Machine Learning (CSE 446): Ensembles

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November 29, 2017

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You should recall from the voted perceptron that letting a collection of classifiers **vote** can offer a richer decision boundary than any individual classifier offers.

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Where do we get different datasets?

Data: $D = \langle (x_n, y_n) \rangle_{n=1}^{N}$, number of bootstrap samples required B
Result: Resampled datasets $\tilde{D}_1, \ldots, \tilde{D}_B$
for $b \in \{1, \ldots, B\}$ **do**
    for $n \in \{1, \ldots, N\}$ **do**
        sample i uniformly at random from $\{1, \ldots, N\}$;
        add (x_i, y_i) to $\tilde{D}_b$;
    end
end
return $\tilde{D}_1, \ldots, \tilde{D}_B$;

Algorithm 1: BootstrapResampling

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Why is it called "bootstrap"?

Bagging: Ensembles of the *Same* Learner

1. Apply $\textsc{BootstrapResampling}$ to D, creating datasets $\tilde{D}_1, \ldots, \tilde{D}_B$.
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You can think of bagging as a variance-reducing technique; it tends to have similar benefits to regularization.

Boosting

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We'll consider one particular method called AdaBoost (for "adaptive boosting") that has some interesting theoretical properties *and* is widely used.

Data: $D = \langle (x_n, y_n) \rangle_{n=1}^{N}$, number of epochs E, weighted learner $\mathcal{W}$
Result: classifier
$\boldsymbol{\beta}^{(0)} = \langle \frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N} \rangle$; # initialize example weights
for $e \in \{1, \ldots, E\}$ **do**
    $f^{(e)} \leftarrow \mathcal{W}(D, \boldsymbol{\beta}^{(e-1)})$; # train the classifier on the weighted data
    $\hat{\epsilon}^{(e)} \leftarrow \sum_{n=1}^{N} \beta_n^{(e-1)} \cdot [\![f^{(e)}(x_n) \neq y_n]\!]$; # weighted error rate
    $\alpha^{(e)} \leftarrow \frac{1}{2} \log \left(\frac{1-\hat{\epsilon}^{(e)}}{\hat{\epsilon}^{(e)}} \right)$; # "adaptive" weight for $f^{(e)}$
    for $n \in \{1, \ldots, N\}$ **do**
        $\beta_n^{(e)} \leftarrow \frac{1}{Z^{(e)}} \cdot \beta_n^{(e-1)} \cdot \exp\left(-\alpha^{(e)} \cdot y_n \cdot f^{(e)}(x_n)\right)$; # update example weights
        ($Z^{(e)}$ is a normalization constant)
    end
end
return $f_{\text{boost}}(\cdot) = \text{sign}\left(\sum_{e=1}^{E} \alpha^{(e)} \cdot f^{(e)}(\cdot) \right)$;

Algorithm 2: AdaBoost

Notes about AdaBoost

- Typically, $\mathcal{W}$ is a shallow decision tree, or a linear classifier. In the literature, it is often called a **weak** learner (definition comes later).

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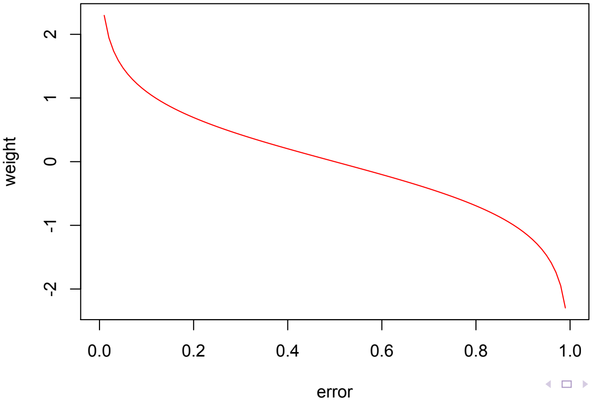

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- For examples we get right ($f^{(e)}(x_n) = y_n$), the weight β_n will *decrease*; we increase the weights of examples we get wrong.
  - See the book for more insight on what happens on the first epoch with a *very* simple $\mathcal{W}$. Each successive $f^{(e)}$ is intended to work harder wherever previous classifiers have been failing (hence, "adaptive").

Formula for $\beta_n^{(e)}$

$$\beta_n^{(e)} \leftarrow \frac{1}{Z^{(e)}} \cdot \beta_n^{(e-1)} \cdot \exp\left(-\alpha^{(e)} \cdot y_n \cdot f^{(e)}(x_n)\right)$$

$$\text{where } Z^{(e)} = \sum_{i=1}^{N} \beta_i^{(e-1)} \cdot \exp\left(-\alpha^{(e)} \cdot y_i \cdot f^{(e)}(x_i)\right)$$

($Z^{(e)}$ forces the sum $\sum_{n=1}^{N} \beta_n^{(e)} = 1$.)