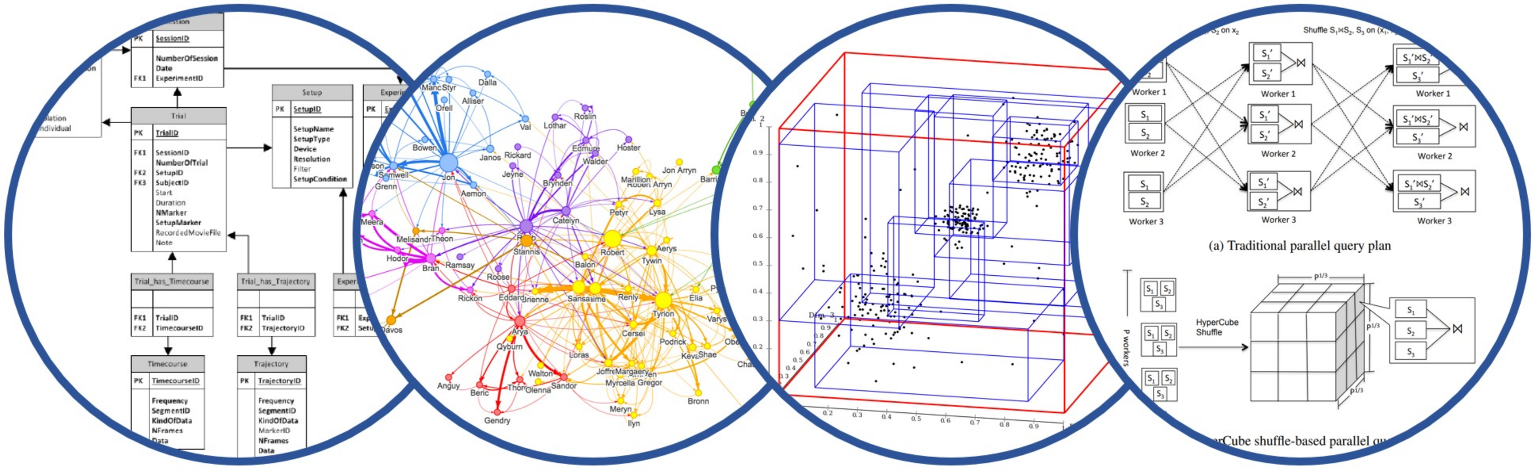

Database System Internals

External Memory Algorithms (part 3)

Paul G. Allen School of Computer Science and Engineering
University of Washington, Seattle

Index Based Selection

- Example:

B(R) = 2000
T(R) = 100,000
V(R, a) = 20

cost of $\sigma_{a=v}(R) = ?$

- Table scan:
- Index based selection:

Index Based Selection

- Example:

B(R) = 2000
T(R) = 100,000
V(R, a) = 20

cost of $\sigma_{a=v}(R) = ?$

- Table scan: B(R) = 2,000 I/Os
- Index based selection:

Index Based Selection

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T(R) = 100,000
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  - If index is clustered:
  - If index is unclustered:

Index Based Selection

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B(R) = 2000
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- Table scan: B(R) = 2,000 I/Os
- Index based selection:
  - If index is clustered:
  - If index is unclustered: T(R)/V(R,a) = 5,000 I/Os

Index Based Selection

- Example:

B(R) = 2000
T(R) = 100,000
V(R, a) = 20

cost of $\sigma_{a=v}(R)$ = ?

- Table scan: B(R) = 2,000 I/Os
- Index based selection:
  - If index is clustered: B(R)/V(R,a) = 100 I/Os
  - If index is unclustered: T(R)/V(R,a) = 5,000 I/Os

Index Based Selection

- Example:

B(R) = 2000
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cost of $\sigma_{a=v}(R) = ?$

- Table scan: B(R) = 2,000 I/Os!
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Index Based Selection

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- Index based selection:
  - If index is clustered: B(R)/V(R,a) = 100 I/Os
  - If index is unclustered: T(R)/V(R,a) = 5,000 I/Os!

