

Database System Internals

External Memory Algorithms (part 3)

Paul G. Allen School of Computer Science and Engineering
University of Washington, Seattle

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:**

- **Index based selection:**

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- Table scan: $B(R) = 2,000$ I/Os
- Index based selection:

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:** $B(R) = 2,000$ I/Os
- **Index based selection:**
 - If index is clustered:
 - If index is unclustered:

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:** $B(R) = 2,000$ I/Os
- **Index based selection:**
 - If index is clustered:
 - If index is unclustered: $T(R)/V(R, a) = 5,000$ I/Os

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:** $B(R) = 2,000$ I/Os
- **Index based selection:**
 - If index is clustered: $B(R)/V(R,a) = 100$ I/Os
 - If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:** $B(R) = 2,000$ I/Os!
- **Index based selection:**
 - If index is clustered: $B(R)/V(R,a) = 100$ I/Os
 - If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os!

Index Based Selection

- **Example:**

$$\begin{aligned} B(R) &= 2000 \\ T(R) &= 100,000 \\ V(R, a) &= 20 \end{aligned}$$

$$\text{cost of } \sigma_{a=v}(R) = ?$$

- **Table scan:** $B(R) = 2,000$ I/Os!
- **Index based selection:**
 - If index is clustered: $B(R)/V(R,a) = 100$ I/Os
 - If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os!

Lesson: Don't build unclustered indexes when $V(R,a)$ is small !

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R , for each tuple fetch corresponding tuple(s) from S

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R , for each tuple fetch corresponding tuple(s) from S

- Cost of nested loop join
 - $B(R) + T(R) * B(S)$
- **Cost of Index Nested Loop Join:**
 - If index on S is clustered:
 - If index on S is unclustered:

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R , for each tuple fetch corresponding tuple(s) from S
- Cost of nested loop join
 - $B(R) + T(R) * B(S)$
- **Cost of Index Nested Loop Join:**
 - If index on S is clustered:
 - If index on S is unclustered: $B(R) + T(R)T(S)/V(S,a)$

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R , for each tuple fetch corresponding tuple(s) from S
- Cost of nested loop join
 - $B(R) + T(R) * B(S)$
- **Cost of Index Nested Loop Join:**
 - If index on S is clustered: $B(R) + T(R)B(S)/V(S,a)$
 - If index on S is unclustered: $B(R) + T(R)T(S)/V(S,a)$

Outline

- **Join operator algorithms**
 - One-pass algorithms (Sec. 15.2 and 15.3)
 - Index-based algorithms (Sec 15.6)
 - Two-pass algorithms (Sec 15.4 and 15.5)

Two-Pass Algorithms

- Hash-join, merge-join assumed data \leq memory
- Next: algorithm when the data \gg main memory
Called external memory algorithm
- Merge-join
- Partitioned hash-join

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
- What is its runtime?
- What is the best algorithm for sorting a large file of n items on disc?
- What is its runtime?

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
 - Quicksort
- What is its runtime?

- What is the best algorithm for sorting a large file of n items on disc?
- What is its runtime?

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
 - Quicksort
- What is its runtime?
 - $O(n \log n)$
- What is the best algorithm for sorting a large file of n items on disc?
- What is its runtime?

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
 - Quicksort
- What is its runtime?
 - $O(n \log n)$
- What is the best algorithm for sorting a large file of n items on disc?
 - Multi-way Merge sort
- What is its runtime?
 - $O(n \log n)$ CPU time; $O(B \log_M B)$ disk I/O's

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
 - Quicksort
- What is its runtime?
 - $O(n \log n)$
- What is the best algorithm for sorting a large file of n items on disc?
 - **Multi-way** Merge sort
- What is its runtime?
 - $O(n \log n)$ CPU time; $O(B \log_M B)$ disk I/O's

Main memory merge-sort: **2-way**
External memory merge-sort: **multi-way**

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?
 - Quicksort
- What is its runtime?
 - $O(n \log n)$
- What is the best algorithm for sorting a large file of n items on disc?
 - **Multi-way** Merge sort
- What is its runtime?
 - $O(n \log n)$ CPU time; $O(B \log_M B)$ disk I/O's

Main memory merge-sort: **2-way**
External memory merge-sort: **multi-way**

Merge-Join is based on the multi-way merge-sort (next)

Merge-Sort: Basic Terminology

- A **run** in a sequence is an increasing subsequence
- What are the runs?

2, 4, 99, 103, 88, 77, 3, 79, 100, 2, 50

Merge-Sort: Basic Terminology

- A **run** in a sequence is an increasing subsequence
- What are the runs?

2, 4, 99, 103, | 88, | 77, | 3, 79, 100, | 2, 50

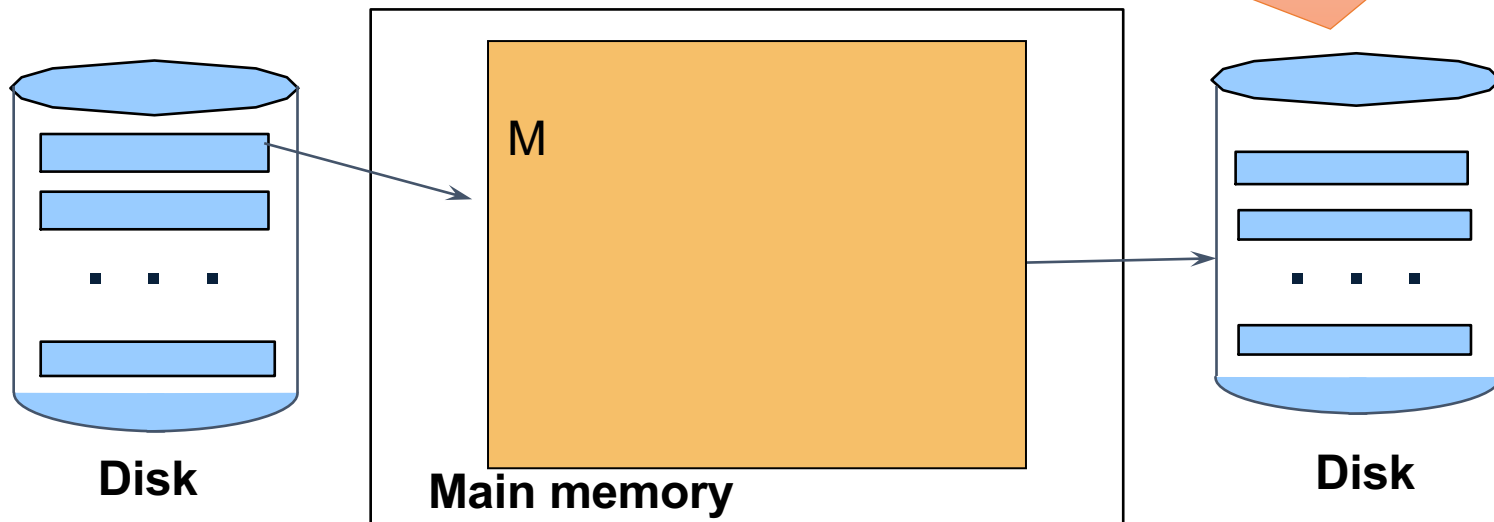
External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

External Merge-Sort: Step 1

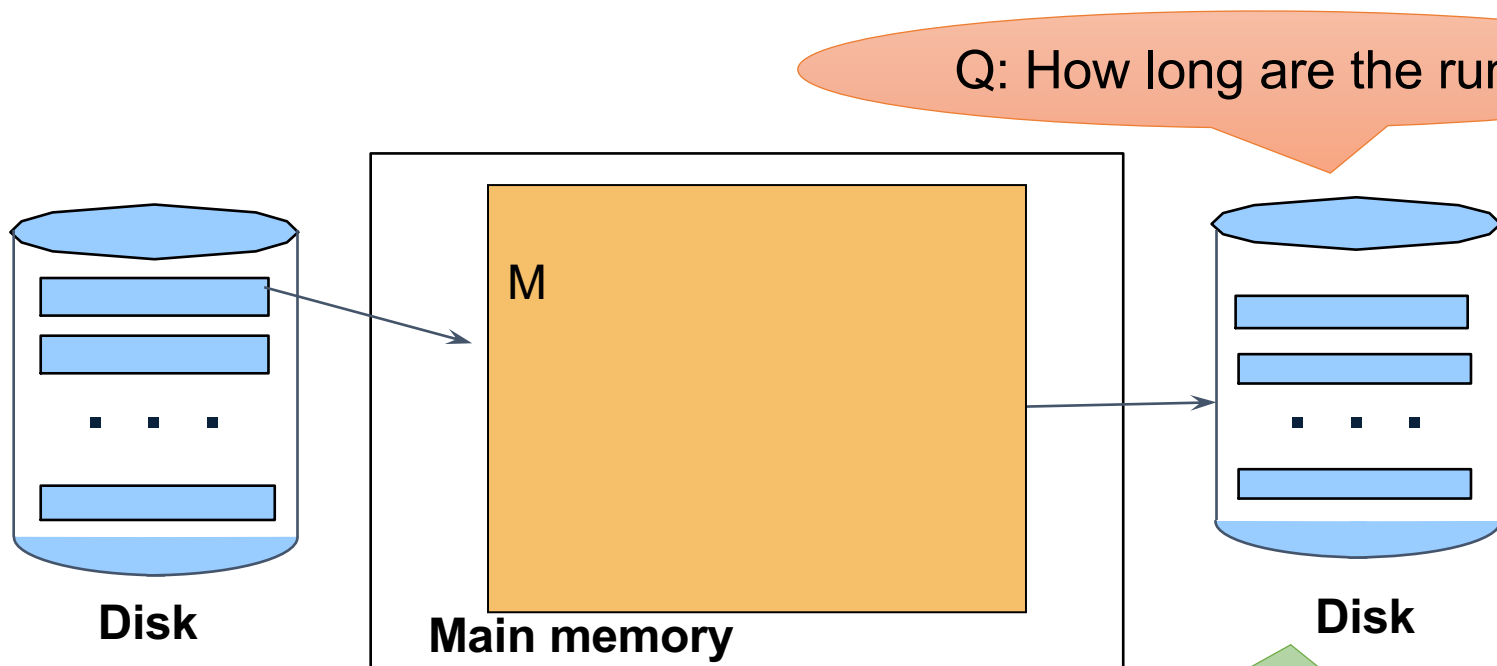
Phase one: load M blocks in memory, sort, send to disk, repeat

Q: How long are the runs?



External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

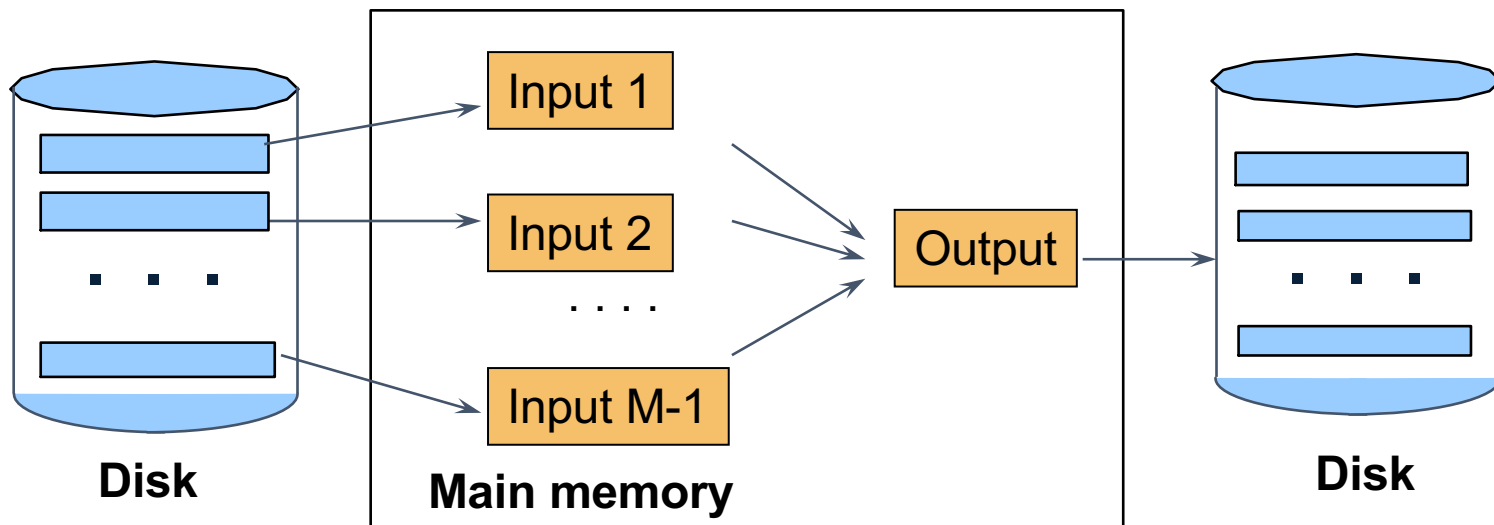


Q: How long are the runs?

A: Length = M blocks

Phase two: merge M runs into a bigger run

- Merge $M - 1$ runs into a new run
- Result: runs of length $M(M - 1) \approx M^2$



Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322

2, 4, 7, 43, 78, 103, 523

1, 6, 9, 12, 33, 52, 88, 320

Output:

0

Example

- Merging three runs to produce a longer run:

0, **14**, 33, 88, 92, 192, 322

2, 4, 7, 43, 78, 103, 523

1, 6, 9, 12, 33, 52, 88, 320

Output:

0, **?**

Example

- Merging three runs to produce a longer run:

0, **14**, 33, 88, 92, 192, 322

2, 4, 7, 43, 78, 103, 523

1, **6**, 9, 12, 33, 52, 88, 320

Output:

0, **1**, **?**

Example

- Merging three runs to produce a longer run:

0, **14**, **33**, **88**, **92**, **192**, **322**

2, 4, 7, **43**, **78**, **103**, **523**

1, 6, **9**, **12**, **33**, **52**, **88**, **320**

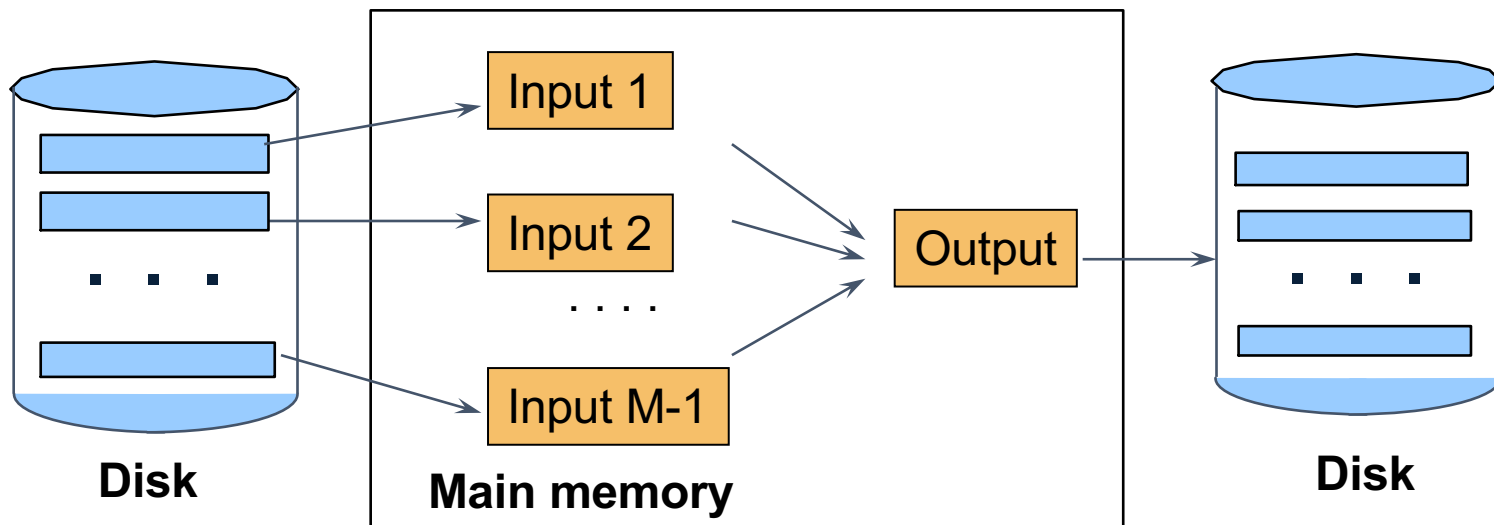
Output:

0, **1**, **2**, **4**, **6**, **7**, **?**

External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge $M - 1$ runs into a new run
- Result: runs of length $M(M - 1) \approx M^2$



If approx. $B \leq M^2$ then we are done

Cost of External Merge Sort

In theory:

- Number of I/O's: $O(B(R) * \log_M B(R))$

In practice:

- Assumption $B(R) \leq M^2$
- Read+write+read = $3B(R)$

Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?

Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?
- Example:
 - Page size = 32KB
 - Memory size 32GB: $M = 10^6$ -pages

Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?

- Example:
 - Page size = 32KB
 - Memory size 32GB: $M = 10^6$ pages

- R can be as large as 10^{12} pages
 - 32×10^{15} Bytes = 32 PB

Merge-Join

Join $R \bowtie S$

- How?....

Merge-Join

Join $R \bowtie S$

- Step 1a: generate initial runs for R
- Step 1b: generate initial runs for S
- Step 2: merge and join
 - Either merge first and then join
 - Or merge & join at the same time

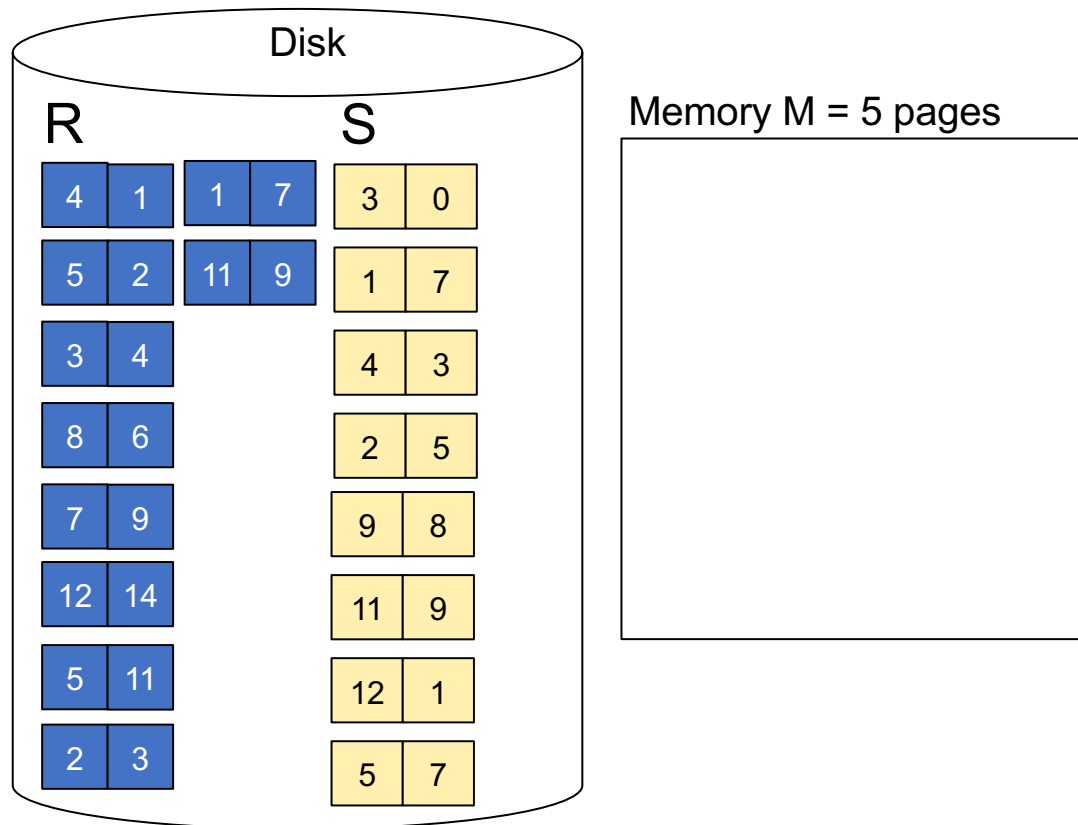
Merge-Join Example

Setup: Want to join R and S

Relation R has 10 pages with 2 tuples per page

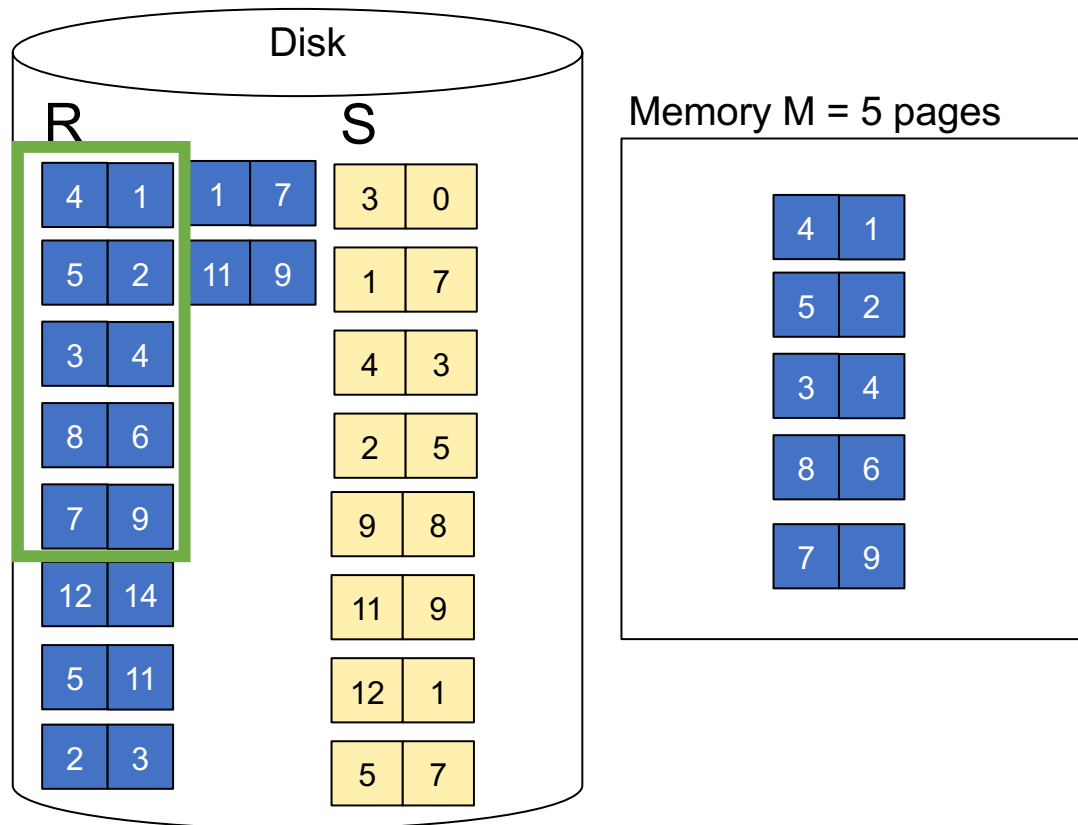
Relation S has 8 pages with 2 tuples per page

Values shown are values of join attribute for each given tuple



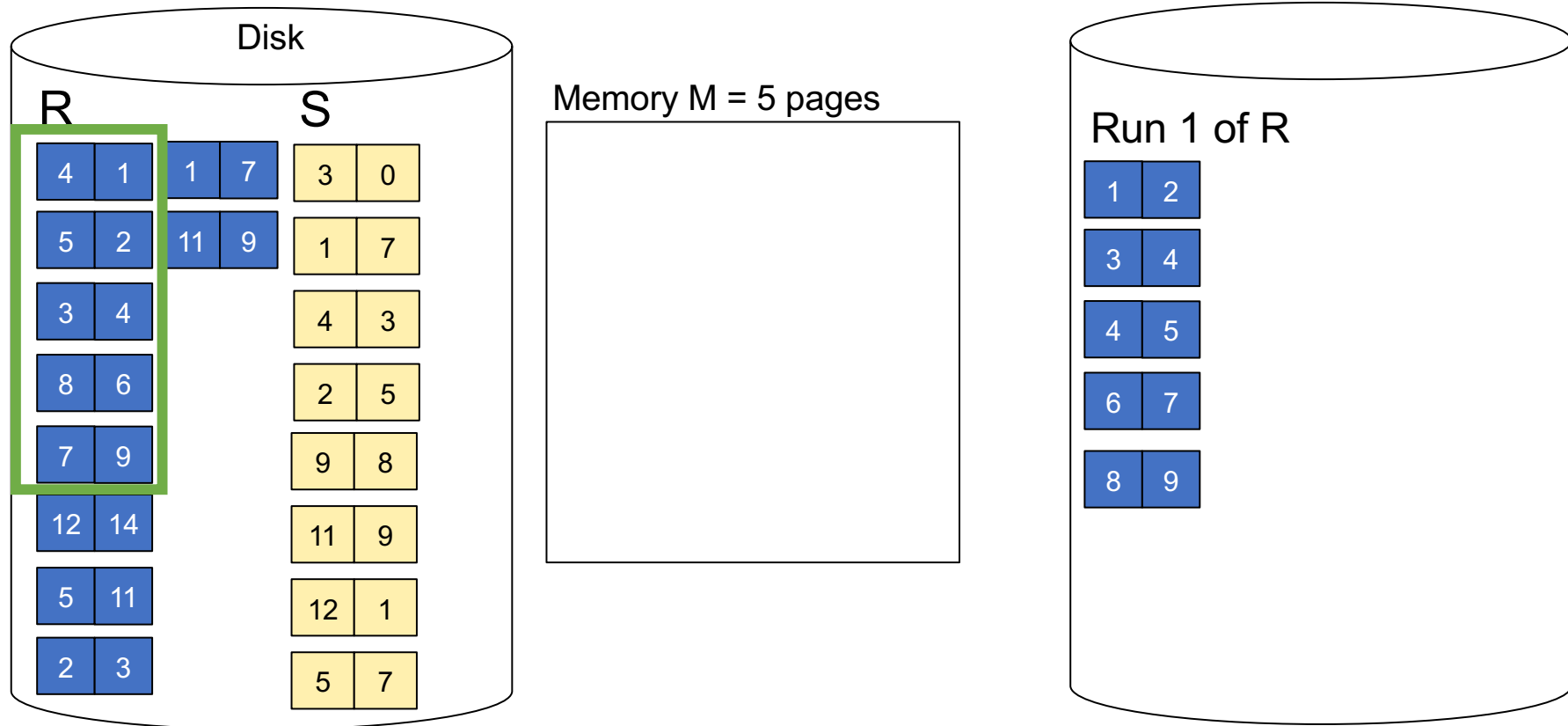
Merge-Join Example

Step 1: Read M pages of R and sort in memory



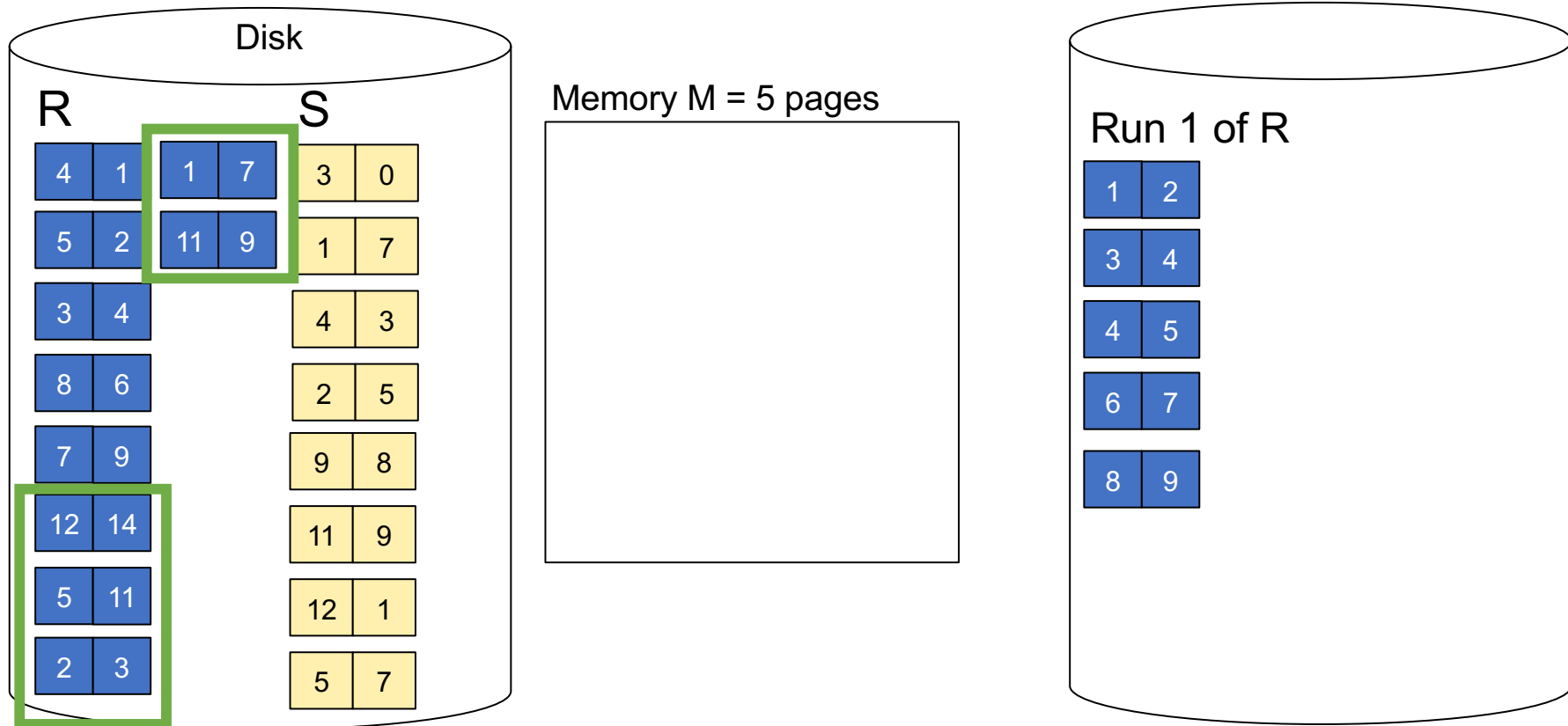
Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk



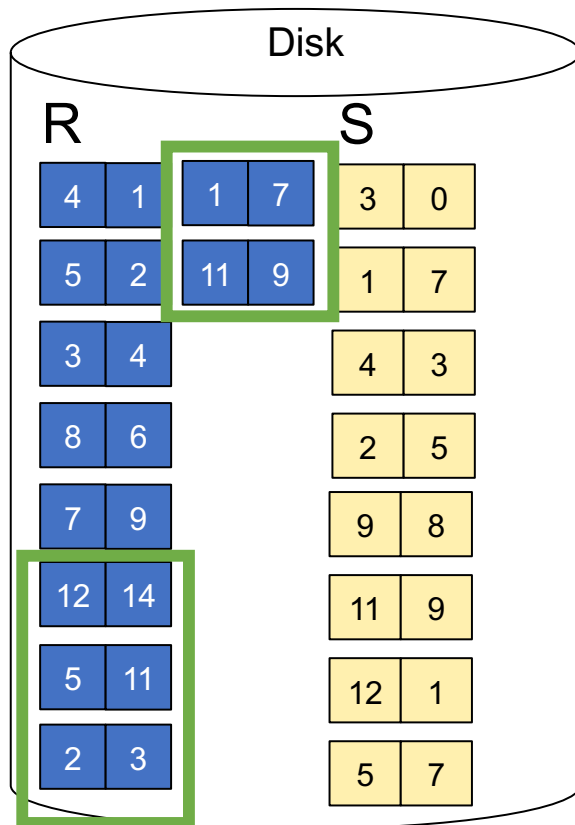
Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk

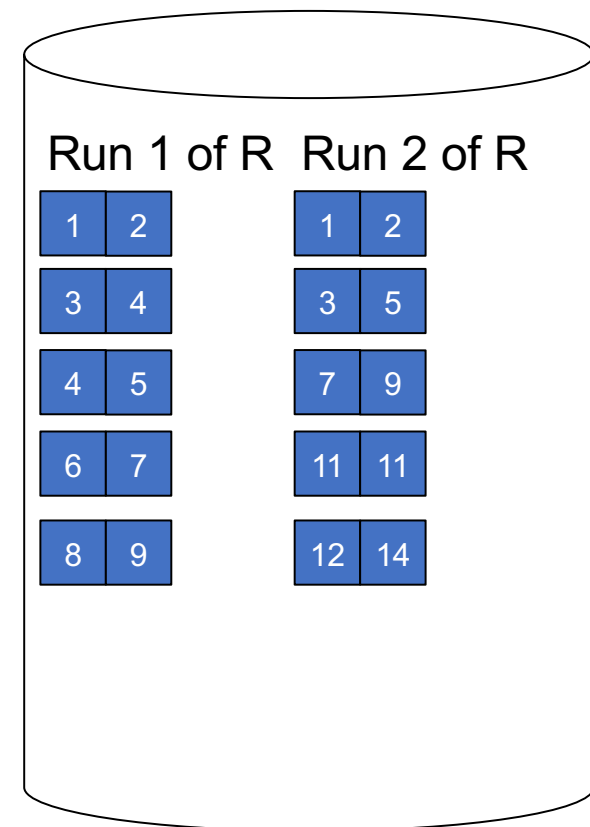


Merge-Join Example

Step 1: Repeat for next M pages until all R is processed

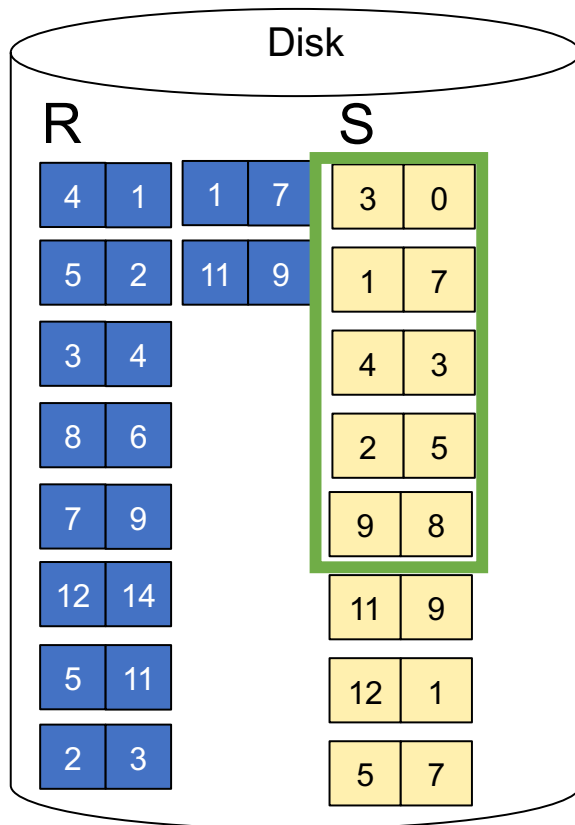


Memory M = 5 pages

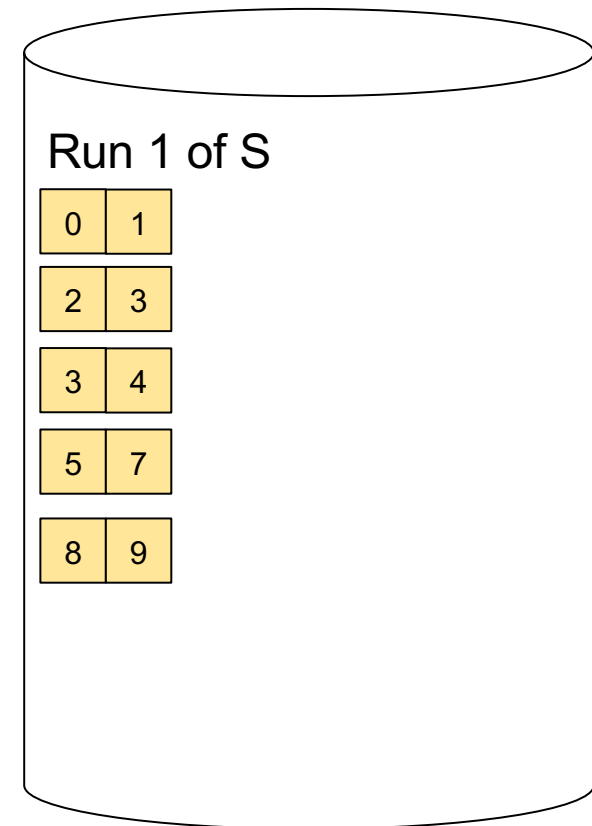


Merge-Join Example

Step 1: Do the same with S

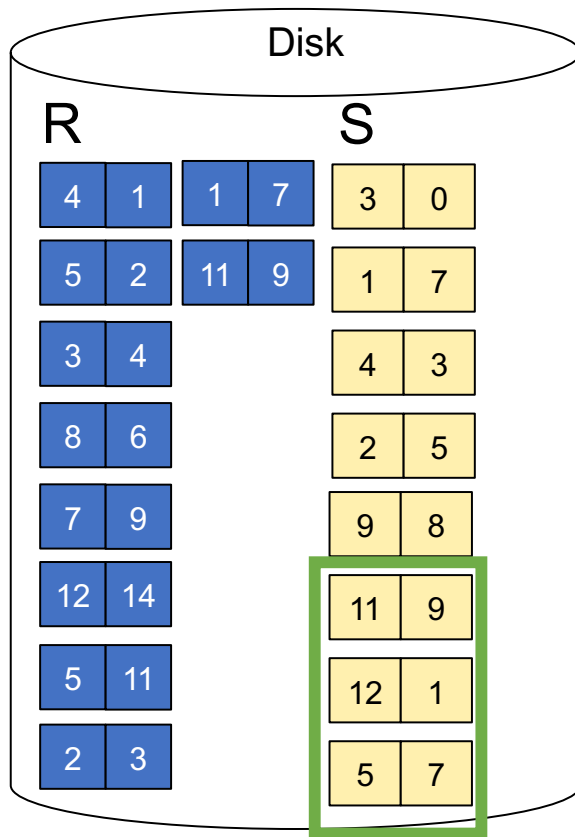


Memory M = 5 pages

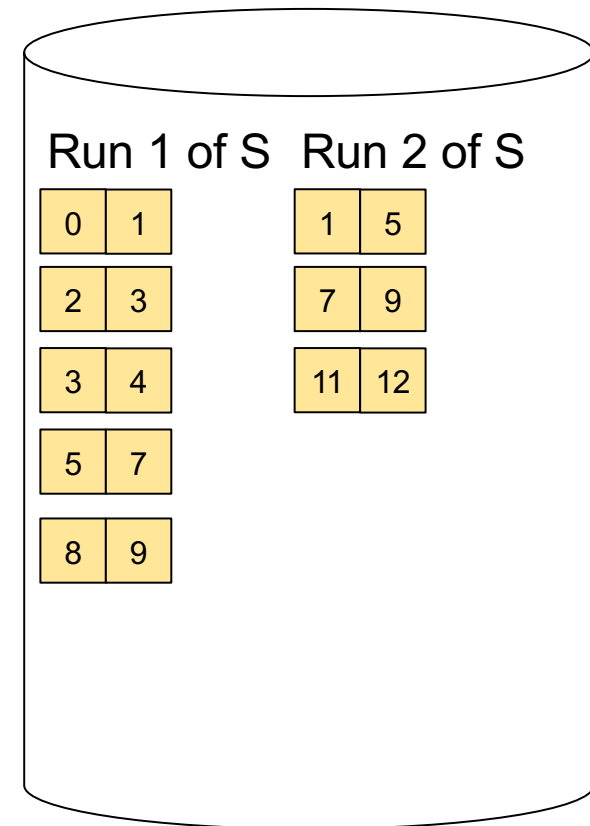


Merge-Join Example

Step 1: Do the same with S

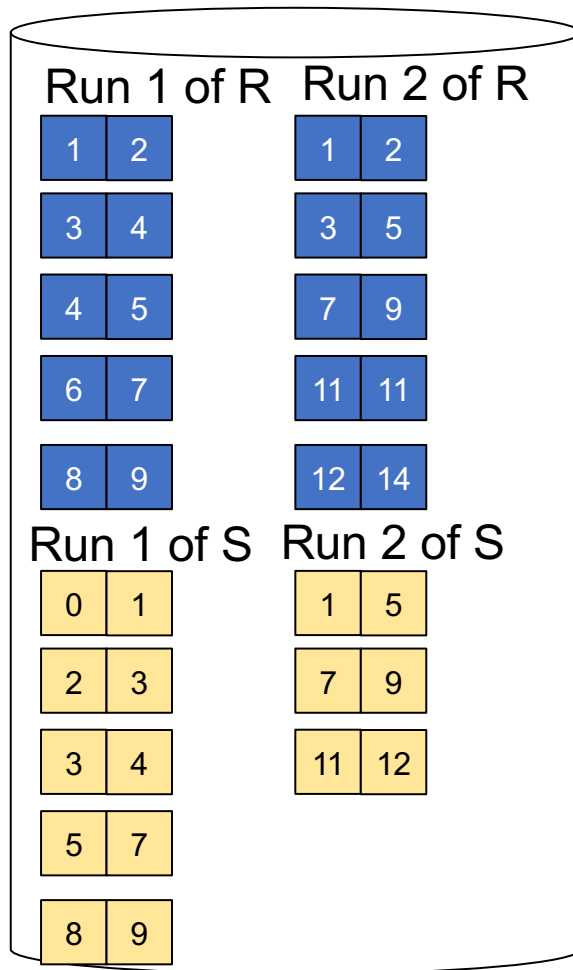


Memory M = 5 pages



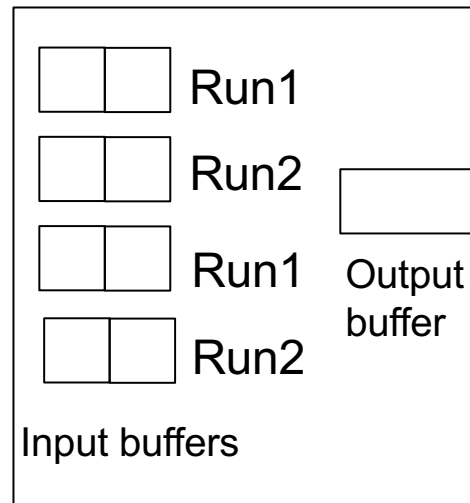
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

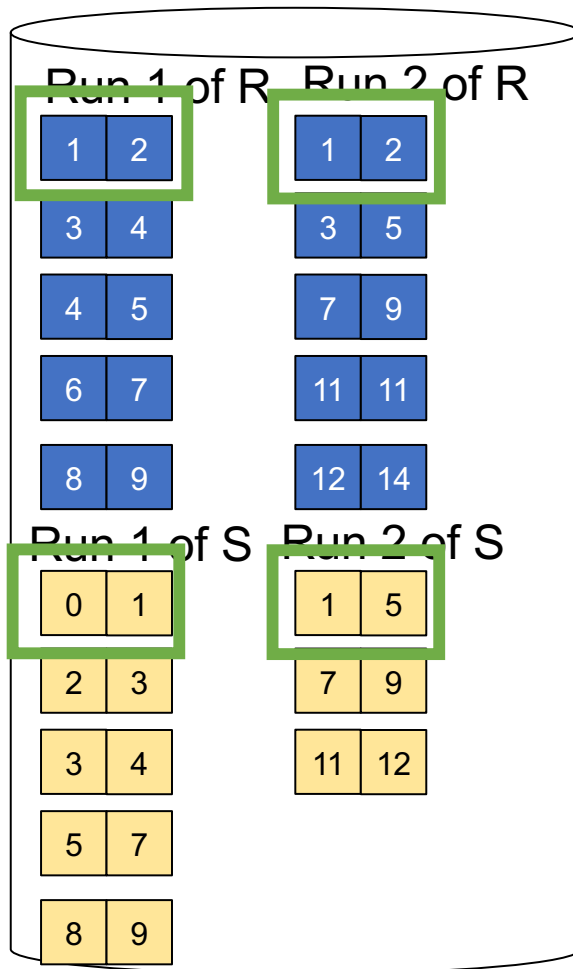
Memory $M = 5$ pages



Step 2: Join while merging
Output tuples

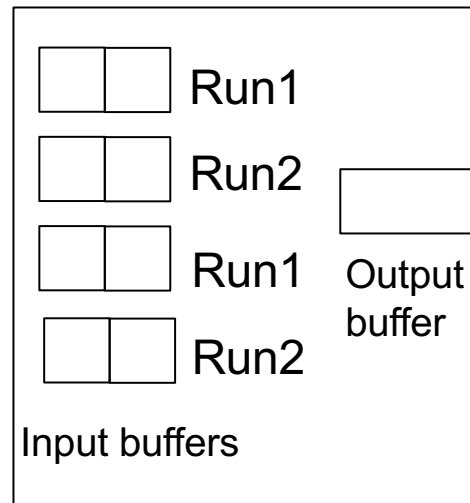
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

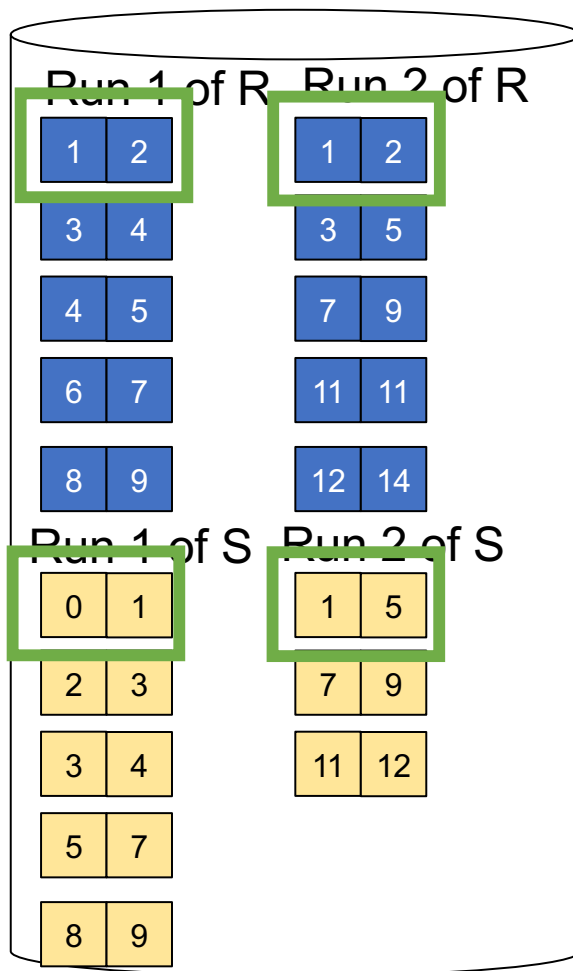
Memory $M = 5$ pages



Step 2: Join while merging
Output tuples

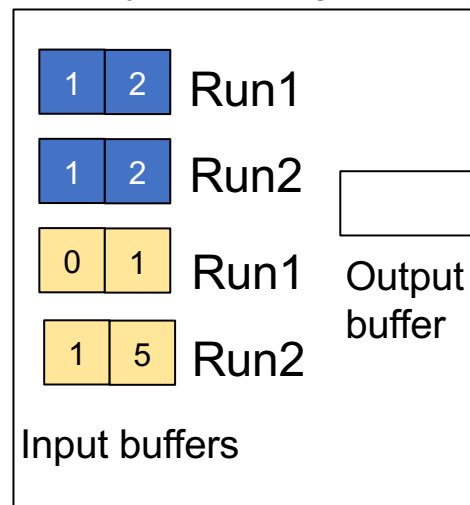
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

