Machine Learning (CSE 446): Decision Trees

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Announcements

- First assignment posted. Due Thurs, Jan 18th.
  Remember the late policy (see the website).
- TA office hours posted.
  (Please check website before you go, just in case of changes.)
- Midterm: Weds, Feb 7.
- Today: Decision Trees, the supervised learning

Features (a conceptual point)

Let ϕ be (one such) function that maps from inputs x to values. There could be many such functions, sometimes we write $\Phi(x)$ for the feature "vector" (it's really a "tuple").

- If ϕ maps to $\{0, 1\}$, we call it a "binary feature (function)."
- If ϕ maps to $\mathbb{R}$, we call it a "real-valued feature (function)."
- ϕ could map to categorical values.
- ordinal values, integers, ...

Often, there isn't much of a difference between x and the tuple of features.

Features

categorical

Data derived from `https://archive.ics.uci.edu/ml/datasets/Auto+MPG`

mpg; cylinders; displacement; horsepower; weight; acceleration; year; origin

18.0	8	307.0	130.0	3504.	12.0	70	1
15.0	8	350.0	165.0	3693.	11.5	70	1
18.0	8	318.0	150.0	3436.	11.0	70	1
16.0	8	304.0	150.0	3433.	12.0	70	1
17.0	8	302.0	140.0	3449.	10.5	70	1
15.0	8	429.0	198.0	4341.	10.0	70	1
14.0	8	454.0	220.0	4354.	9.0	70	1
14.0	8	440.0	215.0	4312.	8.5	70	1
14.0	8	455.0	225.0	4425.	10.0	70	1
15.0	8	390.0	190.0	3850.	8.5	70	1
15.0	8	383.0	170.0	3563.	10.0	70	1
14.0	8	340.0	160.0	3609.	8.0	70	1
15.0	8	400.0	150.0	3761.	9.5	70	1
14.0	8	455.0	225.0	3086.	10.0	70	1
24.0	4	113.0	95.00	2372.	15.0	70	3
22.0	6	198.0	95.00	2833.	15.5	70	1
18.0	6	199.0	97.00	2774.	15.5	70	1
21.0	6	200.0	85.00	2587.	16.0	70	1
27.0	4	97.00	88.00	2130.	14.5	70	3
26.0	4	97.00	46.00	1835.	20.5	70	2
25.0	4	110.0	87.00	2672.	17.5	70	2
24.0	4	107.0	90.00	2430.	14.5	70	2

Input: a row in this table. a feature mapping corresponds to a column.

Goal: predict whether mpg is < 23 ("bad" $= 0$) or above ("good" $= 1$) given other attributes (other columns).

201 "good" and 197 "bad"; guessing the most frequent class (good) will get 50.5% accuracy.

Let's build a classifier!

- Let's just try to build a classifier.
  (This is our first learning algorithm)
- For now, let's ignore the "test" set and trying to "generalize"
- Let's start with just looking at a simple classifier.
  What is a simple classification rule?

Contingency Table

values of y	values of feature ϕ			
	v_1	v_2	$\cdots$	v_K
0				
1				

Decision Stump Example

y	maker		
	america	europe	asia
0	174	14	9
1	75	56	70
	↓	↓	↓
	0	1	1

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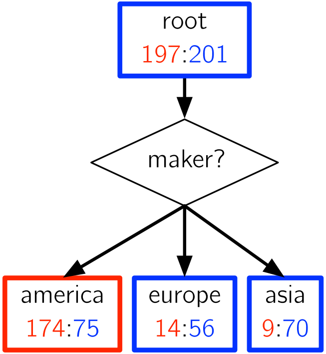

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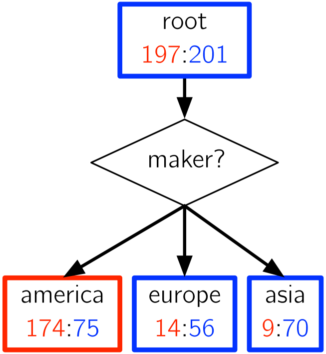
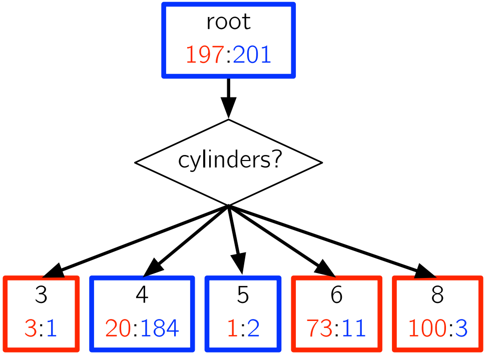