Lesson: Don’t build unclustered indexes when V(R,a) is small !

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S

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Outline

- **Join operator algorithms**
  - One-pass algorithms (Sec. 15.2 and 15.3)
  - Index-based algorithms (Sec 15.6)
  - Two-pass algorithms (Sec 15.4 and 15.5)

Two-Pass Algorithms

- Hash-join, merge-join assumed data <= memory

- Next: algorithm when the data >> main memory
  Called *external memory* algorithm

- Merge-join
- Partitioned hash-join

Questions

- What is the “best” algorithm for sorting an array of n elements in main memory?

- What is its runtime?

- What is the best algorithm for sorting a large file of n items on disc?

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Main memory merge-sort: 2-way
External memory merge-sort: multi-way

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Merge-Join is based on the multi-way merge-sort (next)

Merge-Sort: Basic Terminology

- A run in a sequence is an increasing subsequence

- What are the runs?

  2, 4, 99, 103, 88, 77, 3, 79, 100, 2, 50

Merge-Sort: Basic Terminology

- A run in a sequence is an increasing subsequence

- What are the runs?

  2, 4, 99, 103, | 88, | 77, | 3, 79, 100, | 2, 50

External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

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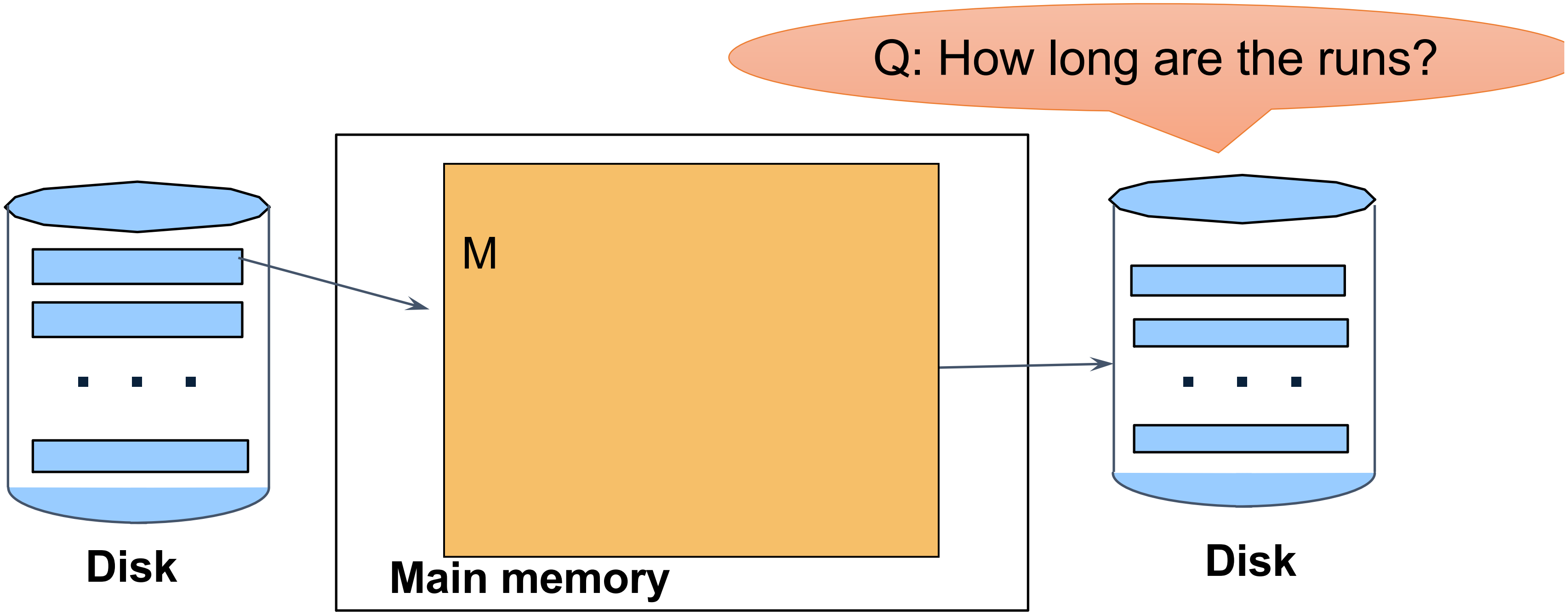
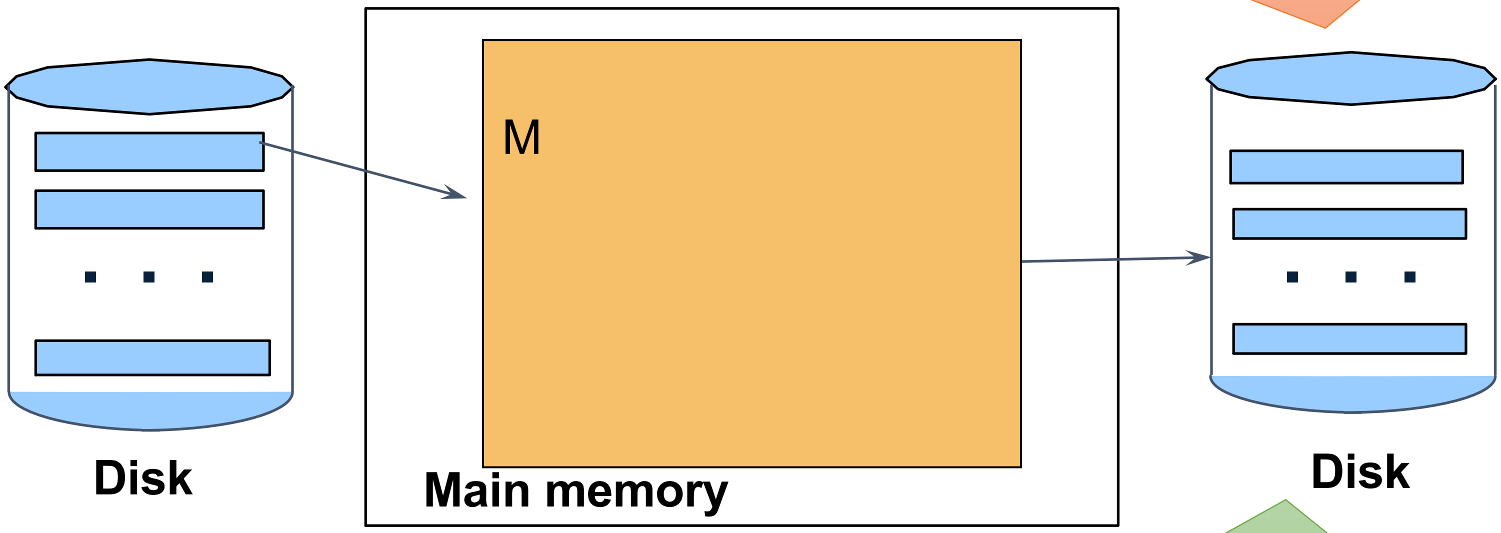


