Machine Learning (CSE 446): Neural Networks

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Classifiers We've Covered So Far

	decision boundary?	difficult part of learning?
decision trees	piecewise-axis-aligned	greedy split decisions
K-nearest neighbors	possibly very complex	indexing training data
perceptron	linear	iterative optimization method required
logistic regression	linear	iterative optimization method required
naïve Bayes	linear (see A4)	none

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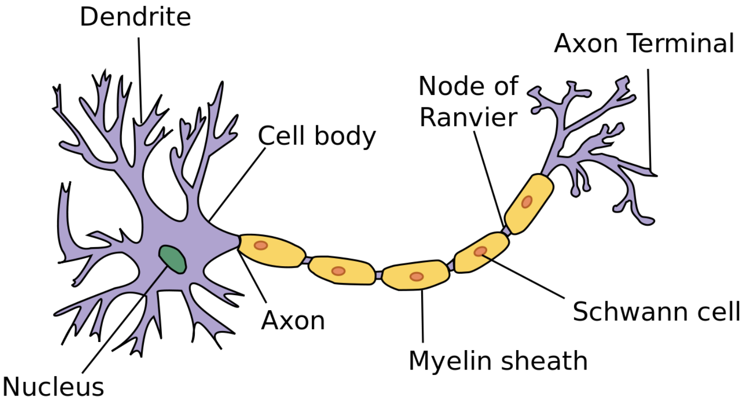
The next methods we'll cover permit **nonlinear** decision boundaries.

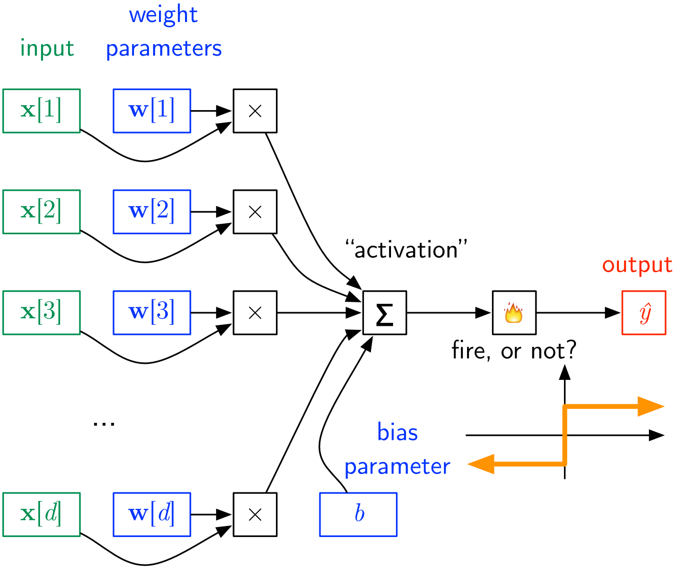
Inspiration from Neurons

Image from Wikimedia Commons.



Input signals come in through dendrites, output signal passes out through the axon.

Neuron-Inspired Classifiers



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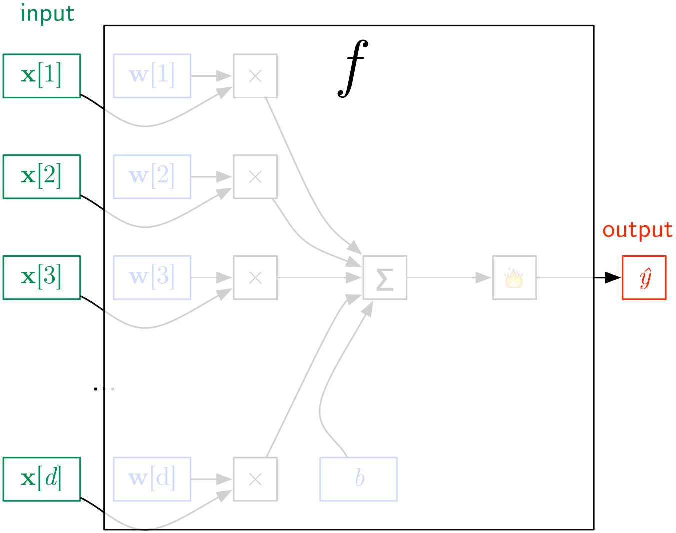

