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Adaboost

1: Initialize a vector of $n$ uniform weights $w_1$
2: for $t = 1, \ldots, T$
3: Train model $h_t$ on $X, y$ with weights $w_t$
4: Compute the weighted training error rate of $h_t$
5: Choose $\beta_t = \frac{1}{2} \ln \left( \frac{1-\epsilon_t}{\epsilon_t} \right)$
6: Update all instance weights:
   $$w_{t+1,i} = w_{t,i} \exp \left( -\beta_t y_i h_t(x_i) \right)$$
7: Normalize $w_{t+1}$ to be a distribution
8: end for
9: Return the hypothesis
   $$H(x) = \text{sign} \left( \sum_{t=1}^{T} \beta_t h_t(x) \right)$$

• Final model is a weighted combination of members
  – Each member weighted by its importance
AdaBoost

[Freund & Schapire, 1997]

**INPUT:** training data $X, y = \{(x_i, y_i)\}_{i=1}^n$, the number of iterations $T$

1: Initialize a vector of $n$ uniform weights $w_1 = [\frac{1}{n}, \ldots, \frac{1}{n}]$
2: for $t = 1, \ldots, T$
3: Train model $h_t$ on $X, y$ with instance weights $w_t$
4: Compute the weighted training error rate of $h_t$:
   \[ \epsilon_t = \sum_{i: y_i \neq h_t(x_i)} w_{t,i} \]
5: Choose $\beta_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$
6: Update all instance weights:
   \[ w_{t+1,i} = w_{t,i} \exp(-\beta_t y_i h_t(x_i)) \quad \forall i = 1, \ldots, n \]
7: Normalize $w_{t+1}$ to be a distribution:
   \[ w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^n w_{t+1,j}} \quad \forall i = 1, \ldots, n \]
8: end for
9: Return the hypothesis
   \[ H(x) = \text{sign} \left( \sum_{t=1}^T \beta_t h_t(x) \right) \]
AdaBoost

**INPUT:** training data $X, y = \{(x_i, y_i)\}_{i=1}^{n}$, the number of iterations $T$

1: Initialize a vector of $n$ uniform weights $w_1 = [\frac{1}{n}, \ldots, \frac{1}{n}]$

2: **for** $t = 1, \ldots, T$

3: Train model $h_t$ on $X, y$ with instance weights $w_t$

4: Compute the weighted training error rate of $h_t$:
   \[ \epsilon_t = \sum_{i : y_i \neq h_t(x_i)} w_{t,i} \]

5: Choose $\beta_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$

6: Update all instance weights:
   \[ w_{t+1,i} = w_{t,i} \exp (-\beta_t y_i h_t(x_i)) \quad \forall i = 1, \ldots, n \]

7: Normalize $w_{t+1}$ to be a distribution:
   \[ w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n \]

8: **end for**

9: **Return** the hypothesis
   \[ H(x) = \text{sign} \left( \sum_{t=1}^{T} \beta_t h_t(x) \right) \]

$w_t$ is a vector of weights over the instances at iteration $t$

All points start with equal weight
**AdaBoost**

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\epsilon_t = \sum_{i: y_i \neq h_t(x_i)} w_{t,i}
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H(x) = \text{sign} \left( \sum_{t=1}^{T} \beta_t h_t(x) \right)
$$

We weight instances differently when learning the model, either in the cost function or by bootstrap replication.
Base Learner Requirements

• AdaBoost works best with “weak” learners
  – Should not be complex
  – Typically high bias classifiers
  – Works even when weak learner has an error rate just slightly under 0.5 (i.e., just slightly better than random)
    • Can prove training error goes to 0 in O(log n) iterations

• Examples:
  – Decision stumps (1 level decision trees)
  – Depth-limited decision trees
  – Linear classifiers
AdaBoost

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2: for $t = 1, \ldots, T$

3: Train model $h_t$ on $X, y$ with instance weights $w_t$

4: Compute the weighted training error rate of $h_t$:

$$
\epsilon_t = \sum_{i: y_i \neq h_t(x_i)} w_{t,i}
$$

5: Choose $\beta_t = \frac{1}{2} \ln \left( \frac{1-\epsilon_t}{\epsilon_t} \right)$

6: Update all instance weights:

$$
w_{t+1,i} = w_{t,i} \exp \left( -\beta_t y_i h_t(x_i) \right) \quad \forall i = 1, \ldots, n
$$

7: Normalize $w_{t+1}$ to be a distribution:

$$
w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^n w_{t+1,j}} \quad \forall i = 1, \ldots, n
$$