External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

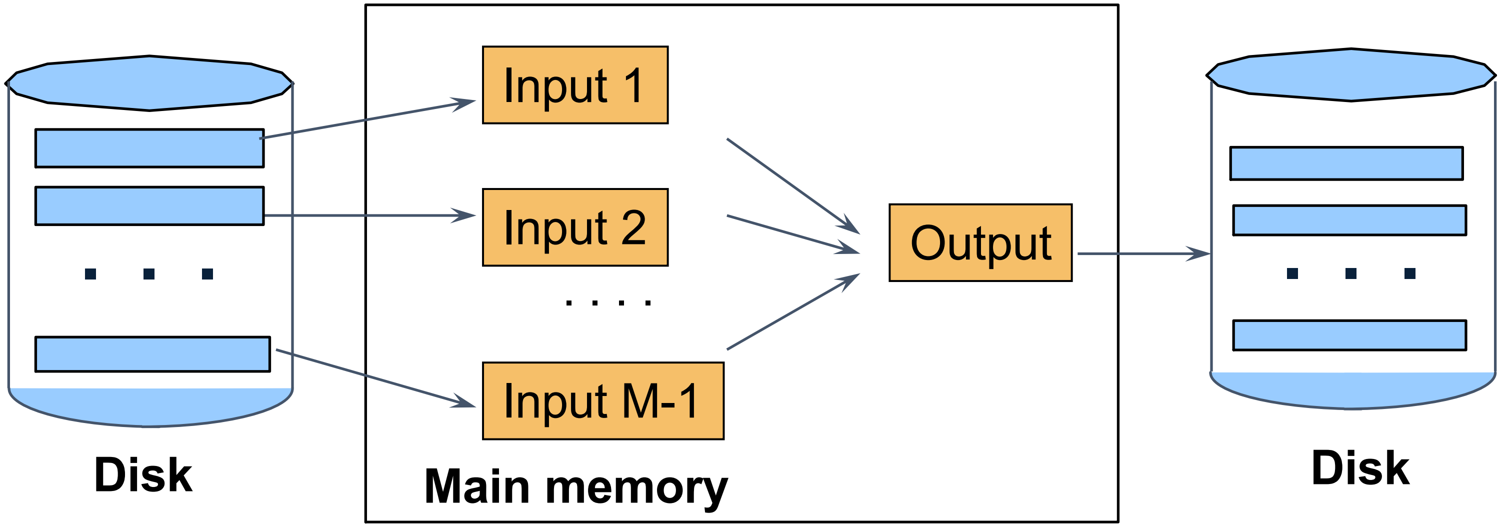


Q: How long are the runs?

A: Length = M blocks

Phase two: merge M runs into a bigger run

- Merge M – 1 runs into a new run
- Result: runs of length M (M – 1) ≈ M²



Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0

Example

- Merging three runs to produce a longer run:

0, **14, 33, 88, 92, 192, 322**
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

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0, ?

Example

- Merging three runs to produce a longer run:

0, **14, 33, 88, 92, 192, 322**
2, 4, 7, 43, 78, 103, 523
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Output:
0, 1, ?

Example

- Merging three runs to produce a longer run:

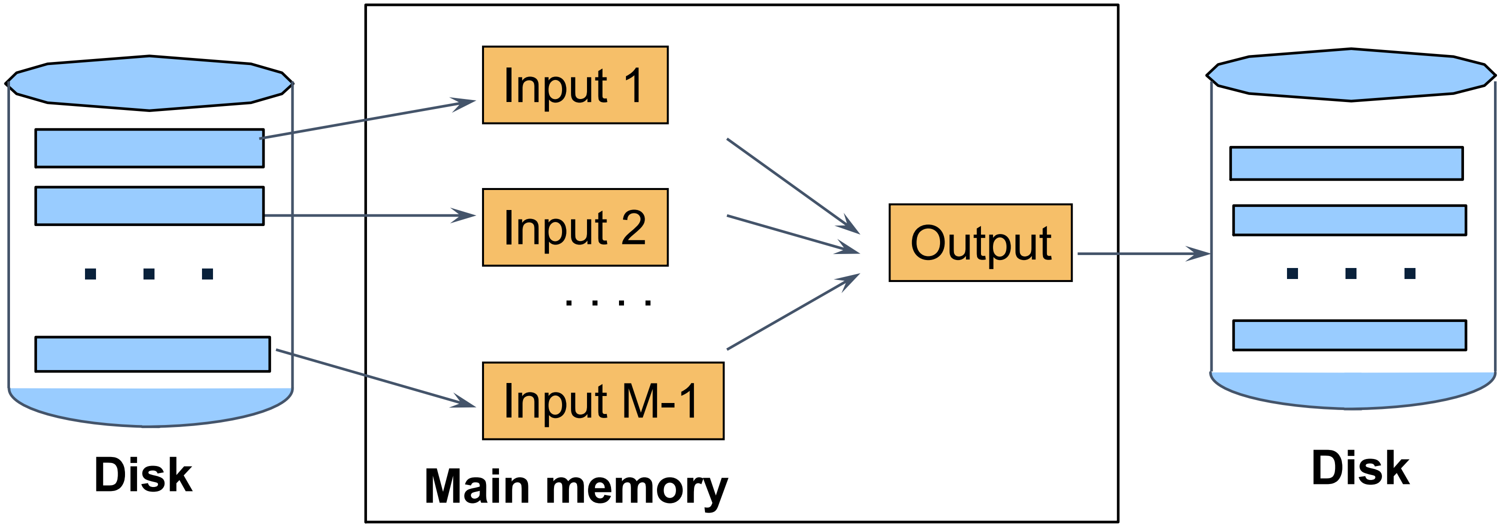
0, **14, 33, 88, 92, 192, 322**
2, 4, 7, **43, 78, 103, 523**
1, 6, **9, 12, 33, 52, 88, 320**

