, as the number of observations $n \rightarrow \infty$ we have $\hat{\theta}_{MLE} \rightarrow \theta_*$

Why is it useful to recover the “true” parameters θ_* of a probabilistic model?

- **Estimation** of the parameters θ_* is the goal
- Help **interpret** or summarize large datasets
- Make **predictions** about future data
- **Generate** new data $X \sim f(\cdot; \hat{\theta}_{MLE})$

Estimation

Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

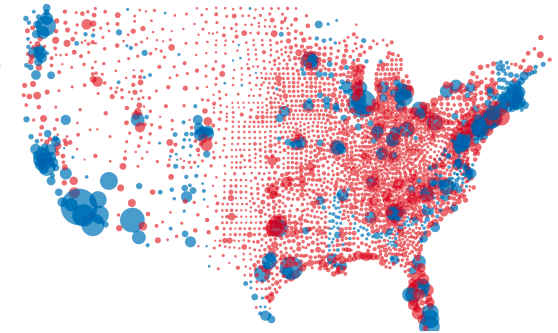
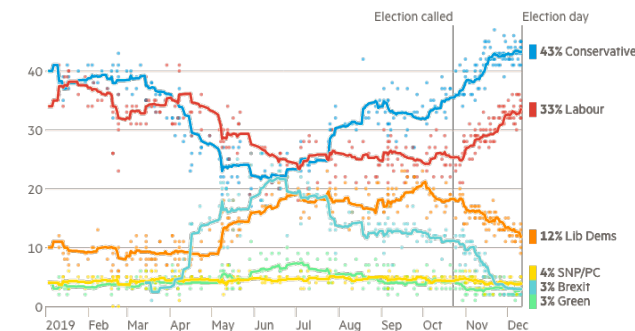
Opinion polls

How does the greater population feel about an issue?
Correct for over-sampling?

- θ_* is “true” average opinion
- X_1, X_2, \dots are sample calls

UK poll tracker

Lines represent weighted averages, points represent polls (%)



A/B testing

How do we figure out which ad results in more click-through?

- θ_* are the “true” average rates
- X_1, X_2, \dots are binary “clicks”

Save on prescription drugs - over \$3,637* a year!

Control

Explore Humana's Medicare plans

Treatment

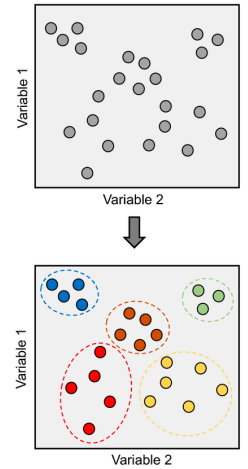
Interpret

Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

Customer segmentation / clustering

Can we identify distinct groups of customers by their behavior?

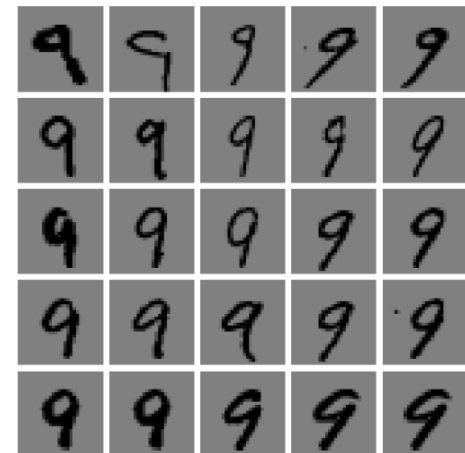
- θ_* describes “center” of distinct groups
- X_1, X_2, \dots are individual customers



Data exploration

What are the degrees of freedom of the dataset?

- θ_* describes the principle directions of variation
- X_1, X_2, \dots are the individual images



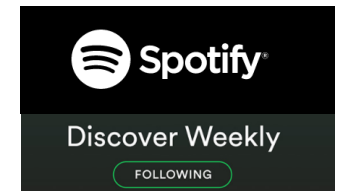
Predict

Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

Content recommendation

Can we predict how much someone will like a movie based on past ratings?

- θ_* describes user’s preferences
- X_1, X_2, \dots are (movie, rating) pairs



Object recognition / classification

Identify a flower given just its picture?

- θ_* describes the characteristics of each kind of flower
- X_1, X_2, \dots are the (image, label) pairs



(a)



(b)



(c)

Figure 1.1: Three types of Iris flowers: Setosa, Versicolor and Virginica. Used with kind permission of Dennis Krumb and SIGNA.

index	sl	sw	pl	pw	label
0	5.1	3.5	1.4	0.2	Setosa
1	4.9	3.0	1.4	0.2	Setosa
...					
50	7.0	3.2	4.7	1.4	Versicolor
...					
149	5.9	3.0	5.1	1.8	Virginica

Generate

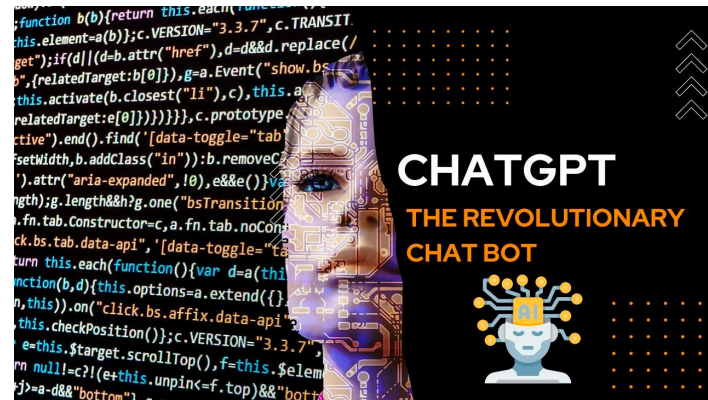
Observe X_1, X_2, \dots, X_n drawn IID from $f(x; \theta)$ for some “true” $\theta = \theta_*$

Text generation

Can AI generate text that could have been written like a human?

- θ_* describes language structure
- X_1, X_2, \dots are text snippets found online

“Kaia the dog wasn't a natural pick to go to mars.
No one could have predicted she would...”



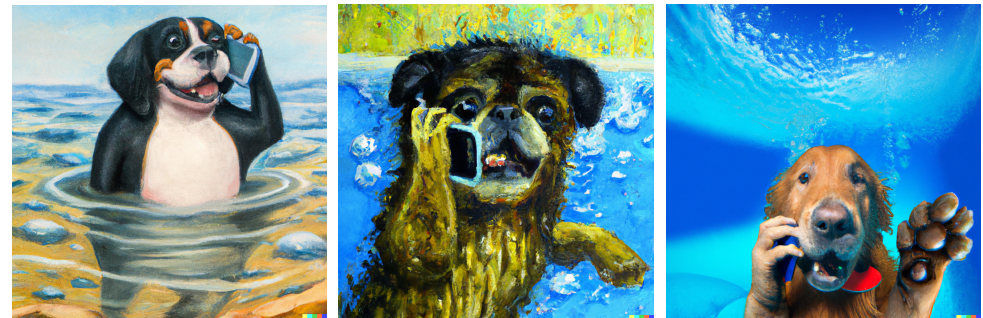
<https://chat.openai.com/chat>

Image to text generation

Can AI generate an image from a prompt?

- θ_* describes the coupled structure of images and text
- X_1, X_2, \dots are the (image, caption) pairs found online

“dog talking on cell phone under water, oil painting”



<https://labs.openai.com/>