Errors: $75 + 14 + 9 = 98$ (about 25%)

Decision Stump Example



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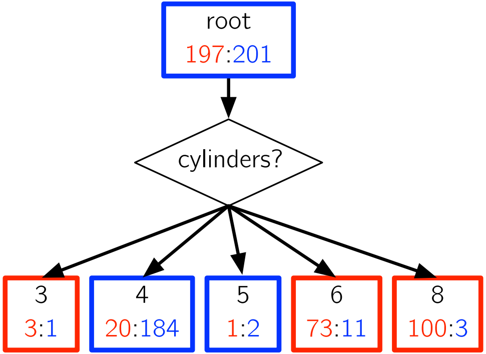


Errors: $1 + 20 + 1 + 11 + 3 = 36$ (about 9%)

Key Idea: Recursion

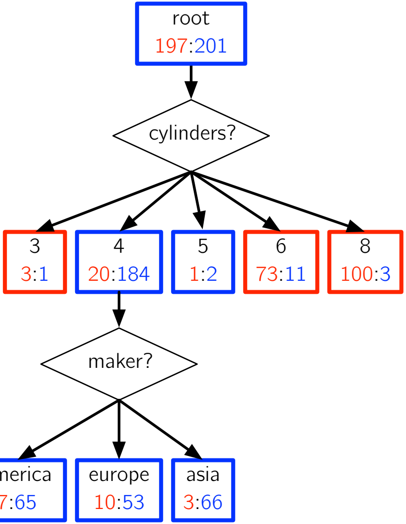
divide & conquer

A single feature **partitions** the data.

For each partition, we could choose another feature and partition further.

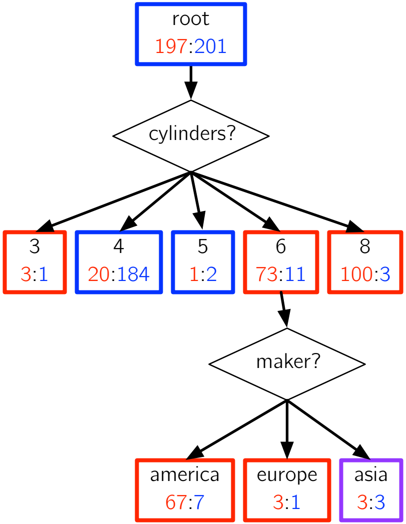
Applying this recursively, we can construct a **decision tree**.

Decision Tree Example



Error reduction compared to the cylinders stump?

Decision Tree Example



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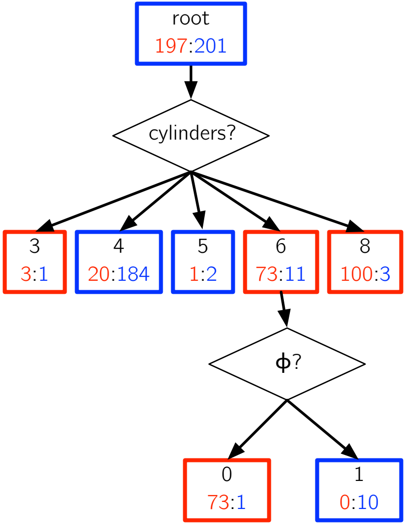

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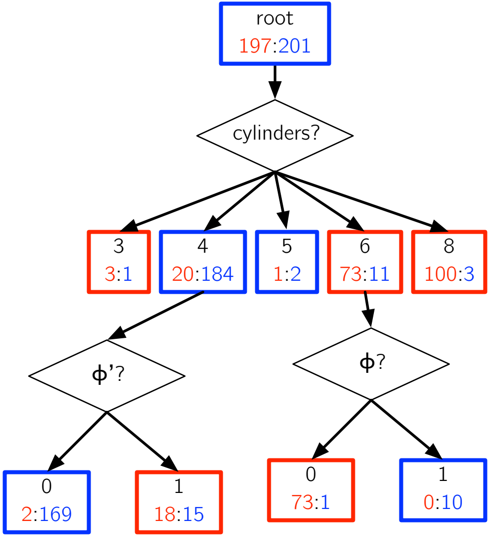

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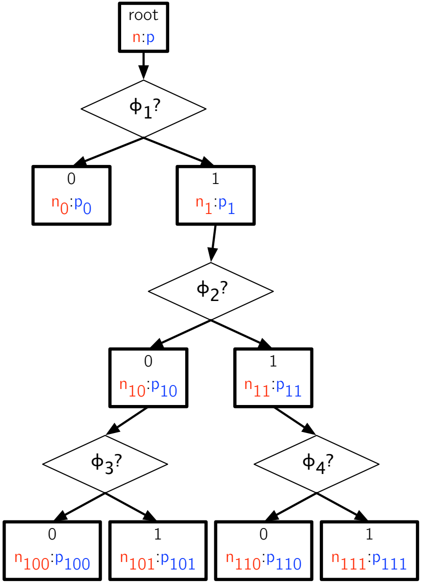
Decision Tree: Making a Prediction