Output:
0, 1, 2, 4, 6, 7, ?

External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge M – 1 runs into a new run
- Result: runs of length M (M – 1) ≈ M^2



If approx. B <= M^2 then we are done

Cost of External Merge Sort

In theory:

- Number of I/O's: $O(B(R) * \log_M B(R))$

In practice:

- Assumption $B(R) <= M^2$
- Read+write+read = $3B(R)$

Discussion

- What does B(R) <= M^2 mean?
- How large can R be?

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- Example:
  - Page size = 32KB
  - Memory size 32GB: $M = 10^6$-pages

Discussion

- What does B(R) <= M^2 mean?
- How large can R be?

- Example:
  - Page size = 32KB
  - Memory size 32GB: M = 10^6 pages

- R can be as large as 10^{12} pages
  - 32 × 10^{15} Bytes = 32 PB

Merge-Join

Join R $\bowtie$ S

- How?....

Merge-Join

Join R $\bowtie$ S

- Step 1a: generate initial runs for R
- Step 1b: generate initial runs for S
- Step 2: merge and join
  - Either merge first and then join
  - Or merge & join at the same time

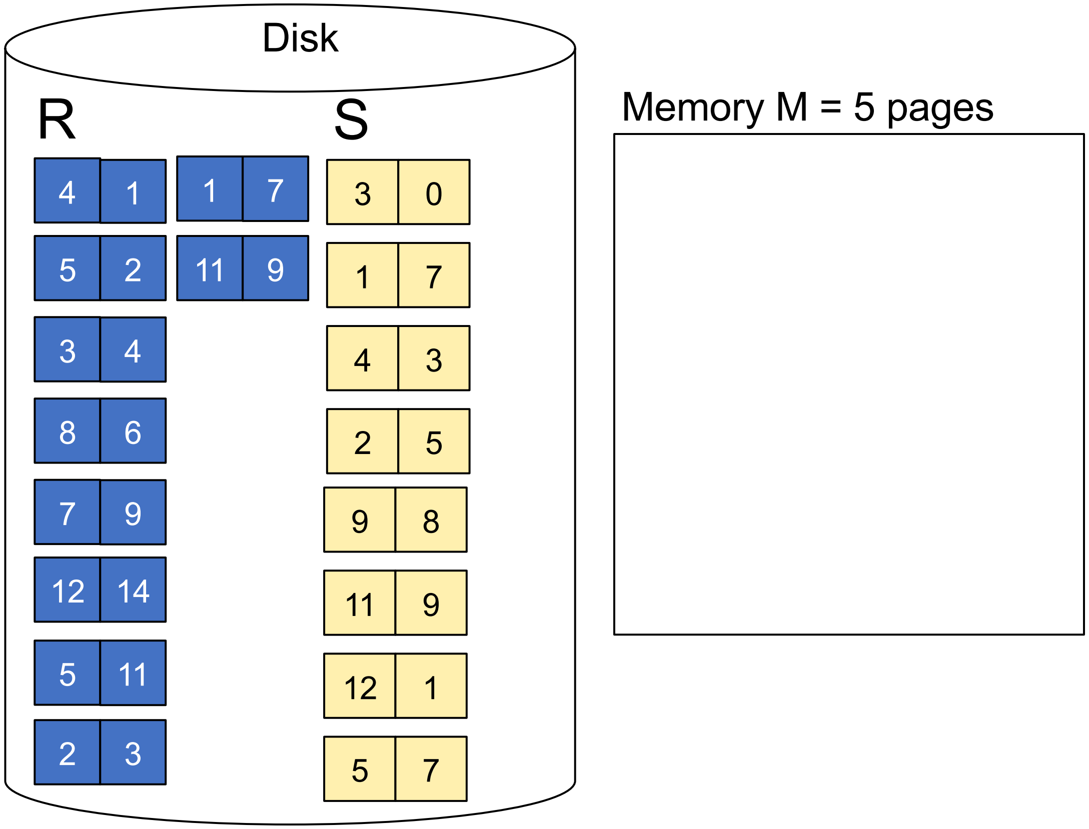
Merge-Join Example

Setup: Want to join R and S

Relation R has 10 pages with 2 tuples per page

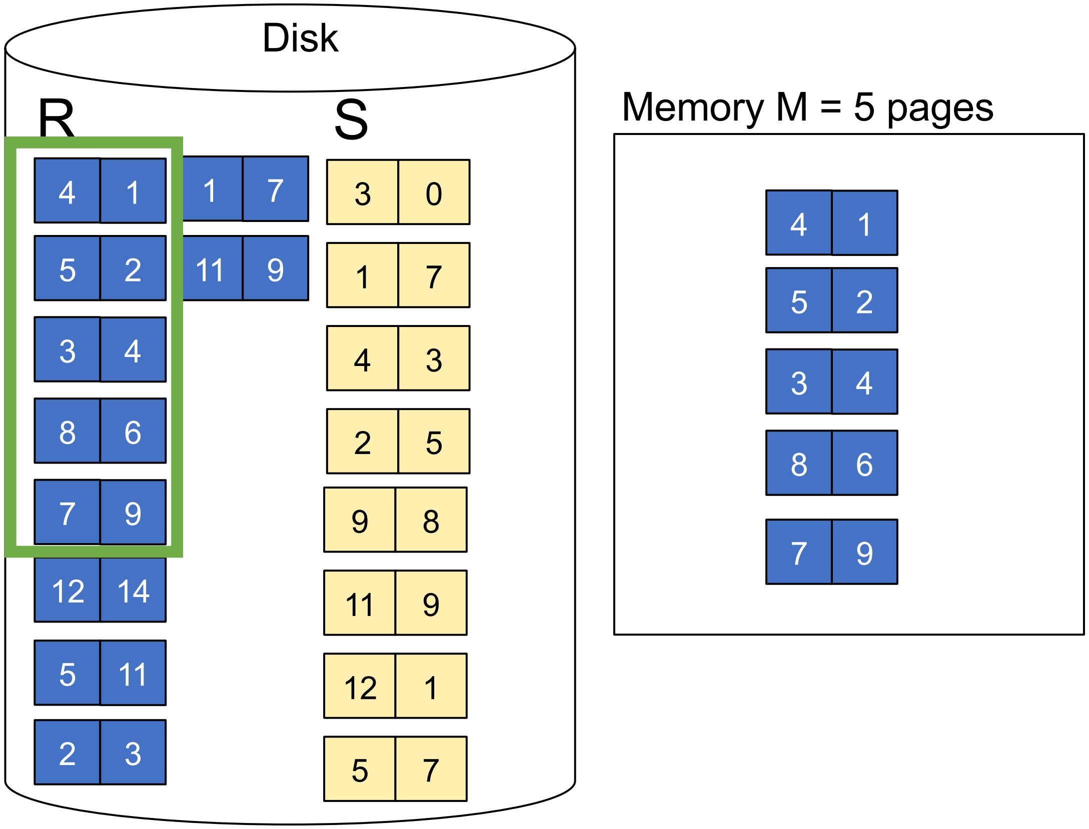
Relation S has 8 pages with 2 tuples per page

