Machine Learning (CSE 446): Decision Trees

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Announcements

- Piazza: If you did not get an email, send us a message.
  Announcements will be made there, so make sure you can see them posted.
- Gradescope:
  - USE YOUR UW ID NUMBER WHEN YOU SIGN UP.
    we need to match your assignments to you.
  - The course code is: 9JKZ4G.
- HW0 posted/Turn in Certification file that you read the website.
- TA office hours posted.
  (Please check website before you go, just in case of changes.)
- Midterm date: Mon, Feb 11.
- Qz section this week.
- Today: an example, Decision Trees

The "i.i.d." Supervised Learning Setup

- Let ℓ be a **loss function**; $\ell(y, \hat{y})$ is our loss by predicting $\hat{y}$ when y is the correct output.
- Let $\mathcal{D}(x, y)$ define the (unknown) underlying probability of input/output pair (x, y), in "nature."
- The training data $D = \langle (x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N) \rangle$ are assumed to be **identical, independently, distributed** (i.i.d.) samples from $\mathcal{D}$.
- We care about our **expected error (i.e. the expected loss, the "true" expected loss, ...)** with regards to the underlying distribution $\mathcal{D}$.
- Goal: find a **hypothesis** which as has "low" expected error, using the training set.

The loss

- Fix a classifier f on (x, y), the "$0/1$ loss" is:

$$\mathbf{1}\{y \neq f(x)\}$$

- Classifier f's **true** expected loss:

$$\epsilon(f) = \sum_{(x,y)} \mathcal{D}(x, y)\mathbf{1}\{y \neq f(x)\} = \mathbb{E}[\mathbf{1}\{y \neq f(x)\}]$$

- Classifier f's average loss on **training data**:

$$\widehat{\epsilon}(f) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{1}\{y_i \neq f(x_i)\}$$

A Toy Data Set

Data derived from `https://archive.ics.uci.edu/ml/datasets/Auto+MPG`

mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
18.0	8	307.0	130.0	3504.	12.0	70	1
15.0	8	350.0	165.0	3693.	11.5	70	1
18.0	8	318.0	150.0	3436.	11.0	70	1
16.0	8	304.0	150.0	3433.	12.0	70	1
17.0	8	302.0	140.0	3449.	10.5	70	1
15.0	8	429.0	198.0	4341.	10.0	70	1
14.0	8	454.0	220.0	4354.	9.0	70	1
14.0	8	440.0	215.0	4312.	8.5	70	1
14.0	8	455.0	225.0	4425.	10.0	70	1
15.0	8	390.0	190.0	3850.	8.5	70	1
15.0	8	383.0	170.0	3563.	10.0	70	1
14.0	8	340.0	160.0	3609.	8.0	70	1
15.0	8	400.0	150.0	3761.	9.5	70	1
14.0	8	455.0	225.0	3086.	10.0	70	1
24.0	4	113.0	95.00	2372.	15.0	70	3
22.0	6	198.0	95.00	2833.	15.5	70	1
18.0	6	199.0	97.00	2774.	15.5	70	1
21.0	6	200.0	85.00	2587.	16.0	70	1
27.0	4	97.00	88.00	2130.	14.5	70	3
26.0	4	97.00	46.00	1835.	20.5	70	2
25.0	4	110.0	87.00	2672.	17.5	70	2
24.0	4	107.0	90.00	2430.	14.5	70	2

Input: a row in this table; "features" are columns.

Goal: predict whether mpg is < 23 ("bad" $= 0$) or above ("good" $= 1$) given other attributes (other columns).

201 "good" and 197 "bad"; guessing the most frequent class (good) will get 50.5% accuracy.

Let's build a classifier!

- Let's just try to build a classifier "by hand".
  (This is our first learning algorithm.)
- For now, let's ignore the true loss/trying to "generalize"
- Let's start with just looking at a simple classifier.
  What is a simple classification rule?
- Conceptual point: Let $\Phi(x) = (\phi_1(x), \phi_2(x), \ldots \phi_d(x))$ be a function that maps from inputs x to a vector of values. Each component function $\phi(x) = \phi_i(x)$ could be:
  - If ϕ maps to $\{0, 1\}$, we call it a "binary feature (function)."
  - If ϕ maps to $\mathbb{R}$, we call it a "real-valued feature (function)."
  - ϕ could map to categorical values, integers, ...

Sometimes we write $\Phi(x)$ to refer to a vector of features of x.