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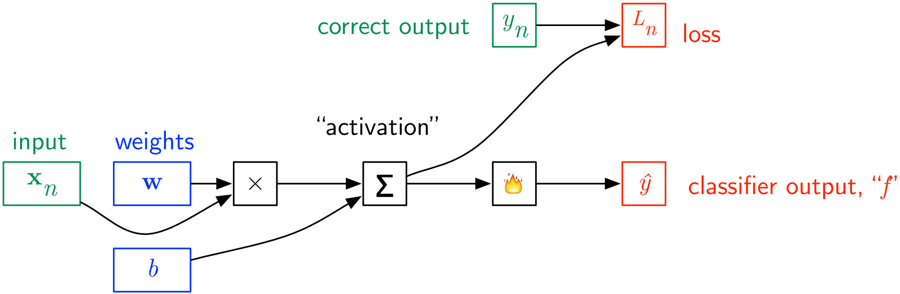

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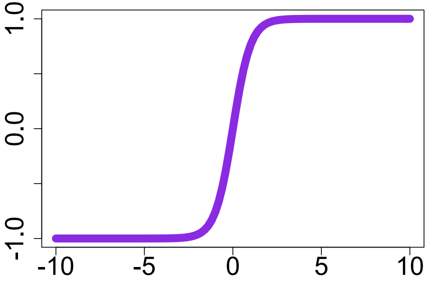

Hyperbolic tangent function, $\tanh(z) = \dfrac{e^z - e^{-z}}{e^z + e^{-z}}$.

Generalization: apply elementwise to a vector, so that $\tanh : \mathbb{R}^k \to (-1, 1)^k$.

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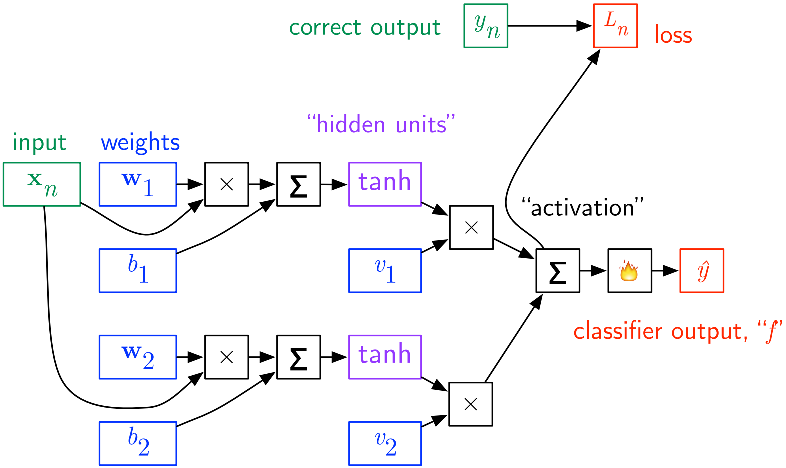

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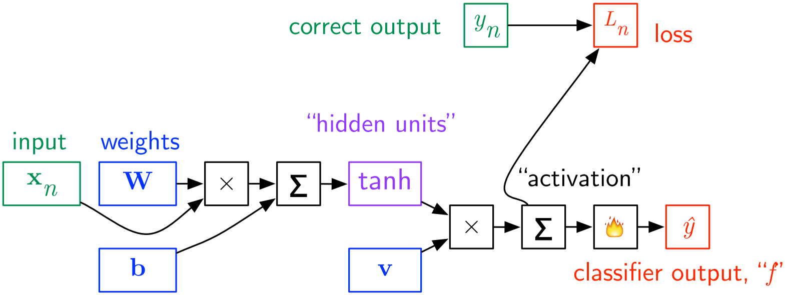

Two-Layer Neural Network

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\begin{aligned}
f(\mathbf{x}) &= \mathsf{sign}\left(\sum_{h=1}^{H} v_h \cdot \tanh\left(\mathbf{w}_h \cdot \mathbf{x} + b_h\right)\right) \\
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- The number of hidden units affects how complicated your decision boundary can be and how easily you will overfit.

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- Special name for (S)GD with chain rule invocations: **backpropagation**.