Values shown are values of join attribute for each given tuple



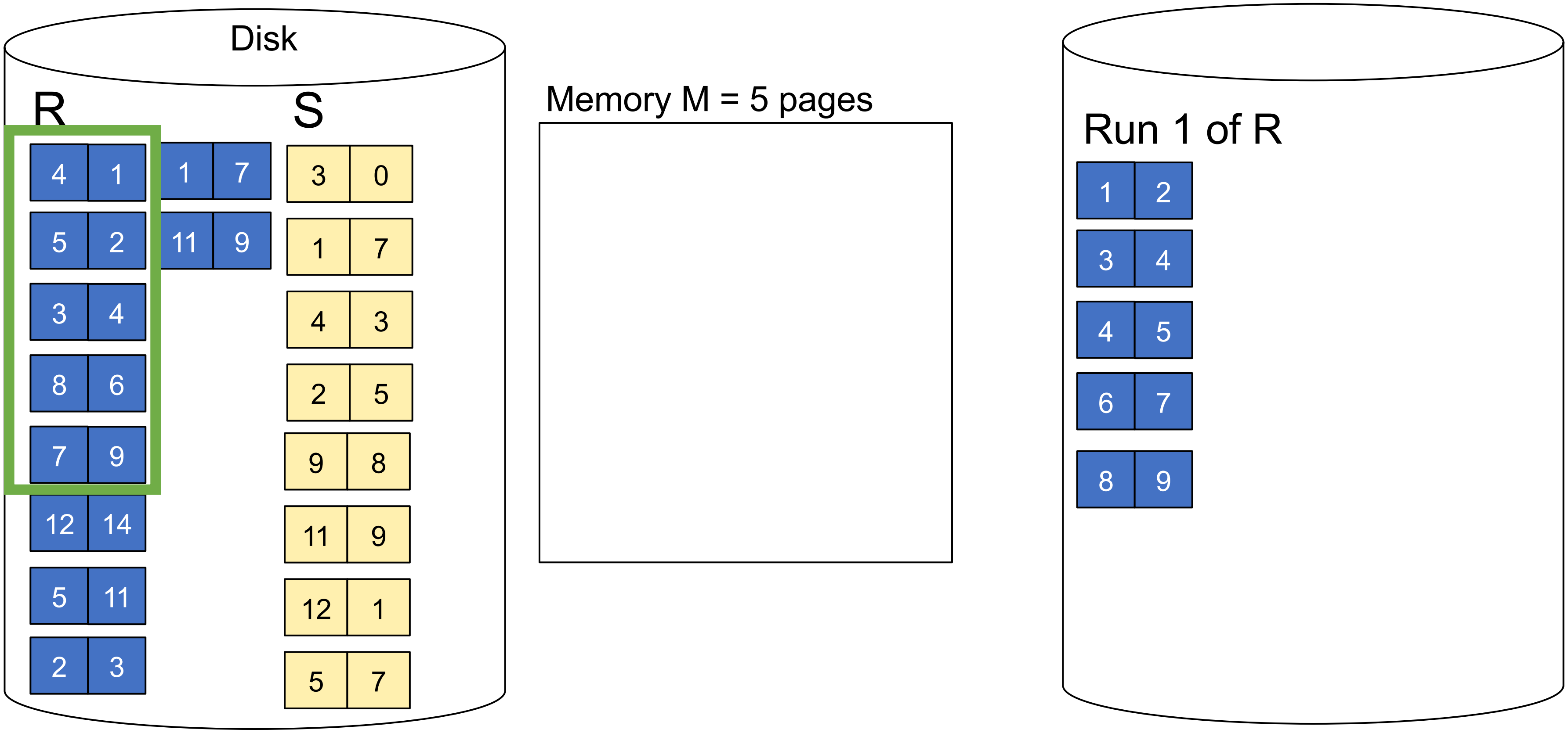
Merge-Join Example