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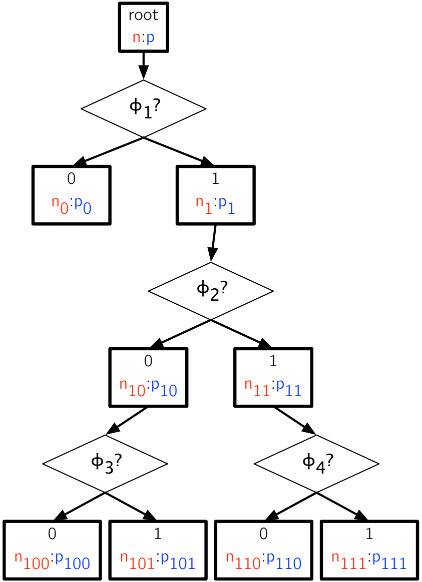
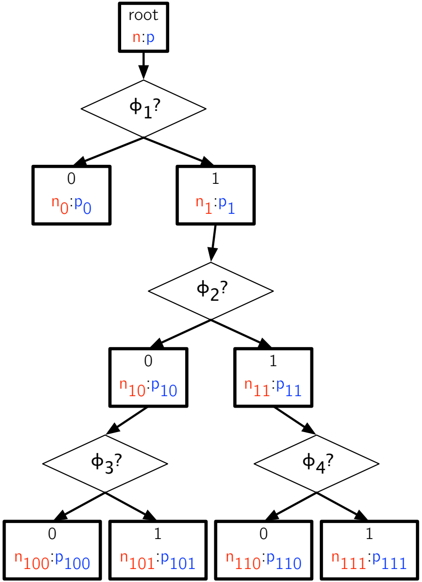


Data: decision tree t, input example x
Result: predicted class
if t *has the form* $\textsc{Leaf}(y)$ **then**
  return y;
else
  # $t.\phi$ is the feature associated with t;
  # t.child(v) is the subtree for value v;
  return $\textsc{DTreeTest}(t.\text{child}(t.\phi(x)), x)$;
end

Algorithm 1: $\textsc{DTreeTest}$

Decision Tree: Making a Prediction



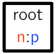
Equivalent boolean formulas:

$$(\phi_1 = 0) \Rightarrow [\![n_0 < p_0]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 0) \Rightarrow [\![n_{100} < p_{100}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 0) \wedge (\phi_3 = 1) \Rightarrow [\![n_{101} < p_{101}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 0) \Rightarrow [\![n_{110} < p_{110}]\!]$$
$$(\phi_1 = 1) \wedge (\phi_2 = 1) \wedge (\phi_4 = 1) \Rightarrow [\![n_{111} < p_{111}]\!]$$

Tangent: How Many Formulas?

- Assume we have D binary features.
- Each feature could be set to 0, or set to 1, or excluded (wildcard/don't care).
- 3^D formulas.

Building a Decision Tree



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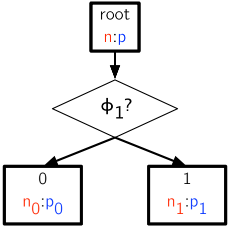
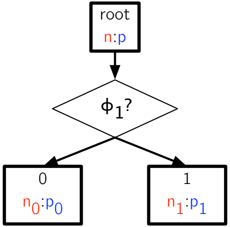


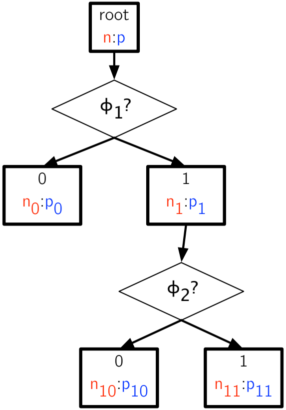
We chose feature ϕ_1. Note that $n = n_0 + n_1$ and $p = p_0 + p_1$.

Building a Decision Tree



We chose not to split the left partition. Why not?