8: end for

9: Return the hypothesis

$$
H(x) = \text{sign} \left( \sum_{t=1}^T \beta_t h_t(x) \right)
$$

Error is the sum the weights of all misclassified instances
AdaBoost

**INPUT:** training data $X, y = \{ (x_i, y_i) \}_{i=1}^n$, the number of iterations $T$

1. Initialize a vector of $n$ uniform weights $w_1 = [\frac{1}{n}, \ldots, \frac{1}{n}]$

2. for $t = 1, \ldots, T$

3. Train model $h_t$ on $X, y$ with instance weights $w_t$

4. Compute the weighted training error rate of $h_t$:
   \[ \epsilon_t = \sum_{i : y_i \neq h_t(x_i)} w_{t,i} \]

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6. Update all instance weights:
   \[ w_{t+1,i} = w_{t,i} \exp (-\beta_t y_i h_t(x_i)) \]

7. Normalize $w_{t+1}$ to be a distribution:
   \[ w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^n w_{t+1,j}} \quad \forall i \]

8. end for

9. Return the hypothesis
   \[ H(x) = \text{sign} \left( \sum_{t=1}^T \beta_t h_t(x) \right) \]

- $\beta_t$ measures the importance of $h_t$
- If $\epsilon_t \leq 0.5$, then $\beta_t \geq 0$
  - Trivial, otherwise flip $h_t$’s predictions
- $\beta_t$ grows as $h_t$’s error shrinks
AdaBoost

INPUT: training data \((X, y)\), the number of iterations \(T\)

1: Initialize a vector of uniform weights
2: for \(t = 1, \ldots, T\) do
3: Train model with current \(w\)
4: Compute the weighted training error rate of \(h_t\)
5: Choose \(\beta_t = \frac{2\ln(1/\epsilon_t)}{\epsilon_t}\)
6: Update all instance weights:
   
   \[
   w_{t+1,i} = w_{t,i} \exp(-\beta_t y_i h_t(x_i)) \quad \forall i = 1, \ldots, n
   \]
7: Normalize \(w_{t+1}\) to be a distribution:
   
   \[
   w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^n w_{t+1,j}} \quad \forall i = 1, \ldots, n
   \]
8: end for
9: Return the hypothesis
   
   \[
   H(x) = \text{sign}\left(\sum_{t=1}^T \beta_t h_t(x)\right)
   \]

This is the same as:

\[
\begin{aligned}
  w_{t+1,i} &= w_{t,i} \times \\
  &\begin{cases}
    e^{-\beta_t} & \text{if } h_t(x_i) = y_i \\
    e^{\beta_t} & \text{if } h_t(x_i) \neq y_i
  \end{cases}
\end{aligned}
\]

will be \(\leq 1\)

will be \(\geq 1\)

Essentially this emphasizes misclassified instances.
**Adaboost**

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2: for $t = 1, \ldots, T$

3: Train model $h_t$ on $X, y$ with instance weights $w_t$

4: Compute the weighted training error rate of $h_t$:

$$\epsilon_t = \sum_{i : y_i \neq h_t(x_i)} w_{t,i}$$

5: Choose $\beta_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$

6: Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp \left( -\beta_t y_i h_t(x_i) \right) \quad \forall i = 1, \ldots, n$$

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$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^n w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

8: end for

9: **Return** the hypothesis

$$H(x) = \text{sign} \left( \sum_{t=1}^T \beta_t h_t(x) \right)$$
Adaboost

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H(x) = \text{sign} \left( \sum_{t=1}^T \beta_t h_t(x) \right)
$$

Member classifiers with less error are given more weight in the final ensemble hypothesis

Final prediction is a weighted combination of each member’s prediction
Dynamic Behavior of AdaBoost

• If a point is repeatedly misclassified...
  – Each time, its weight is increased
  – Eventually it will be emphasized enough to generate a hypothesis that correctly predicts it

• Successive member hypotheses focus on the hardest parts of the instance space
  – Instances with highest weight are often outliers
AdaBoost and Overfitting

• VC Theory predicts that AdaBoost will overfit as the number of weak classifiers $T$ grows large
  – Hypothesis keeps growing more complex

• In practice, AdaBoost often does not overfit, performing significantly better than VC theory suggests

• AdaBoost does not explicitly regularize the model, and yet generalizes well
Explaining Why AdaBoost Works

- Empirically, boosting resists overfitting
- Note that it continues to drive down the test error even AFTER the training error reaches zero
- Boosting maximizing confidence
AdaBoost in Practice

Strengths:
• Fast and simple to program
• No parameters to tune (besides T)
• No assumptions on weak learner

When boosting can fail:
• Given insufficient data
• Overly complex weak hypotheses
• Can be susceptible to noise
• When there are a large number of outliers
Boosted Decision Trees

- Boosted decision trees are one of the best “off-the-shelf” classifiers – i.e., no parameter tuning
- Limit member hypothesis complexity by limiting tree depth
- Boosting methods are typically used with trees in practice

“AdaBoost with trees is the best off-the-shelf classifier in the world” - Breiman, 1996
(Also, see results by Caruana & Niculescu-Mizil, ICML 2006)