Contingency Table

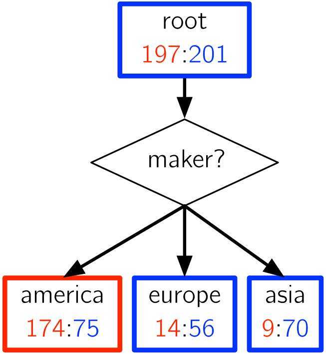
values of y	values of feature ϕ			
	v_1	v_2	$\cdots$	v_K
0				
1				

Decision Stump Example

y	maker		
	america	europe	asia
0	174	14	9
1	75	56	70
	↓	↓	↓
	0	1	1

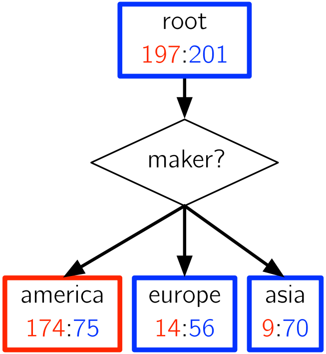
Decision Stump Example

y	maker		
	america	europe	asia
0	174	14	9
1	75	56	70
	$\downarrow$	$\downarrow$	$\downarrow$
	0	1	1



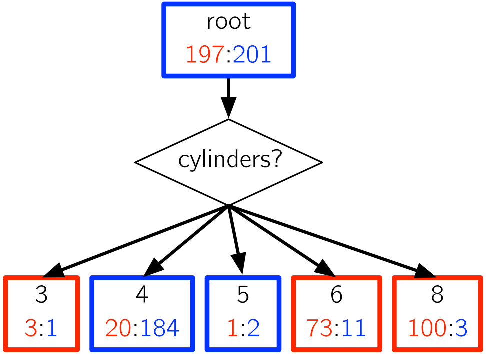
Decision Stump Example

y	maker		
	america	europe	asia
0	174	14	9
1	75	56	70
	$\downarrow$ 0	$\downarrow$ 1	$\downarrow$ 1

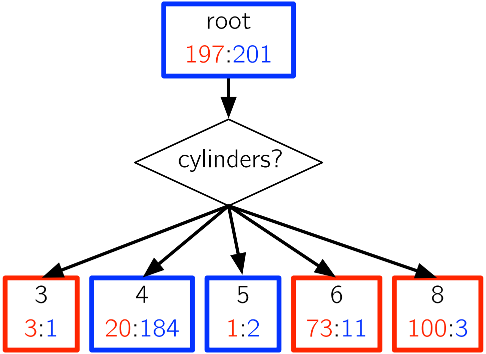


Errors: $75 + 14 + 9 = 98$ (about 25%)

Decision Stump Example



Decision Stump Example



Errors: $1 + 20 + 1 + 11 + 3 = 36$ (about 9%)

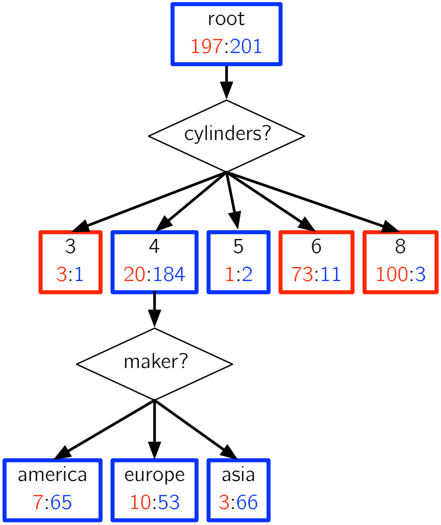
Key Idea: Recursion

A single feature **partitions** the data.

For each partition, we could choose another feature and partition further.

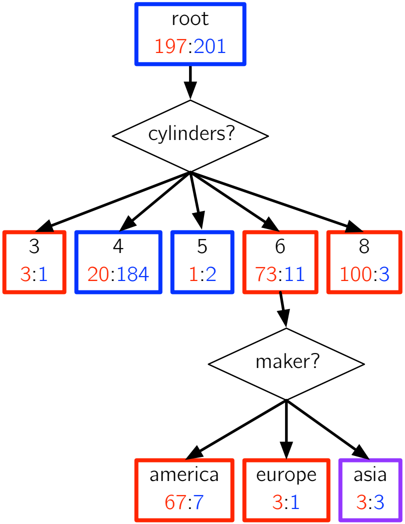
Applying this recursively, we can construct a **decision tree**.

Decision Tree Example



Error reduction compared to the cylinders stump?