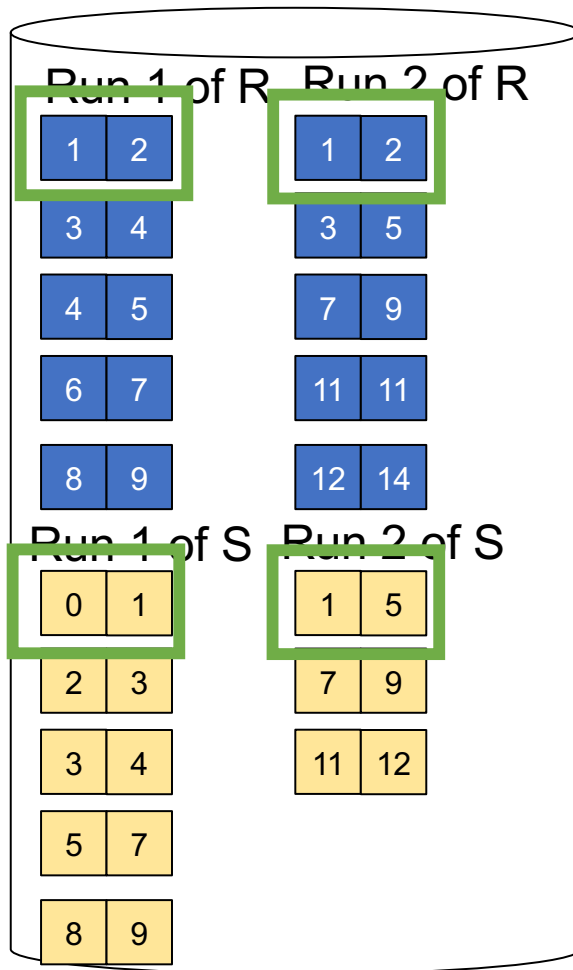
Memory $M = 5$ pages



Step 2: Join while merging
Output tuples

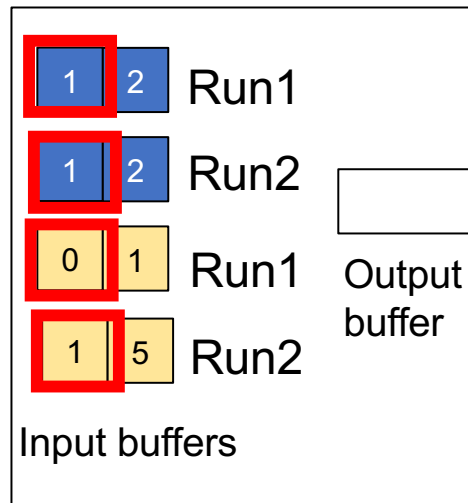
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

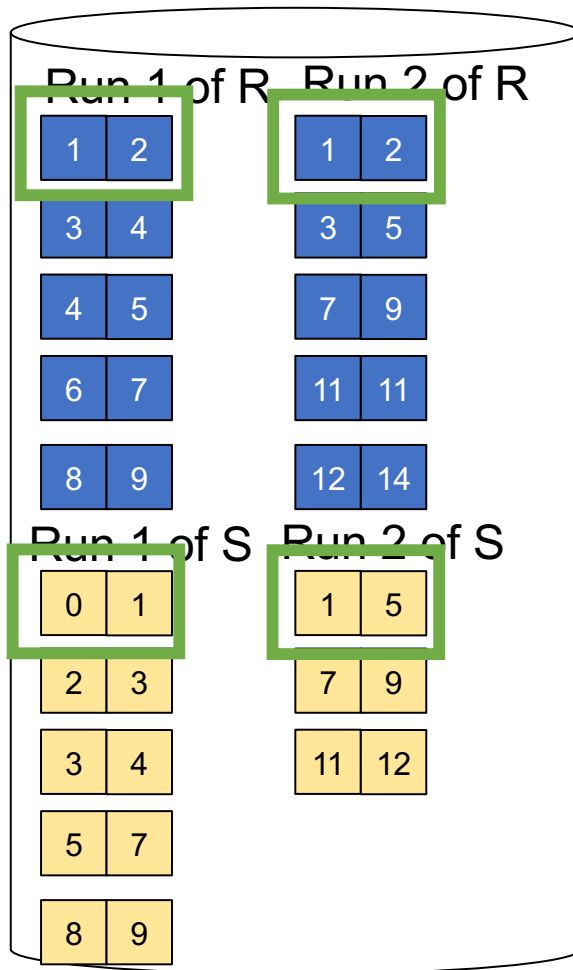
Memory $M = 5$ pages



Step 2: Join while merging Output tuples

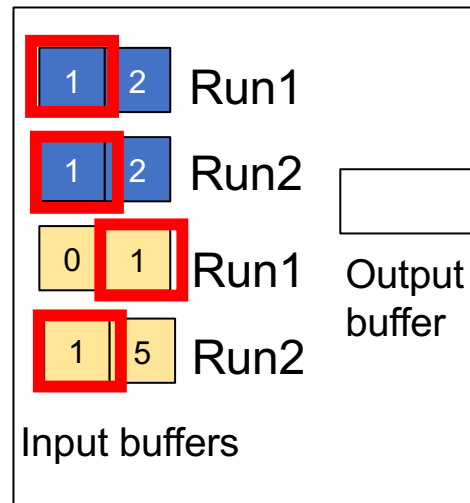
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

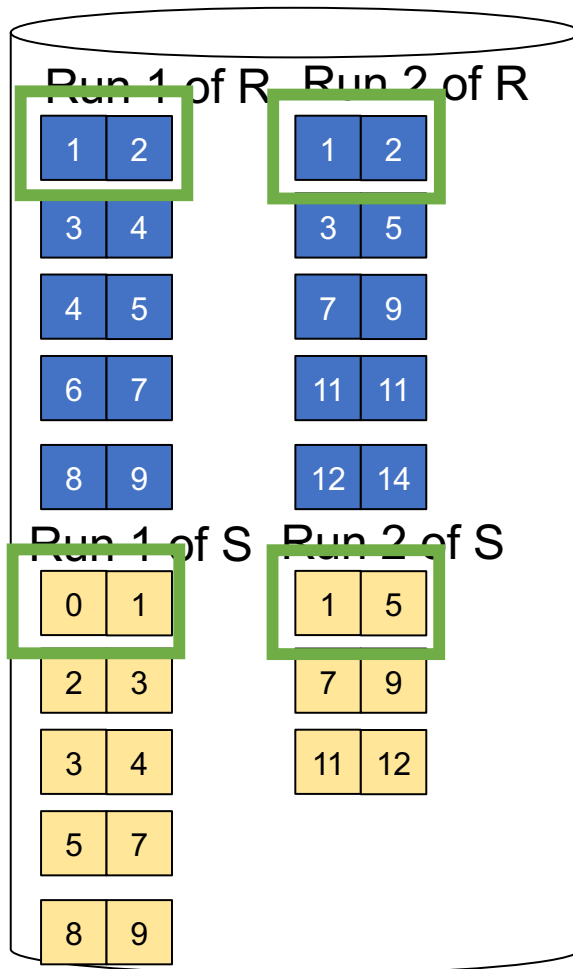
Memory $M = 5$ pages



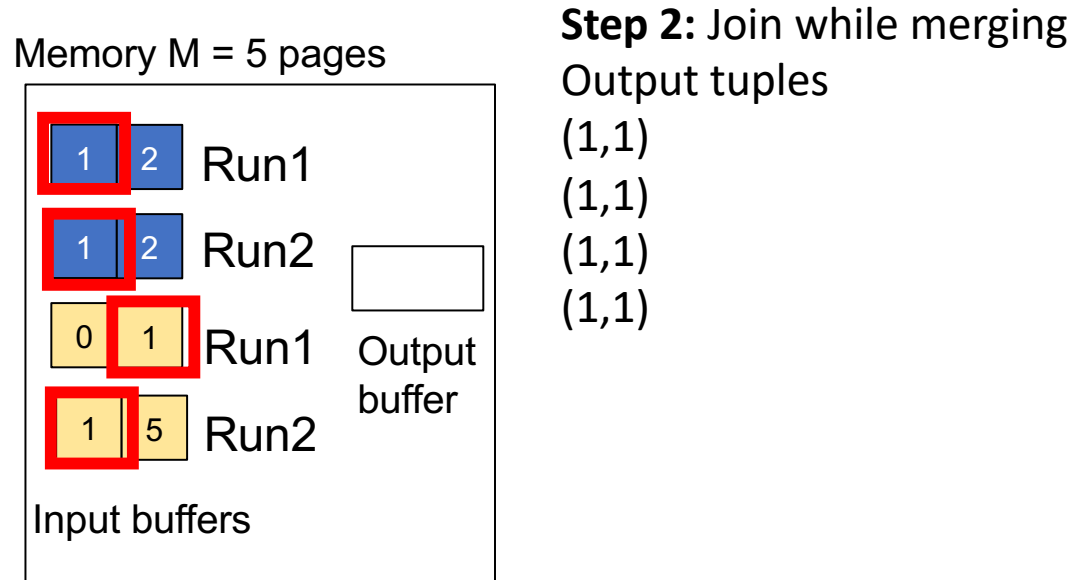
Step 2: Join while merging
Output tuples

Merge-Join Example

Step 2: Join while merging sorted runs

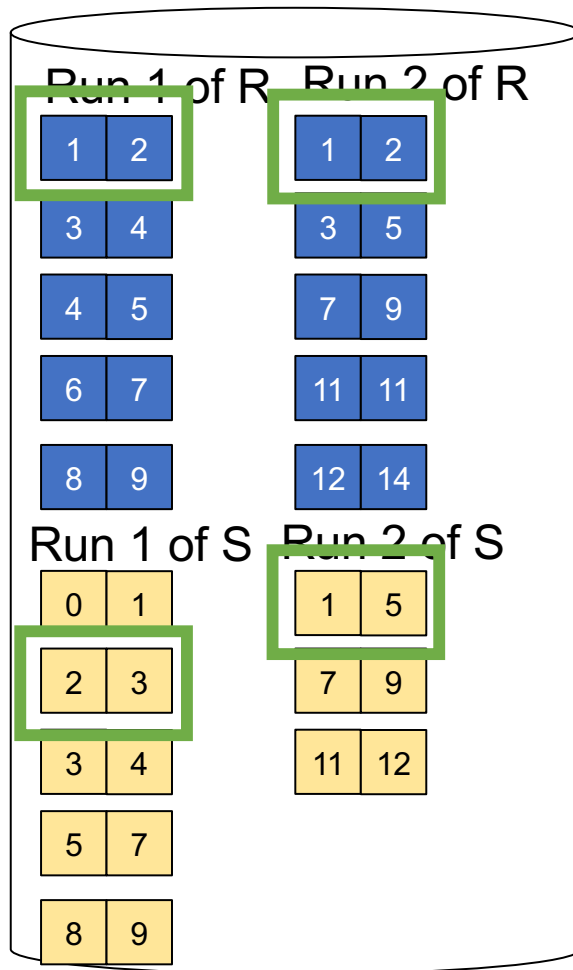


Total cost: $3B(R) + 3B(S)$



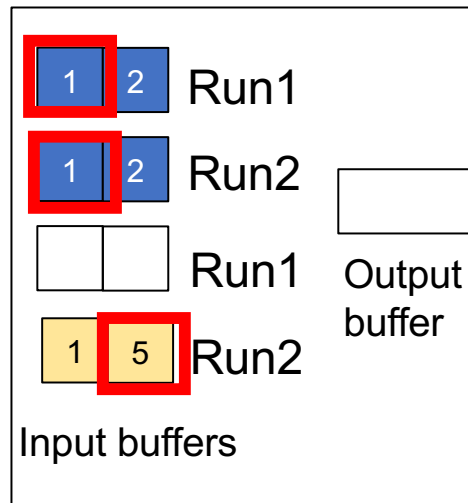
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages



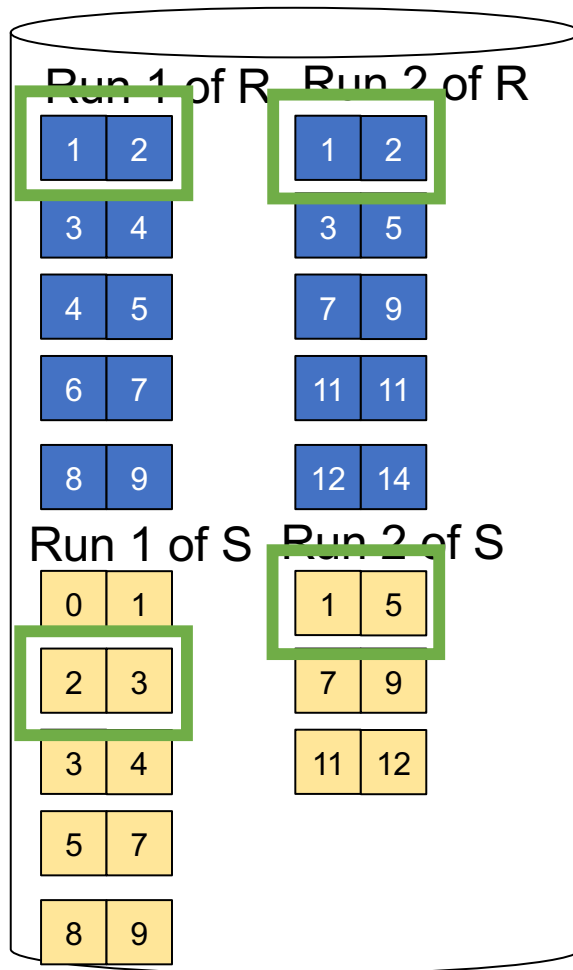
Step 2: Join while merging

Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)

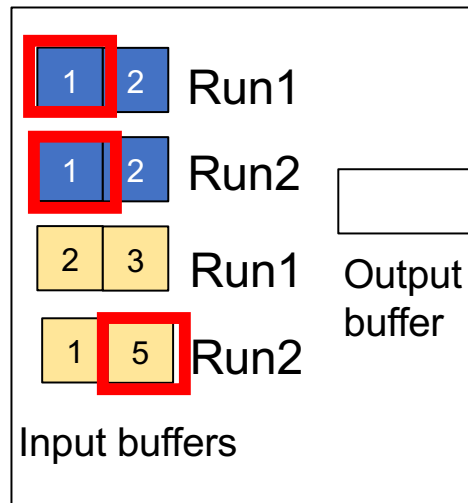
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages



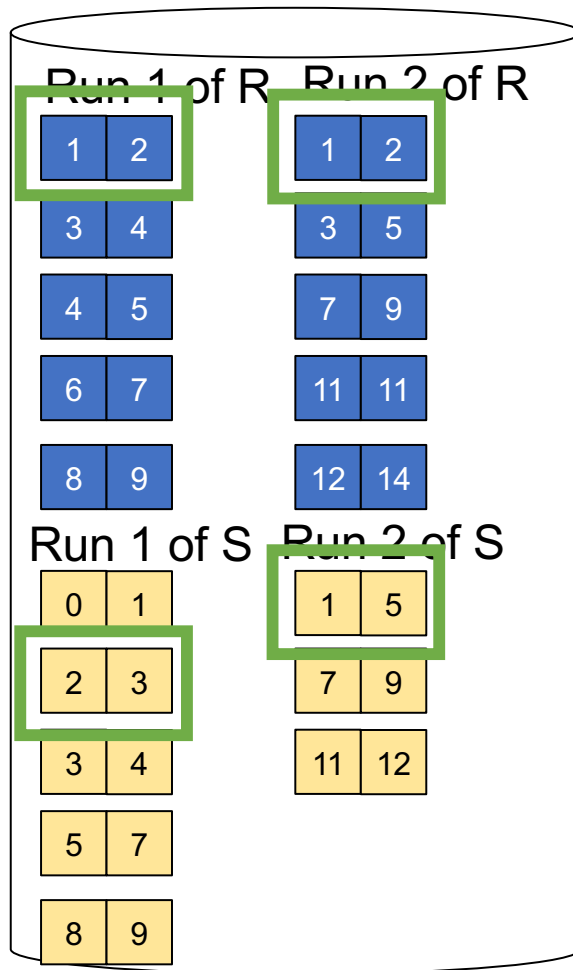
Step 2: Join while merging

Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)

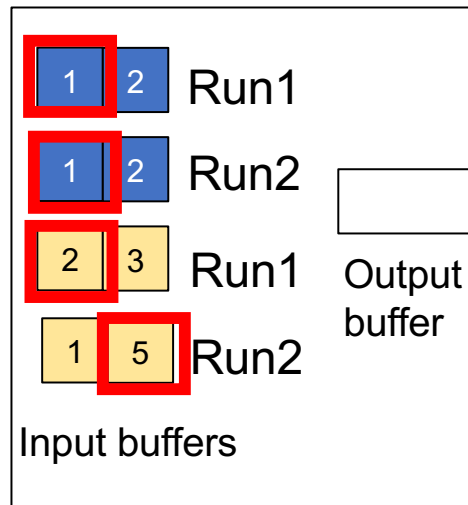
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages



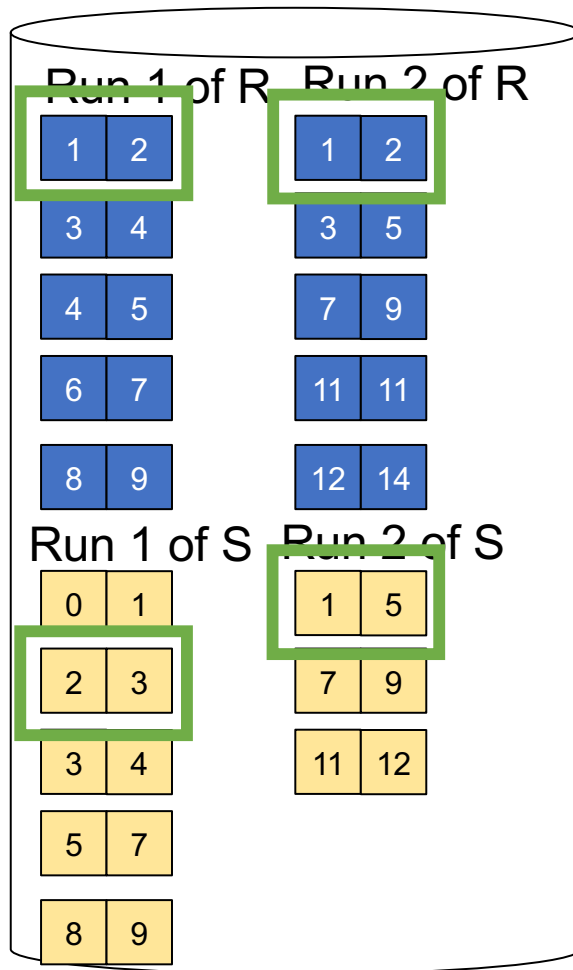
Step 2: Join while merging

Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)

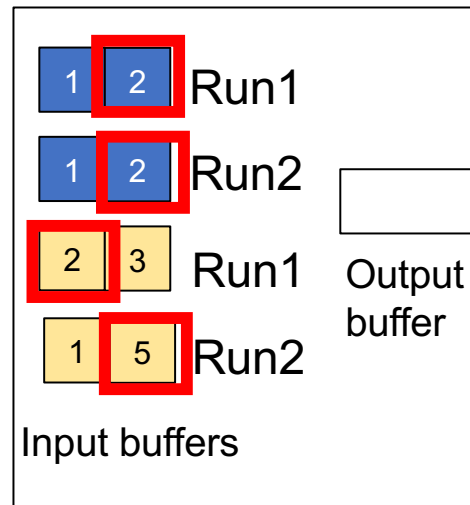
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages



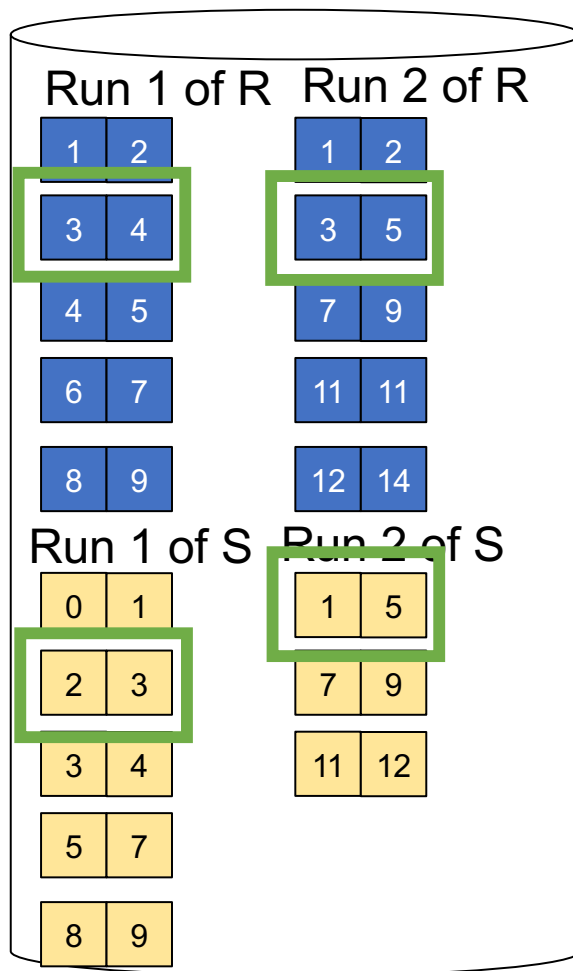
Step 2: Join while merging

Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)
- (2,2)
- (2,2)

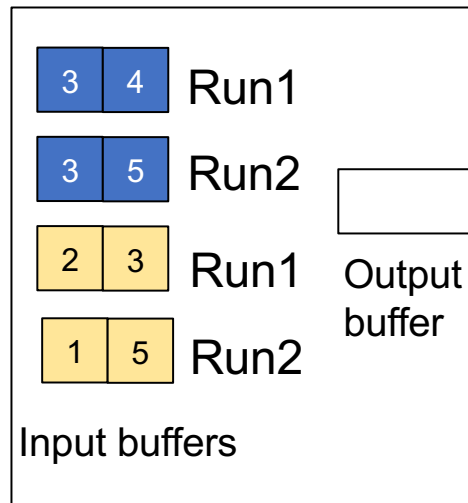
Merge-Join Example

Step 2: Join while merging sorted runs



Total cost: $3B(R) + 3B(S)$

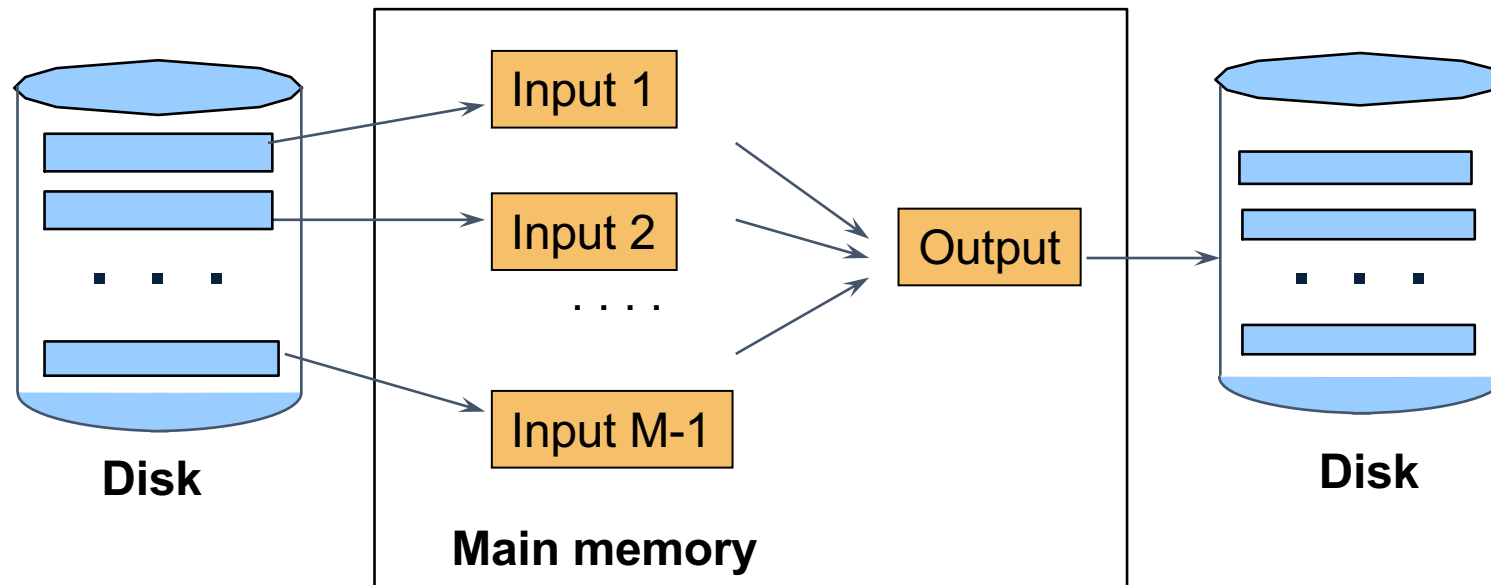
Memory $M = 5$ pages



Step 2: Join while merging
Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)
- (2,2)
- (2,2)
- (3,3)
- (3,3)
- ...

Merge-Join



$M_1 = B(R)/M$ runs for R

$M_2 = B(S)/M$ runs for S

Merge-join $M_1 + M_2$ runs;

need $M_1 + M_2 \leq M$ to process all runs

i.e. $B(R) + B(S) \leq M^2$

Summary of External Join Algorithms

- Block Nested Loop: $B(S) + B(R) \cdot B(S) / (M-1)$
- Index Join:
 - Clustered: $B(R) + T(R)B(S)/V(S,a)$
 - Unclustered: $B(R) + T(R)T(S)/V(S,a)$
- Merge Join: $3B(R) + 3B(S)$
 - $B(R) + B(S) \leq M^2$
- Partitioned Hash Join: (coming up next)

Partitioned Hash Algorithms

- Partition R it into k buckets on disk:
 $R_1, R_2, R_3, \dots, R_k$

Partitioned Hash Algorithms

- Partition R it into k buckets on disk:
 $R_1, R_2, R_3, \dots, R_k$
- Assuming $B(R_1)=B(R_2)=\dots=B(R_k)$, we have
 $B(R_i) = B(R)/k$, for all i

Partitioned Hash Algorithms

- Partition R it into k buckets on disk:
 $R_1, R_2, R_3, \dots, R_k$
- Assuming $B(R_1)=B(R_2)=\dots=B(R_k)$, we have
 $B(R_i) = B(R)/k$, for all i
- Goal: each R_i should fit in main memory:
 $B(R_i) \leq M$

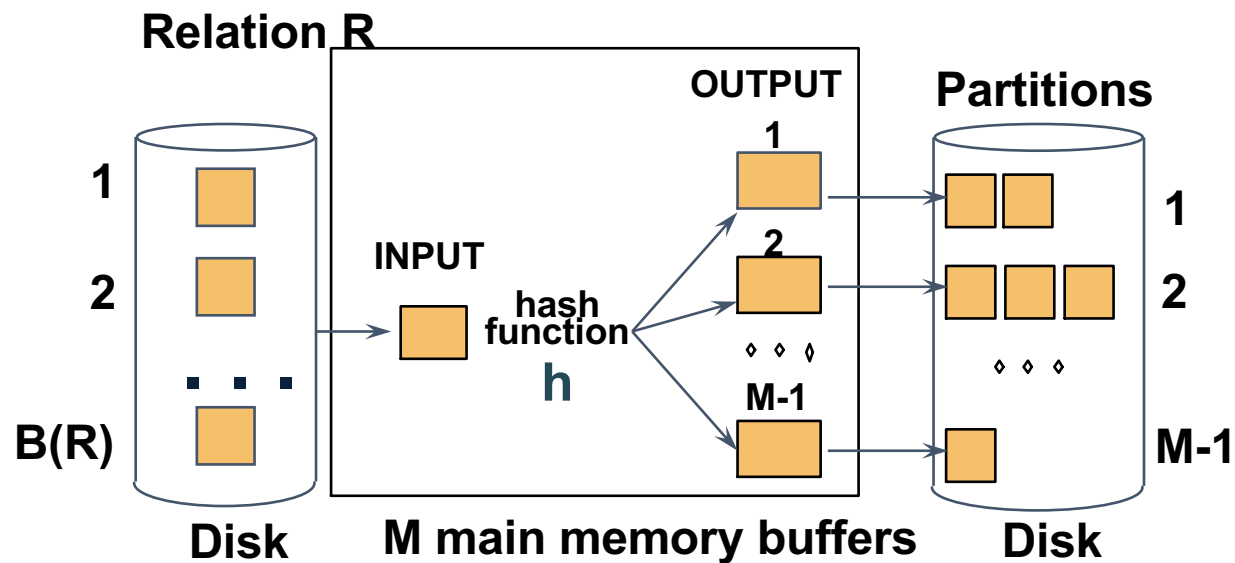
Partitioned Hash Algorithms

- Partition R it into k buckets on disk:
 $R_1, R_2, R_3, \dots, R_k$
- Assuming $B(R_1)=B(R_2)=\dots=B(R_k)$, we have
 $B(R_i) = B(R)/k$, for all i
- Goal: each R_i should fit in main memory:
 $B(R_i) \leq M$

How do we choose k ?

Partitioned Hash Algorithms

- We choose $k = M-1$ Each bucket has size approx. $B(R)/(M-1) \approx B(R)/M$



Assumption: $B(R)/M \leq M$, i.e. $B(R) \leq M^2$

Partitioned Hash Join (Grace-Join)

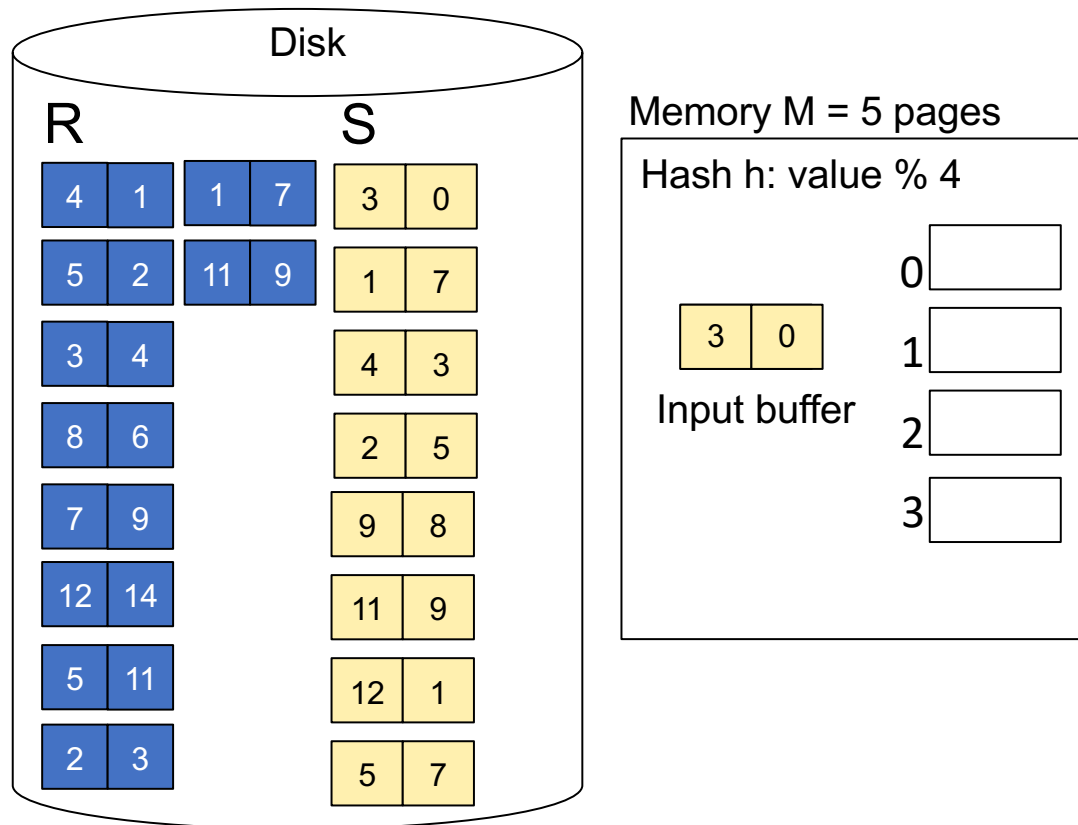
$R \bowtie S$

- Step 1:
 - Hash S into M-1 buckets
 - Send all buckets to disk
- Step 2
 - Hash R into M-1 buckets
 - Send all buckets to disk
- Step 3
 - Join every pair of buckets