Building a Decision Tree



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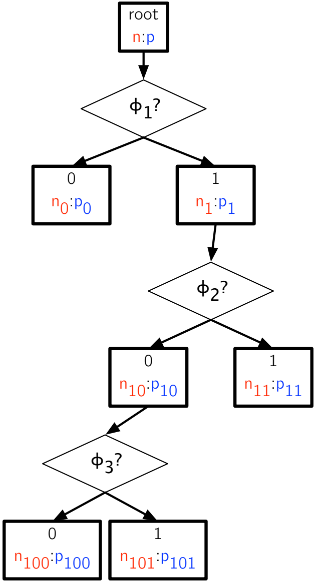

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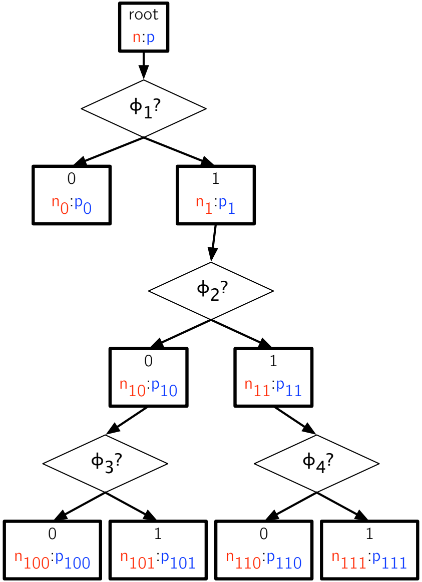


Greedily Building a Decision Tree (Binary Features)

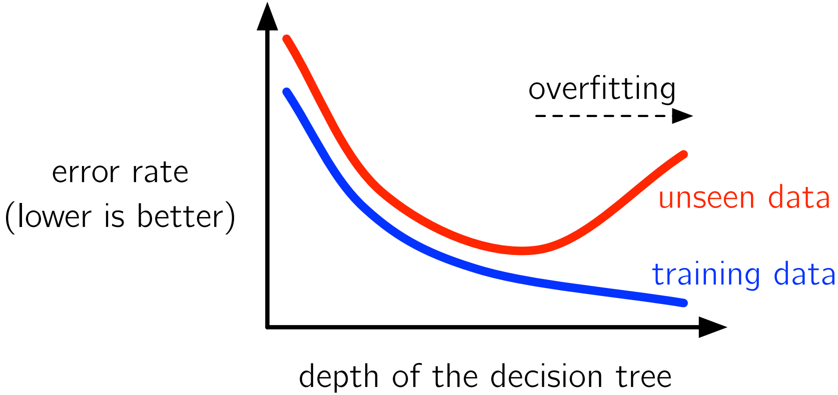
Data: data D, feature set Φ
Result: decision tree
if *all examples in* D *have the same label* y*, or* Φ *is empty and* y *is the best guess* **then**
    return $\textsc{Leaf}(y)$;
else
    for *each feature* ϕ *in* Φ **do**
        partition D into D_0 and D_1 based on ϕ-values;
        let mistakes(ϕ) $=$ (non-majority answers in D_0) $+$ (non-majority answers in D_1);
    end
    let ϕ^* be the feature with the smallest number of mistakes;
    return $\textsc{Node}(\phi^*, \{0 \rightarrow \textsc{DTreeTrain}(D_0, \Phi \setminus \{\phi^*\}), 1 \rightarrow \textsc{DTreeTrain}(D_1, \Phi \setminus \{\phi^*\})\})$;
end

Algorithm 2: $\textsc{DTreeTrain}$

What could go wrong?

- Suppose we split on a variable with many values? (e.g. a continous one like "displacement")
- Suppose we built out our tree to be very deep and wide?

Danger: Overfitting



Detecting Overfitting

If you use all of your data to train, you won't be able to draw the red curve on the preceding slide!

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Solution: hold some out. This data is called **development data**. More terms:

- Decision tree max depth is an example of a **hyperparameter**
- "I used my development data to **tune** the max-depth hyperparameter."

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Better yet, hold out two subsets, one for tuning and one for a true, honest-to-science **test**.

Splitting your data into training/development/test requires careful thinking. Starting point: randomly shuffle examples with an 80%/10%/10% split.

The "i.i.d." Supervised Learning Setup

- Let ℓ be a loss function; $\ell(y, \hat{y})$ is what we lose by outputting $\hat{y}$ when y is the correct output. For classification:

$$\ell(y, \hat{y}) = [\![y \neq \hat{y}]\!]$$

- Let $\mathcal{D}(x, y)$ define the true probability of input/output pair (x, y), in "nature." **We never "know" this distribution.**
- The training data $D = \langle (x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N) \rangle$ are assumed to be **identical, independently, distributed** (i.i.d.) samples from $\mathcal{D}$.
- The test data are also assumed to be i.i.d. samples from $\mathcal{D}$.
- The space of classifiers we're considering is $\mathcal{F}$; f is a classifier from $\mathcal{F}$, chosen by our learning algorithm.