Decision Tree Example



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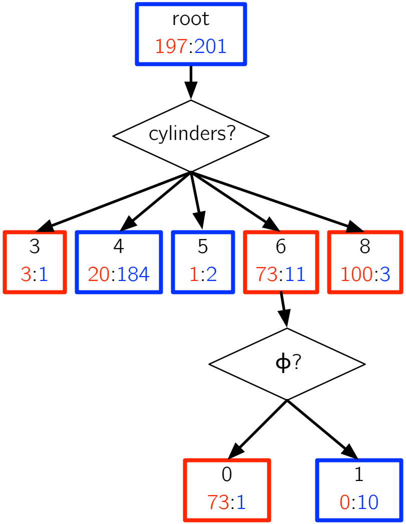
Decision Tree Example



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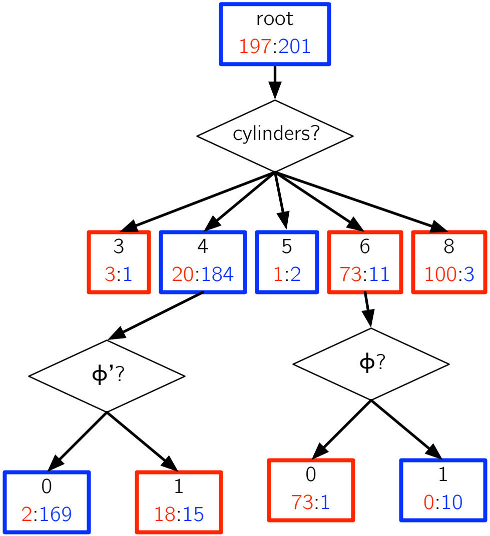
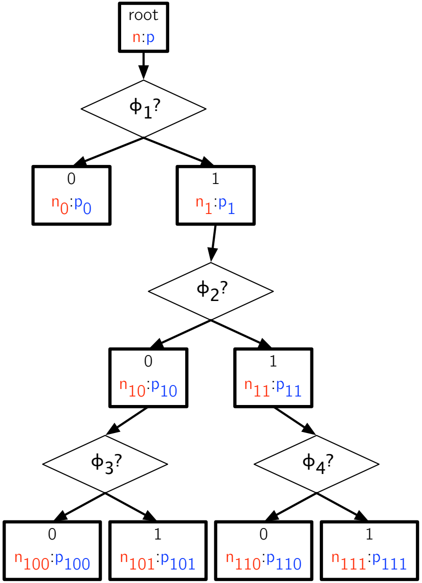
Decision Tree Example

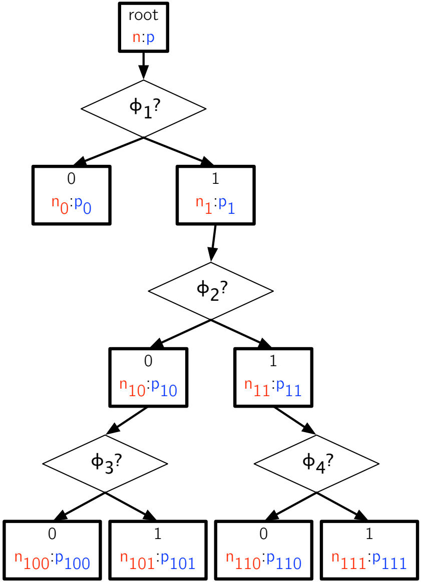


Error reduction compared to the cylinders stump?

Decision Tree: Making a Prediction



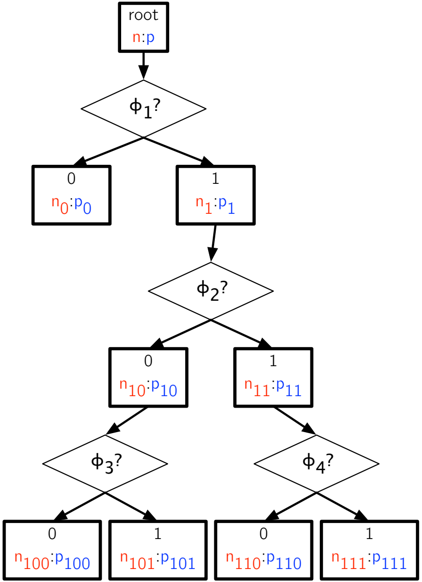
Decision Tree: Making a Prediction



Data: decision tree t, input example x
Result: predicted class
if t *has the form* $\textsc{Leaf}(y)$ **then**
  return y;
else
  $\#$ $t.\phi$ is the feature associated with t;
  $\#$ t.child(v) is the subtree for value v;
  return $\textsc{DTreeTest}(t.\text{child}(t.\phi(x)), x)$;
end