Note: partitioned hash-join
is sometimes called
grace-join

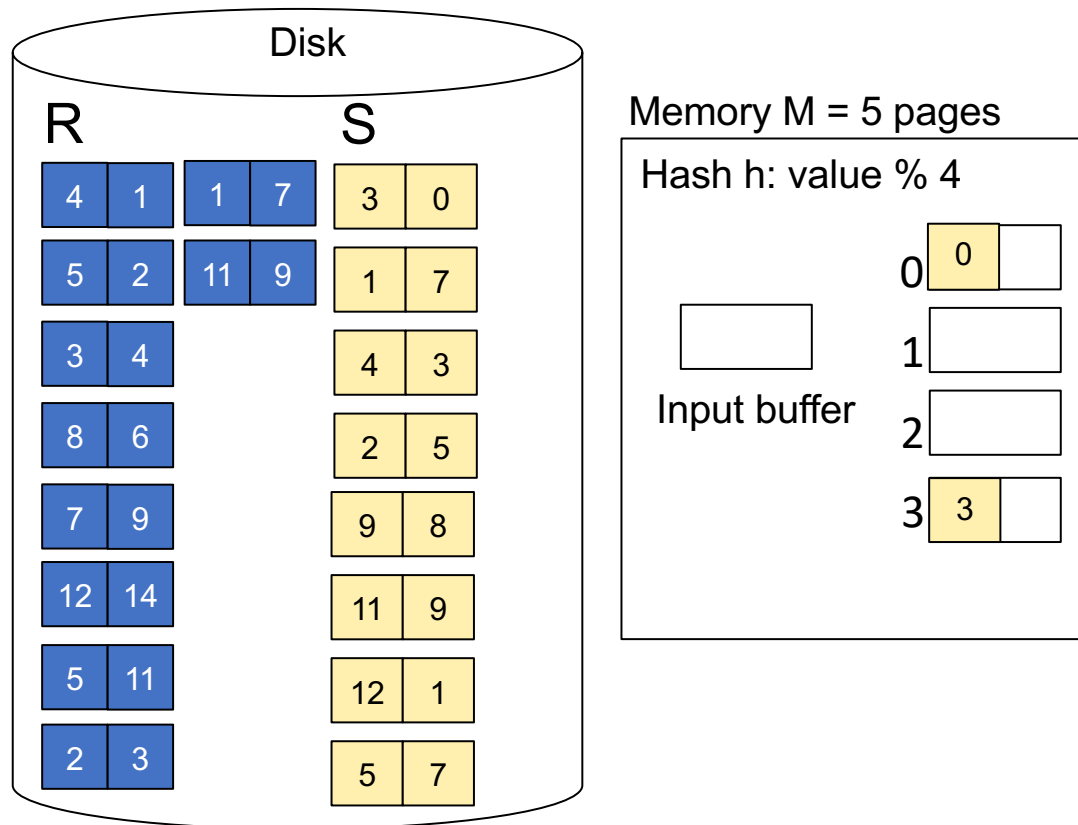
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into $M-1$ (=4 buckets)



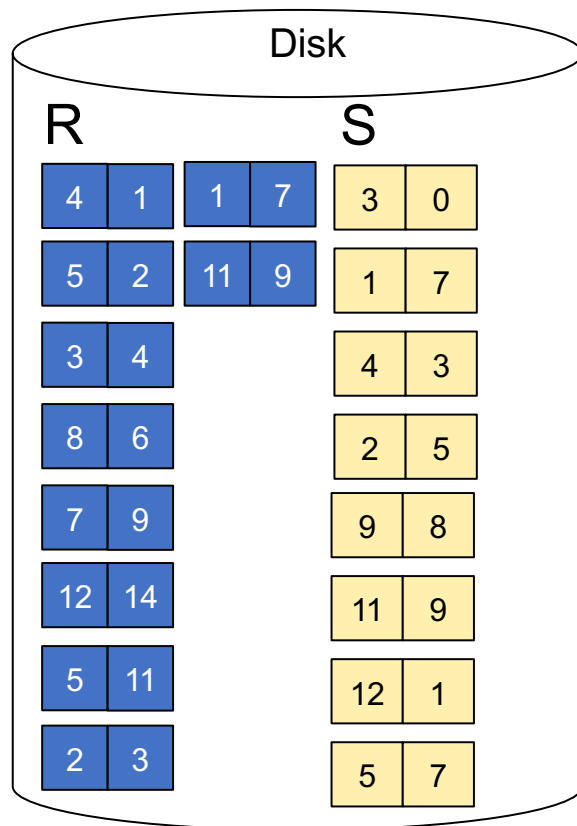
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



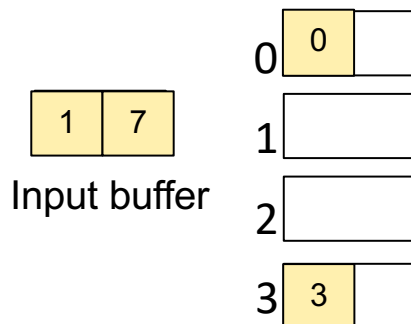
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



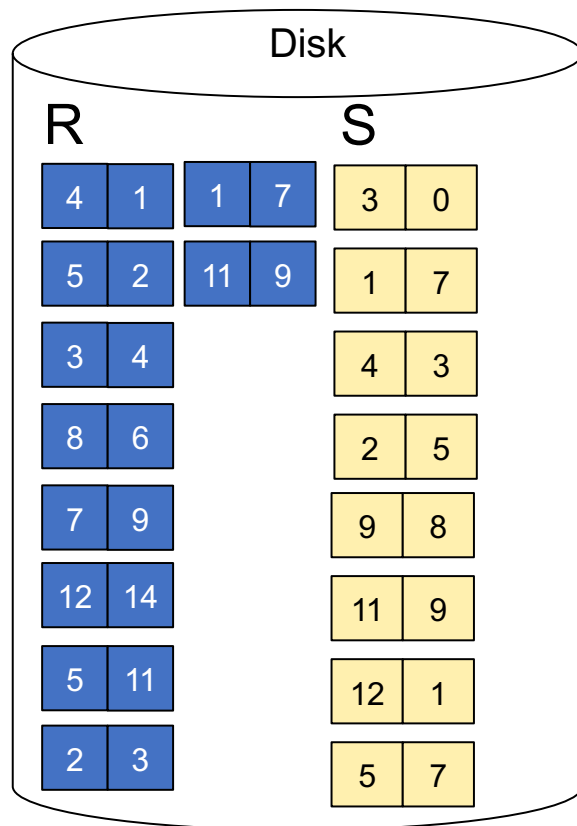
Memory M = 5 pages

Hash h: value % 4



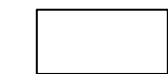
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets

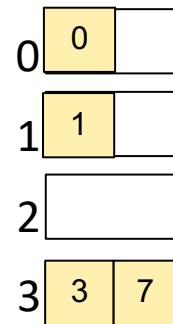


Memory M = 5 pages

Hash h: value % 4

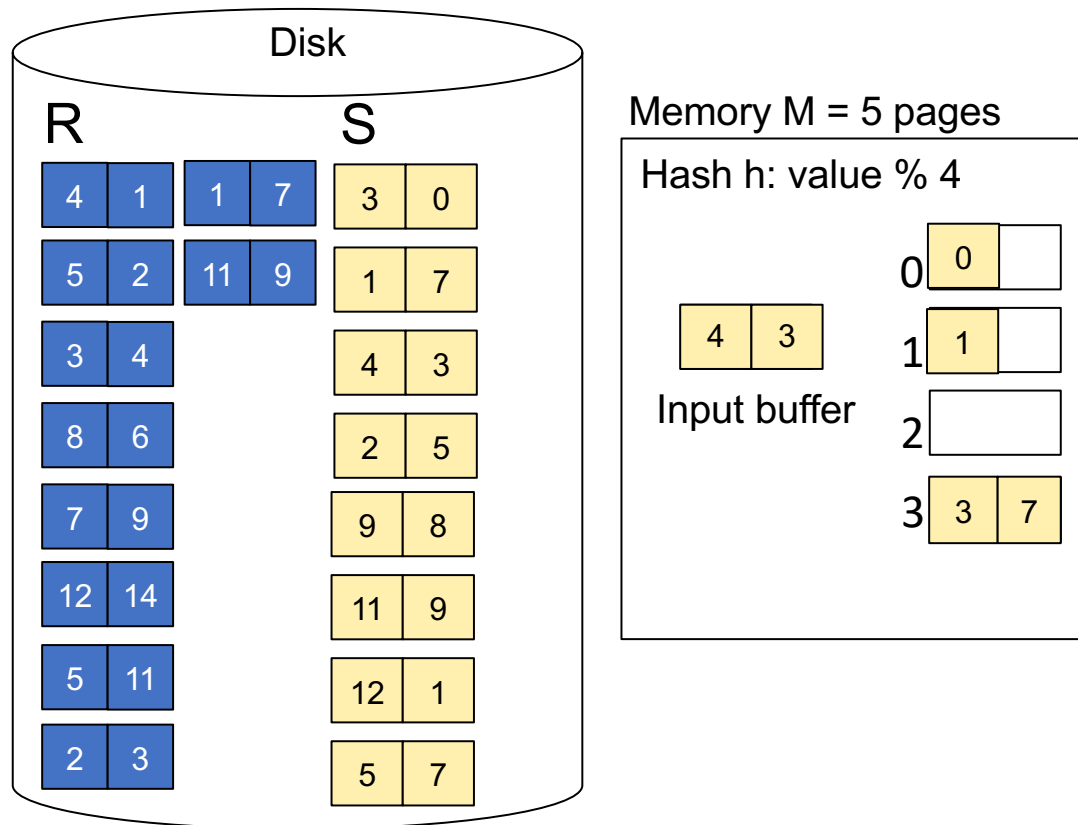


Input buffer



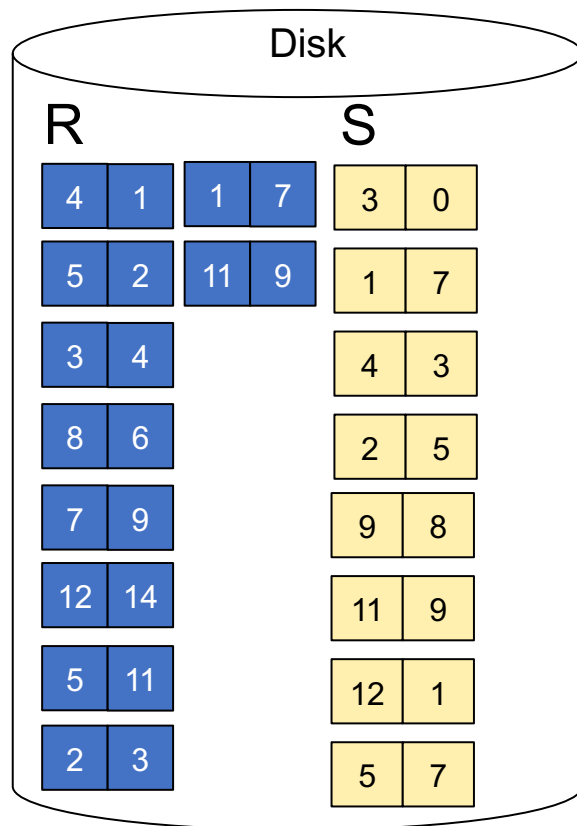
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



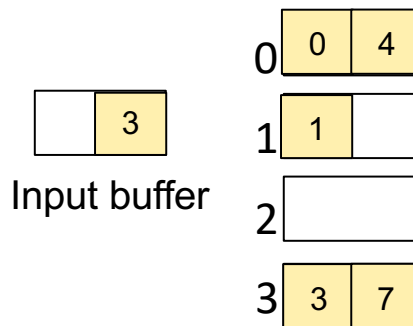
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
When a bucket fills up, flush it to disk



