Machine Learning (CSE 446): Beyond Binary Classification

Noah Smith

University of Washington
nasmith@cs.washington.edu

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2. Down-weight negative examples until you achieve balance. For example,

$$L^{(\text{new})}(\mathbf{x}, y, \text{parameters}) \leftarrow \alpha^{[\![y=+1]\!]} \cdot L^{(\text{old})}(\mathbf{x}, y, \text{parameters})$$

   A similar effect can be achieved in SGD by sampling non-uniformly; assign $\frac{1}{2N_+}$ to positive examples and $\frac{1}{2N_-}$ to negative examples.

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3. Modification to the hinge loss:

$$L_n^{(\text{hinge})}(\mathbf{w}, b) = \max\{0, \overbrace{\text{cost}(y_n)}^{\text{formerly } 1} - y_n \cdot (\mathbf{w} \cdot \mathbf{x}_n + b)\}$$

$$\text{cost}(y_n) = \begin{cases} \alpha & \text{if } y_n = -1 \text{ (false positive)} \\ \beta & \text{if } y_n = +1 \text{ (false negative)} \end{cases}$$

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2. One-versus-all training: train $|\mathcal{Y}|$ binary classifiers, letting each $y \in \mathcal{Y}$ take a turn as the positive class. Let $a^{(y)}$ be the activation function for the classifier where $\{y \to +1, \mathcal{Y} \setminus \{y\} \to -1\}$. Then define the classifier $f : \mathcal{X} \to \mathcal{Y}$ as:

$$f(x) = \operatorname*{argmax}_{y \in \mathcal{Y}} a^{(y)}(x)$$

Theorem: error rate is at most $(|\mathcal{Y}| - 1) \cdot \bar{\epsilon}$, where $\bar{\epsilon}$ is the average error rate among the binary classifiers.

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   Theorem: error rate is at most $(|\mathcal{Y}| - 1) \cdot \bar{\epsilon}$, where $\bar{\epsilon}$ is the average error rate among the binary classifiers. One bad classifier can ruin f; in particular, watch out for the more rare labels, and be sure to tune hyperparameters separately.

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3. All-versus-all ("tournament"): build $\binom{|\mathcal{Y}|}{2}$ classifiers, pairing every $y, y' \in \mathcal{Y}$.

   Theorem: error rate is at most $2(|\mathcal{Y}| - 1) \cdot \bar{\epsilon}$.

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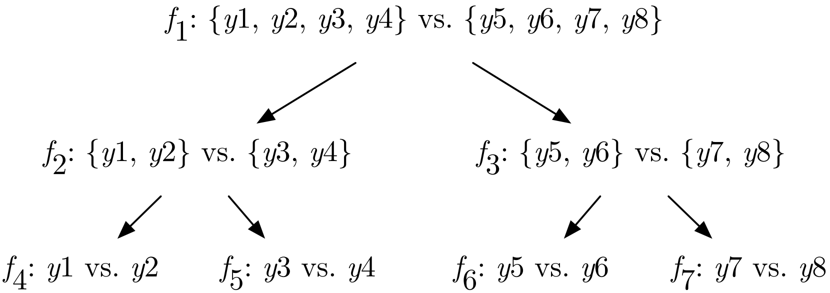
Tree-Structured Tournament for Multiclass Classification



Ranking

Most common setup: $x_n = \langle q_n, \boldsymbol{d} \rangle$, where q_n is a query and $\boldsymbol{d}$ is a (fixed, universal) set of documents $\{d_1, \ldots, d_M\}$. Output $\boldsymbol{y}_n$ is a ranking of $\boldsymbol{d}$.

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Training on the binary problem $\langle(\mathbf{x}_{n,i,j}, y_{n,i,j})\rangle_{n \in \{1,\ldots,N\}; i,j \in \{1,\ldots,M}$ makes sense when the ranking is meant to separate relevant d_i from irrelevant d_i, known as "bipartite" ranking.

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One example:

$$\omega(i, j) = \begin{cases} 1 & \text{if } \min(i, j) \leq 10 \text{ and } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

(More in the book.)

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Loss:

$$\mathbb{E}_{(q,\sigma)\sim\mathcal{D}}\left[\sum_{i,j:i\neq j} [\![\sigma(i) < \sigma(j)]\!] \cdot [\![\hat{\sigma}(i) < \hat{\sigma}(j)]\!] \cdot \omega(i, j)\right]$$

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Deriving a learning algorithm is left as an exercise. (See the book for an example.)