Algorithm 1: $\textsc{DTreeTest}$

Decision Tree: Making a Prediction



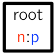
Equivalent boolean formulas:

$$(\phi_1 = 0) \Rightarrow [\![n_0 < p_0]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 0) \Rightarrow [\![n_{100} < p_{100}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 1) \Rightarrow [\![n_{101} < p_{101}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 0) \Rightarrow [\![n_{110} < p_{110}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 1) \Rightarrow [\![n_{111} < p_{111}]\!]$$

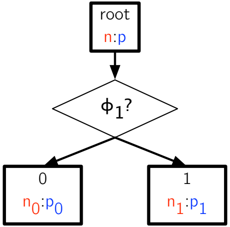
Tangent: How Many Formulas?

- Assume we have D binary features.
- Each feature could be set to 0, or set to 1, or excluded (wildcard/don't care).
- 3^D formulas.

Building a Decision Tree

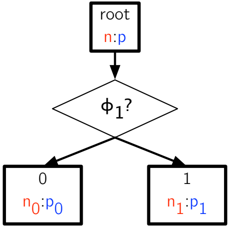


Building a Decision Tree



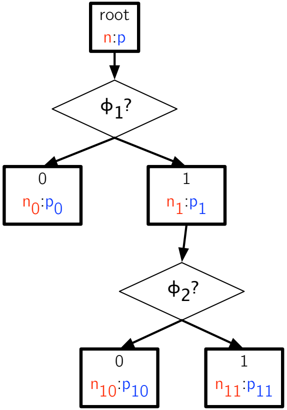
We chose feature ϕ_1. Note that $n = n_0 + n_1$ and $p = p_0 + p_1$.

Building a Decision Tree

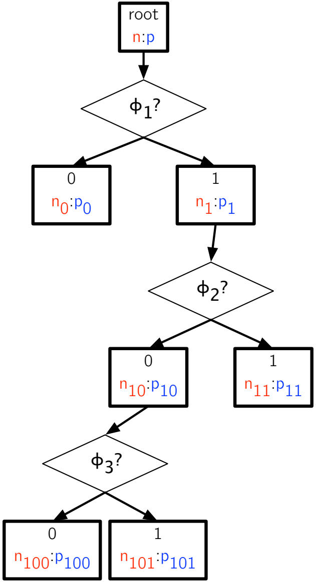


We chose not to split the left partition. Why not?

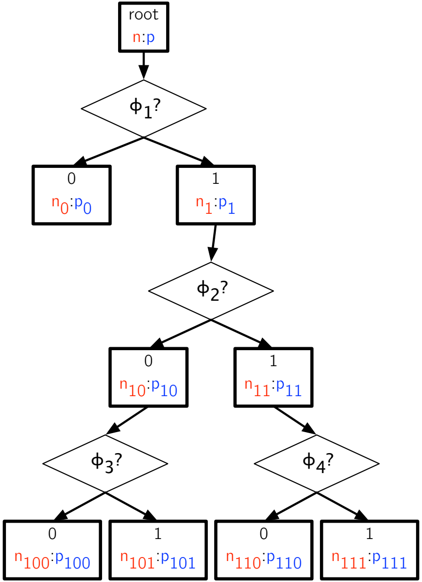
Building a Decision Tree



Building a Decision Tree



Building a Decision Tree



Greedily Building a Decision Tree (Binary Features)

Data: data D, feature set Φ

Result: decision tree

if *all examples in* D *have the same label* y*, or* Φ *is empty and* y *is the best guess* **then**

  return $\textsc{Leaf}(y)$;

else

  for *each feature* ϕ *in* Φ **do**

    partition D into D_0 and D_1 based on ϕ-values;

    let mistakes(ϕ) $=$ (non-majority answers in D_0) $+$ (non-majority answers in D_1);

  end

  let ϕ^* be the feature with the smallest number of mistakes;

  return $\textsc{Node}(\phi^*, \{0 \rightarrow \mathrm{DTreeTrain}(D_0, \Phi \setminus \{\phi^*\}), 1 \rightarrow \mathrm{DTreeTrain}(D_1, \Phi \setminus \{\phi^*\})\})$;

end

Algorithm 2: $\mathrm{DTreeTrain}$

What could go wrong?

- Next class!