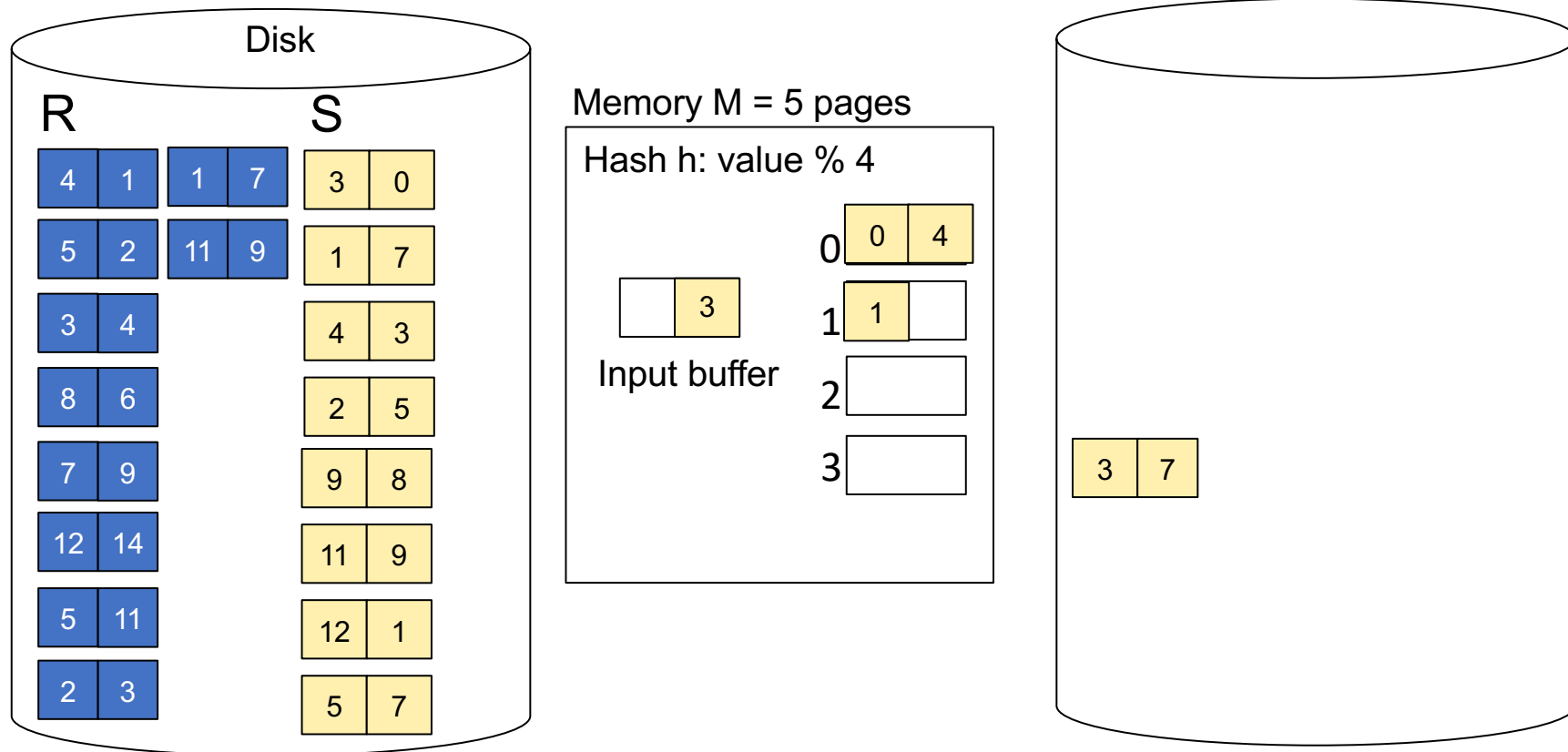
Memory M = 5 pages

Hash h: value % 4



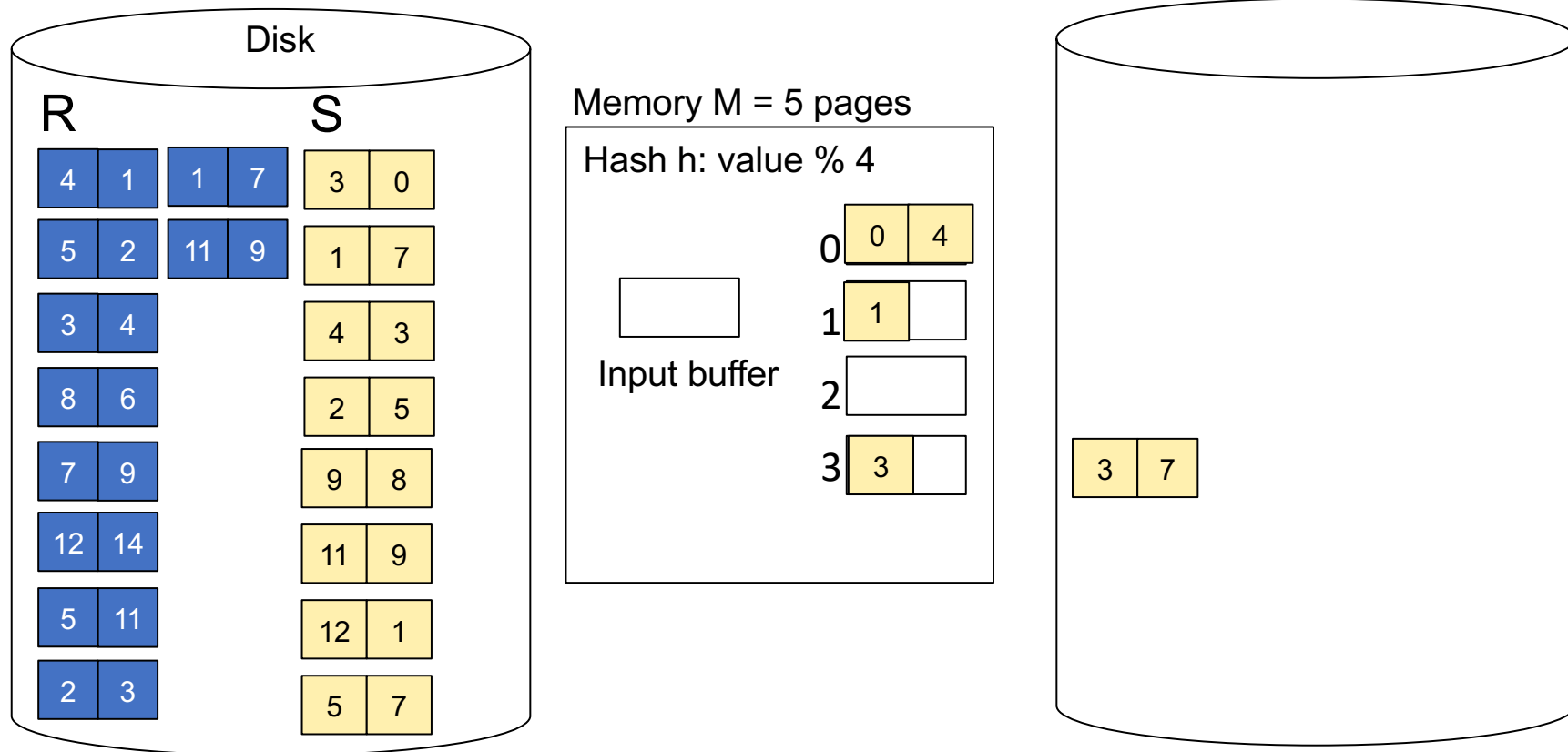
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
When a bucket fills up, flush it to disk



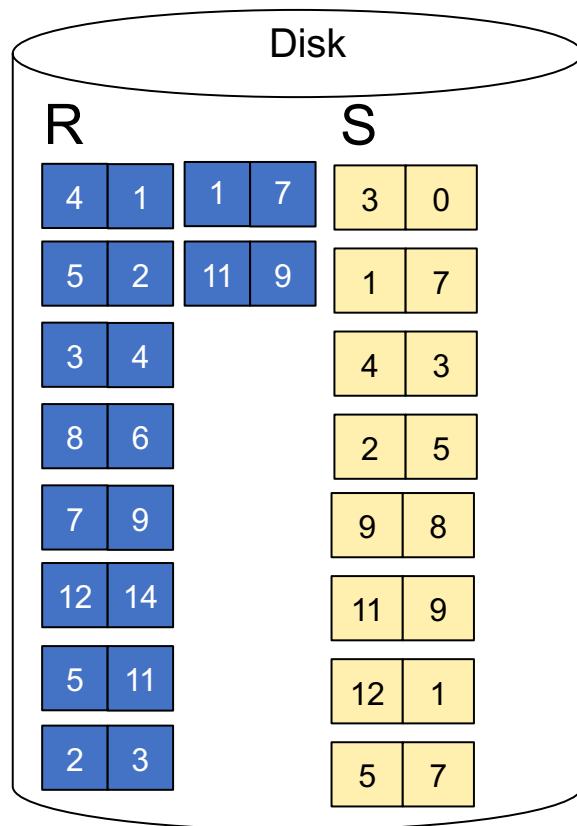
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
When a bucket fills up, flush it to disk



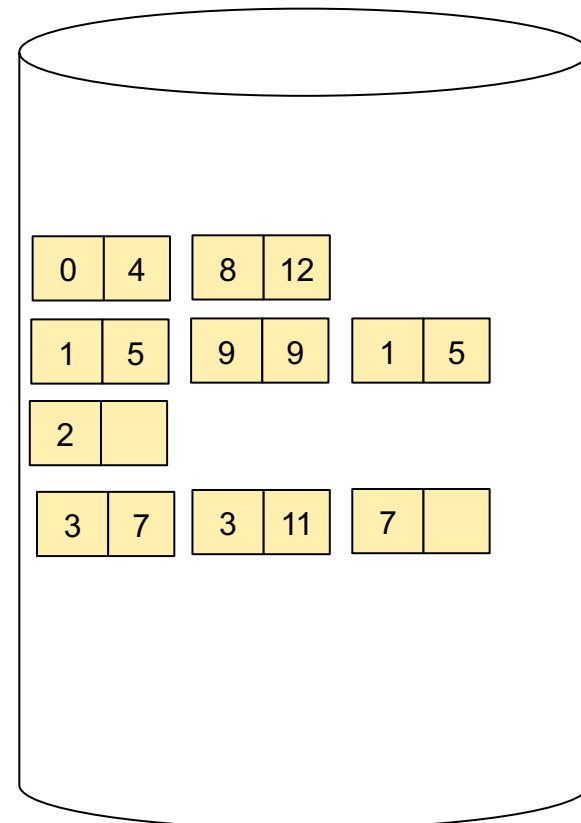
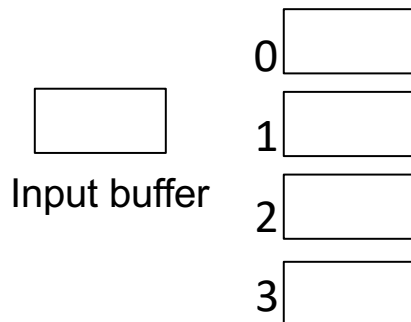
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
At the end, we get relation S back on disk split into 4 buckets



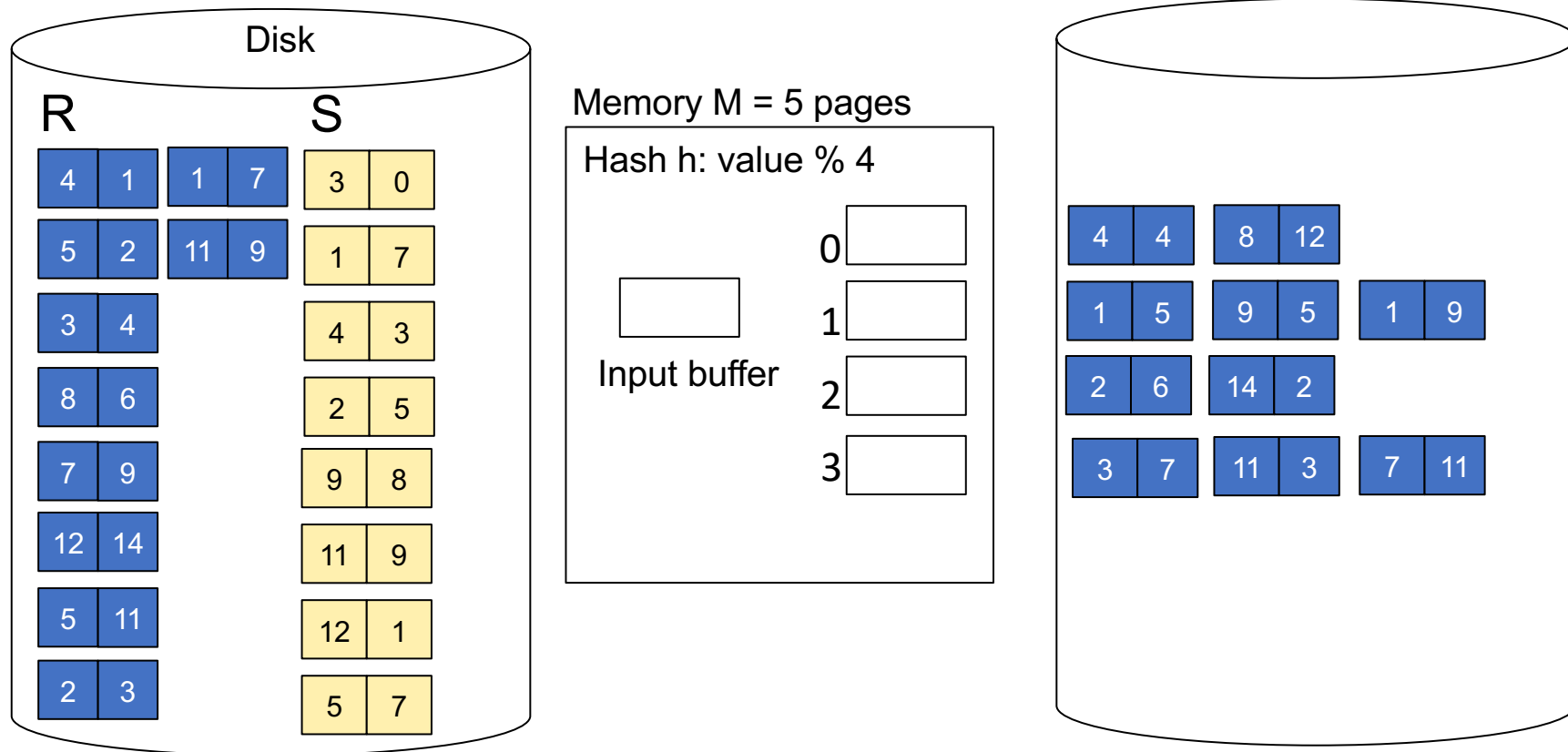
Memory M = 5 pages

Hash h: value % 4



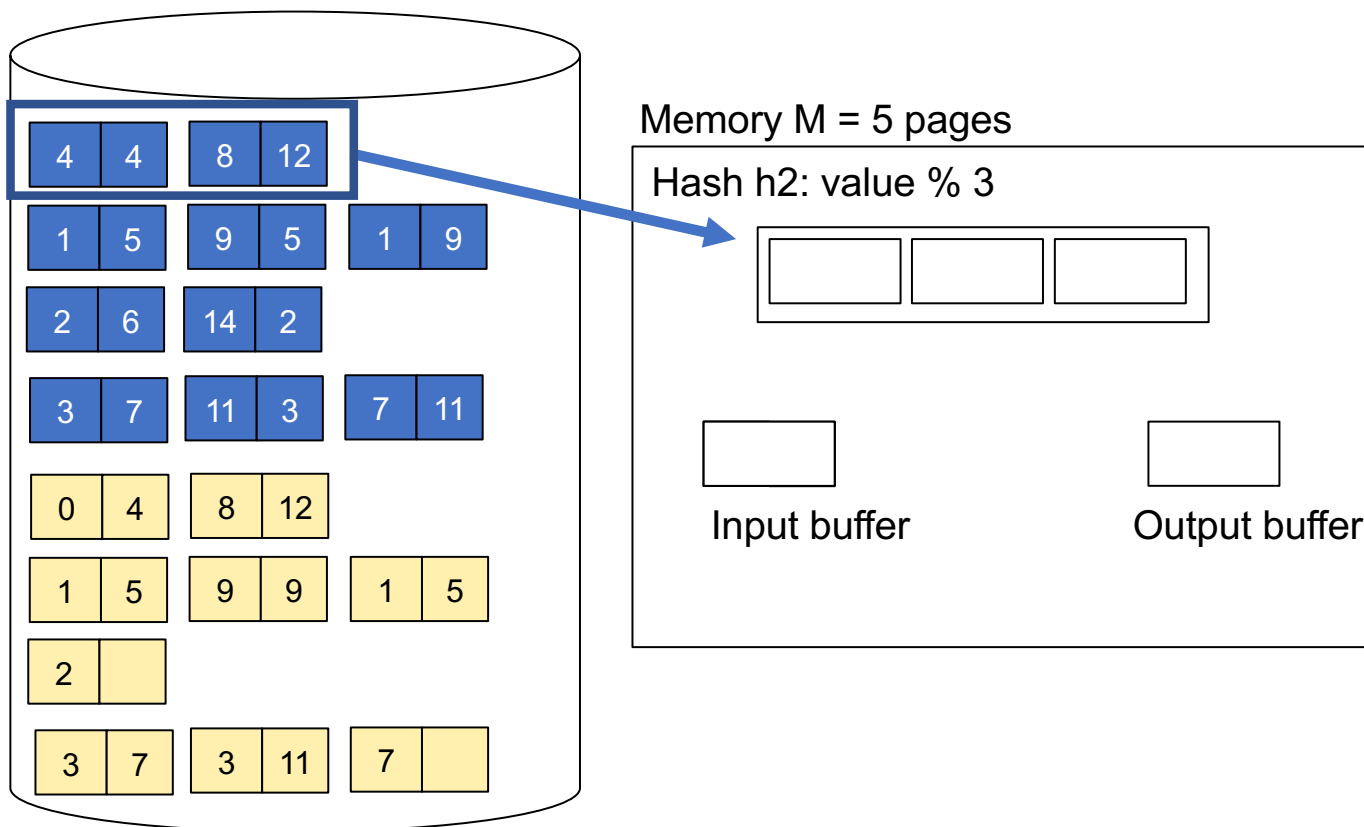
Partitioned Hash-Join Example

Step 2: Read relation R one page at a time and hash into same 4 buckets



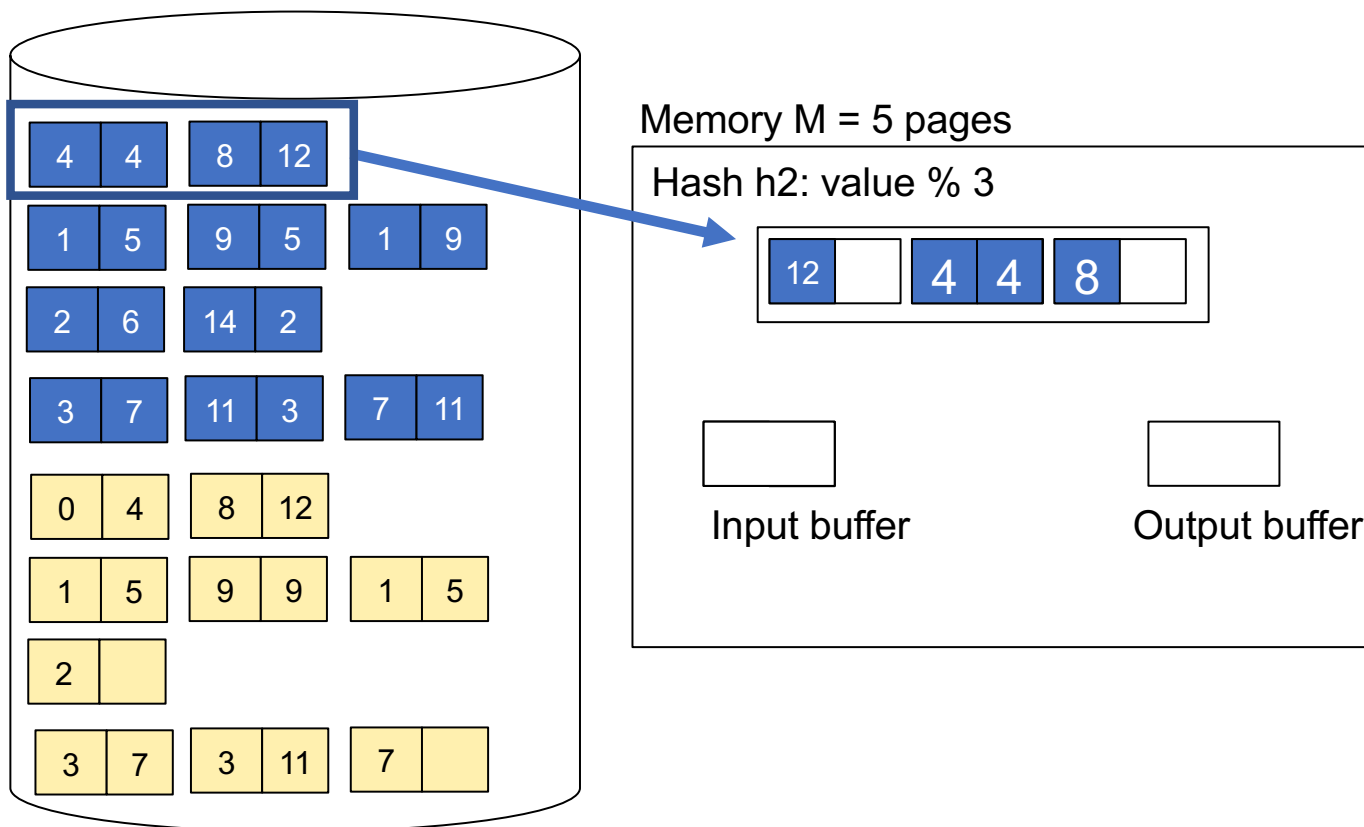
Partitioned Hash-Join Example

Step 3: Read one partition of R and create hash table in memory using a *different* hash function



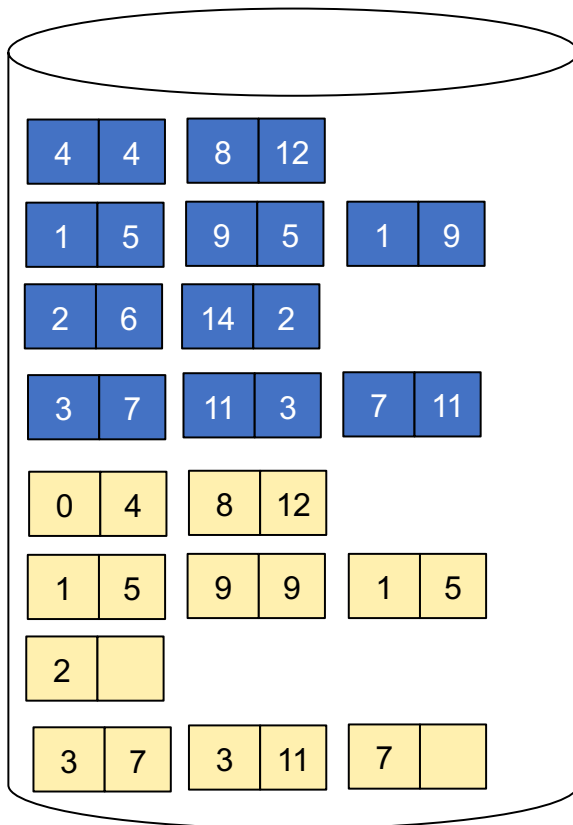
Partitioned Hash-Join Example

Step 3: Read one partition of R and create hash table in memory using a *different* hash function



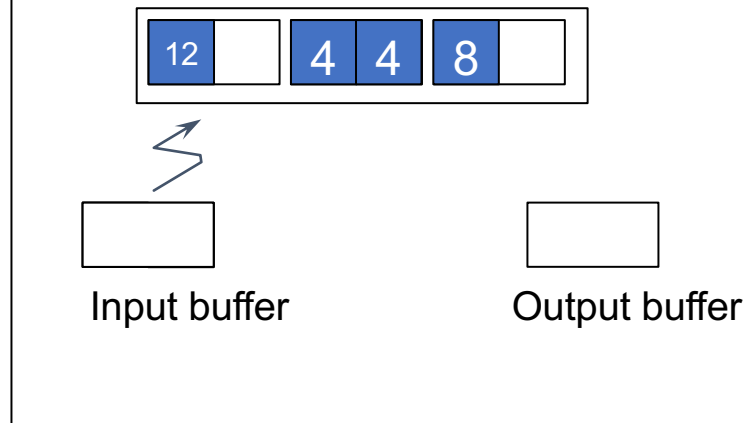
Partitioned Hash-Join Example

Step 3: Read one partition of R and create hash table in memory using a *different* hash function



Memory M = 5 pages

Hash h2: value % 3

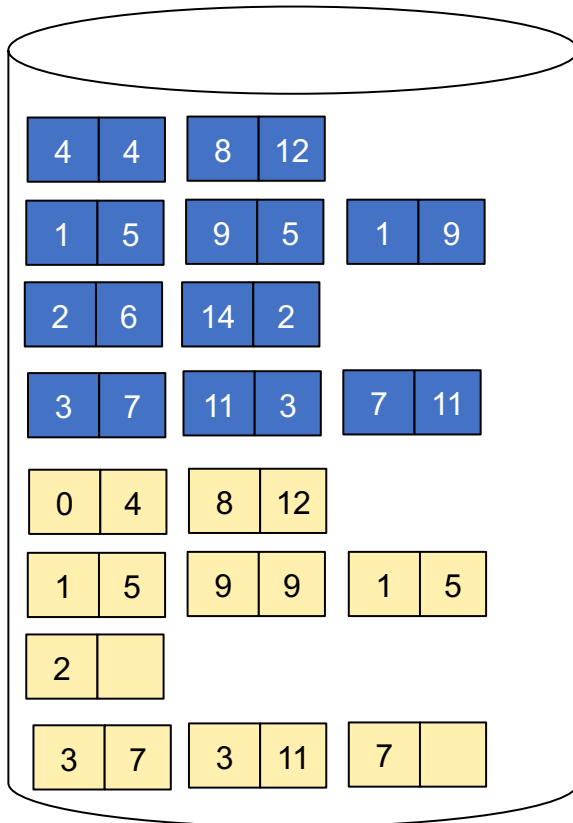


Partitioned Hash-Join Example

Step 4: Scan matching partition of S and probe the hash table

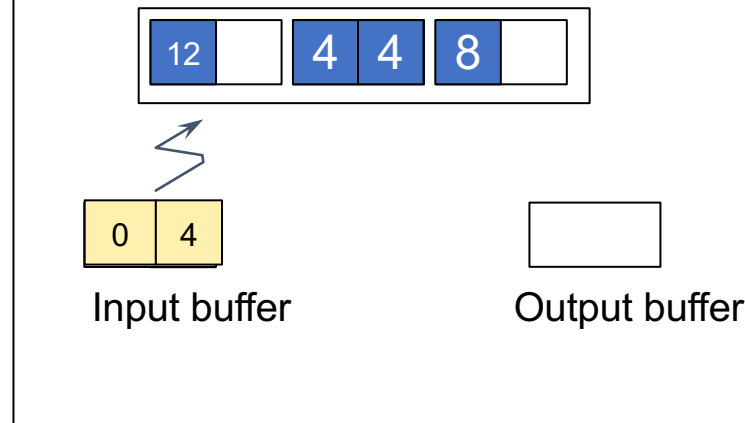
Step 5: Repeat for all the buckets

Total cost: $3B(R) + 3B(S)$



Memory $M = 5$ pages

Hash h_2 : value % 3

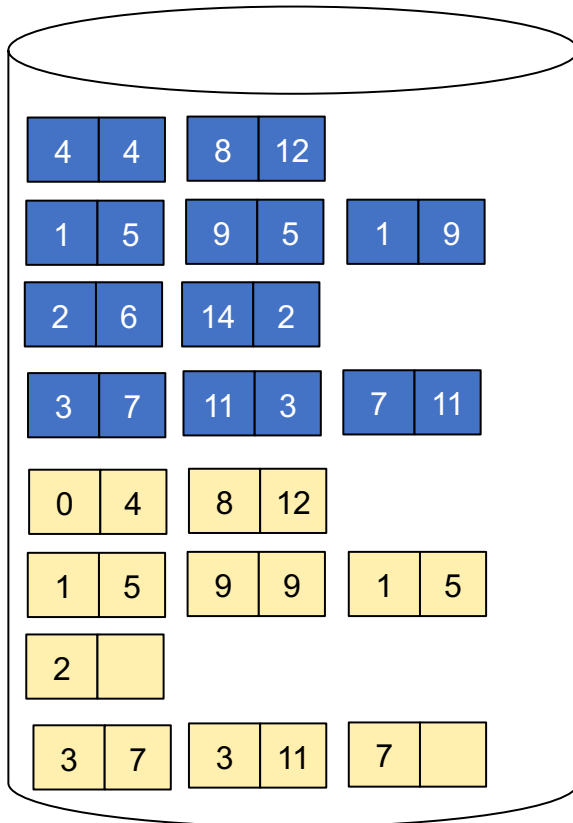


Partitioned Hash-Join Example

Step 4: Scan matching partition of S and probe the hash table

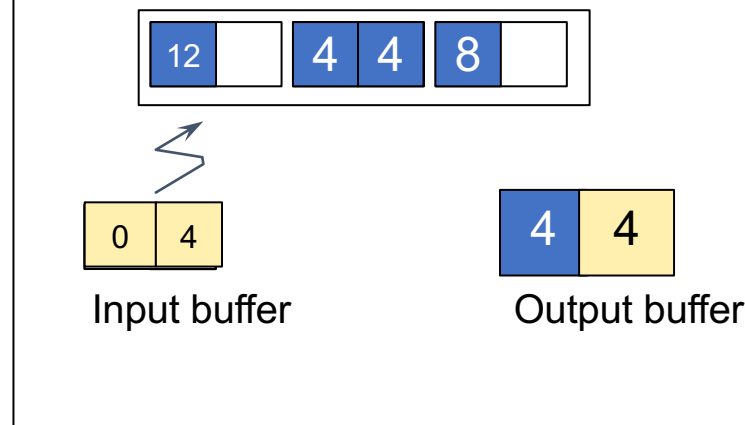
Step 5: Repeat for all the buckets

Total cost: $3B(R) + 3B(S)$



Memory $M = 5$ pages

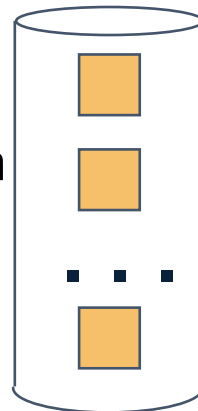
Hash h_2 : value % 3



Partitioned Hash-Join

- Partition both relations using hash fn h : R tuples in partition i will only match S tuples in partition i .

Original Relation



Disk

INPUT



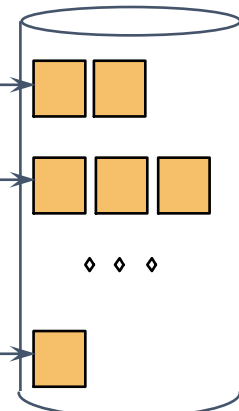
hash function h

OUTPUT



B main memory buffers

Partitions



1

2

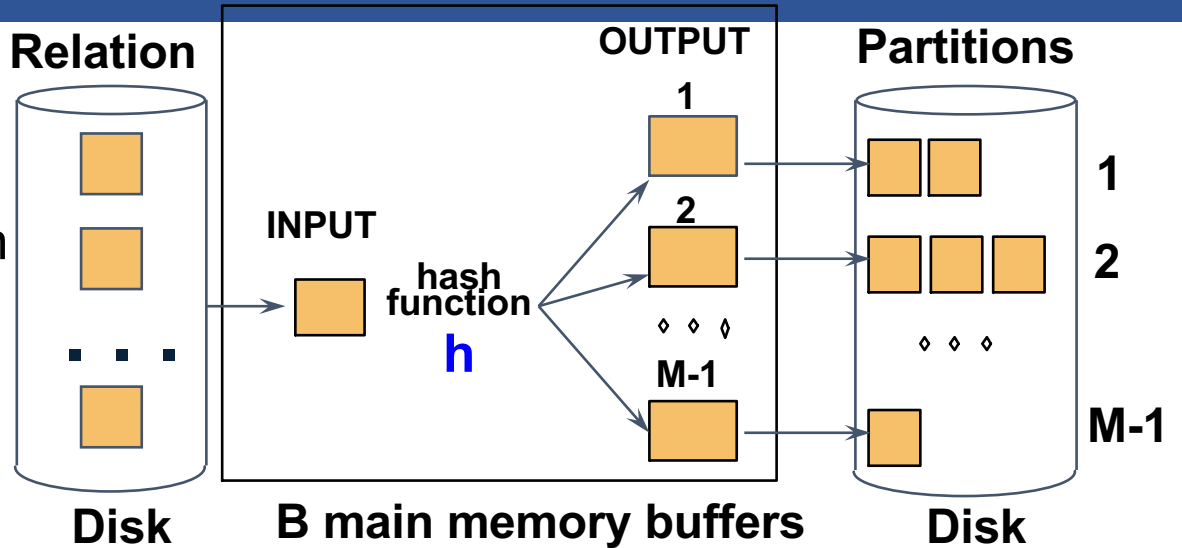
$M-1$

Disk

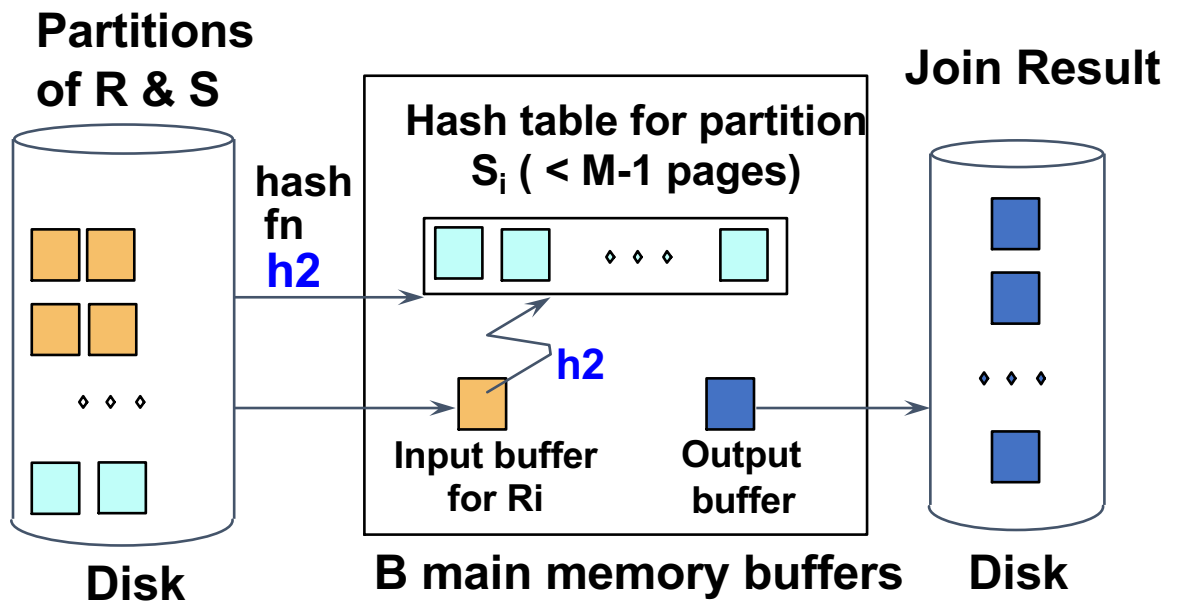
Partitioned Hash-Join

- Partition both relations using hash fn h : R tuples in partition i will only match S tuples in partition i .

Original Relation



- Read in a partition of R , hash it using h_2 ($\neq h!$). Scan matching partition of S , search for matches.



Partitioned Hash-Join

- Cost: $3B(R) + 3B(S)$
- Assumption: $\min(B(R), B(S)) \leq M^2$

Hybrid Hash Join Algorithm (see book)

- Partition S into k buckets
 - t buckets S_1, \dots, S_t stay in memory
 - $k-t$ buckets S_{t+1}, \dots, S_k to disk
- Partition R into k buckets
 - First t buckets join immediately with S
 - Rest $k-t$ buckets go to disk
- Finally, join $k-t$ pairs of buckets:
 $(R_{t+1}, S_{t+1}), (R_{t+2}, S_{t+2}), \dots, (R_k, S_k)$

Summary of External Join Algorithms

- Block Nested Loop: $B(S) + B(R) \cdot B(S) / (M-1)$
- Index Join:
 - Clustered: $B(R) + T(R)B(S)/V(S,a)$
 - Unclustered: $B(R) + T(R)T(S)/V(S,a)$
- Merge Join: $3B(R) + 3B(S)$
 - $B(R) + B(S) \leq M^2$
- Partitioned Hash Join: $3B(R) + 3B(S)$
 - $\min(B(R), B(S)) \leq M^2$