Step 1: Read M pages of R and sort in memory



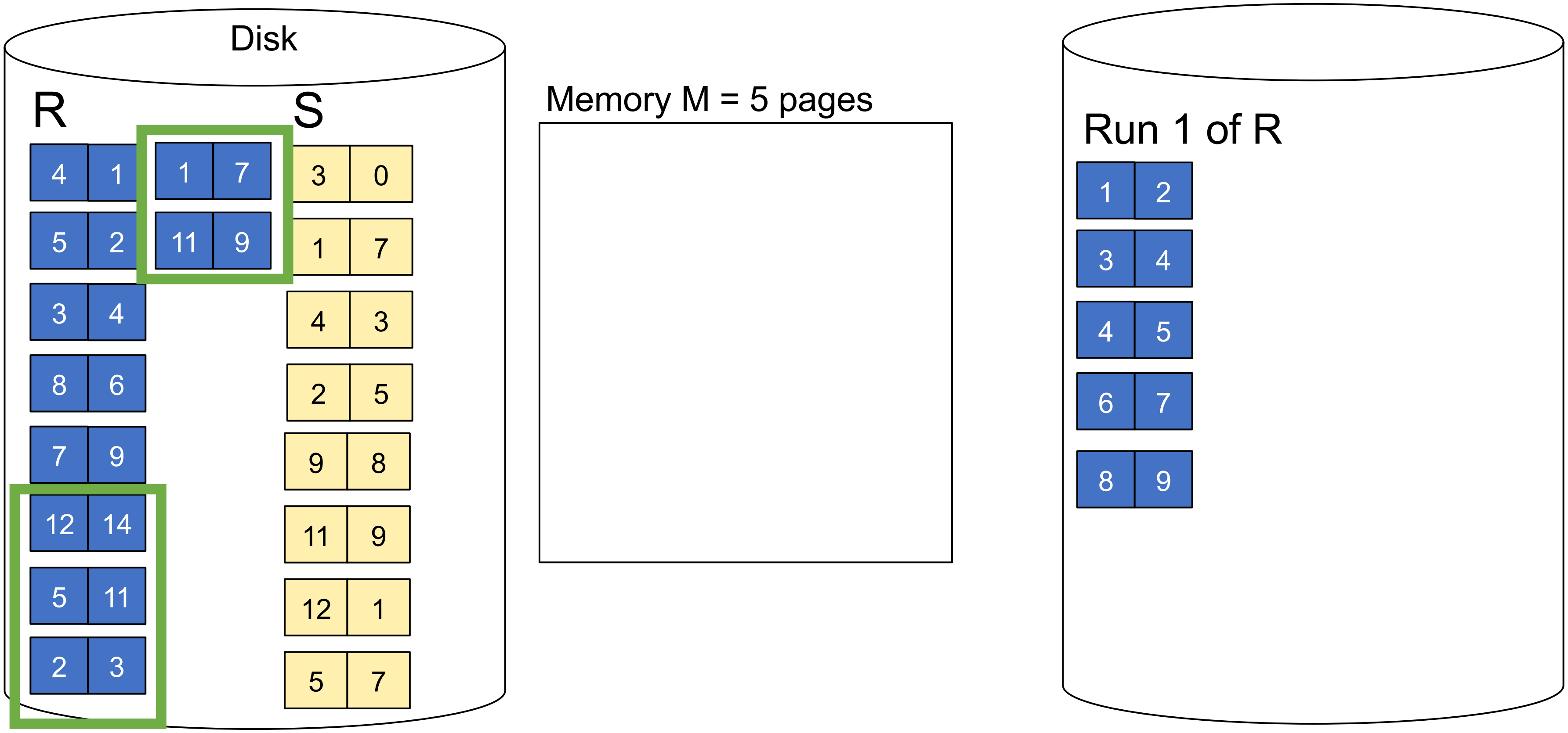
Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk



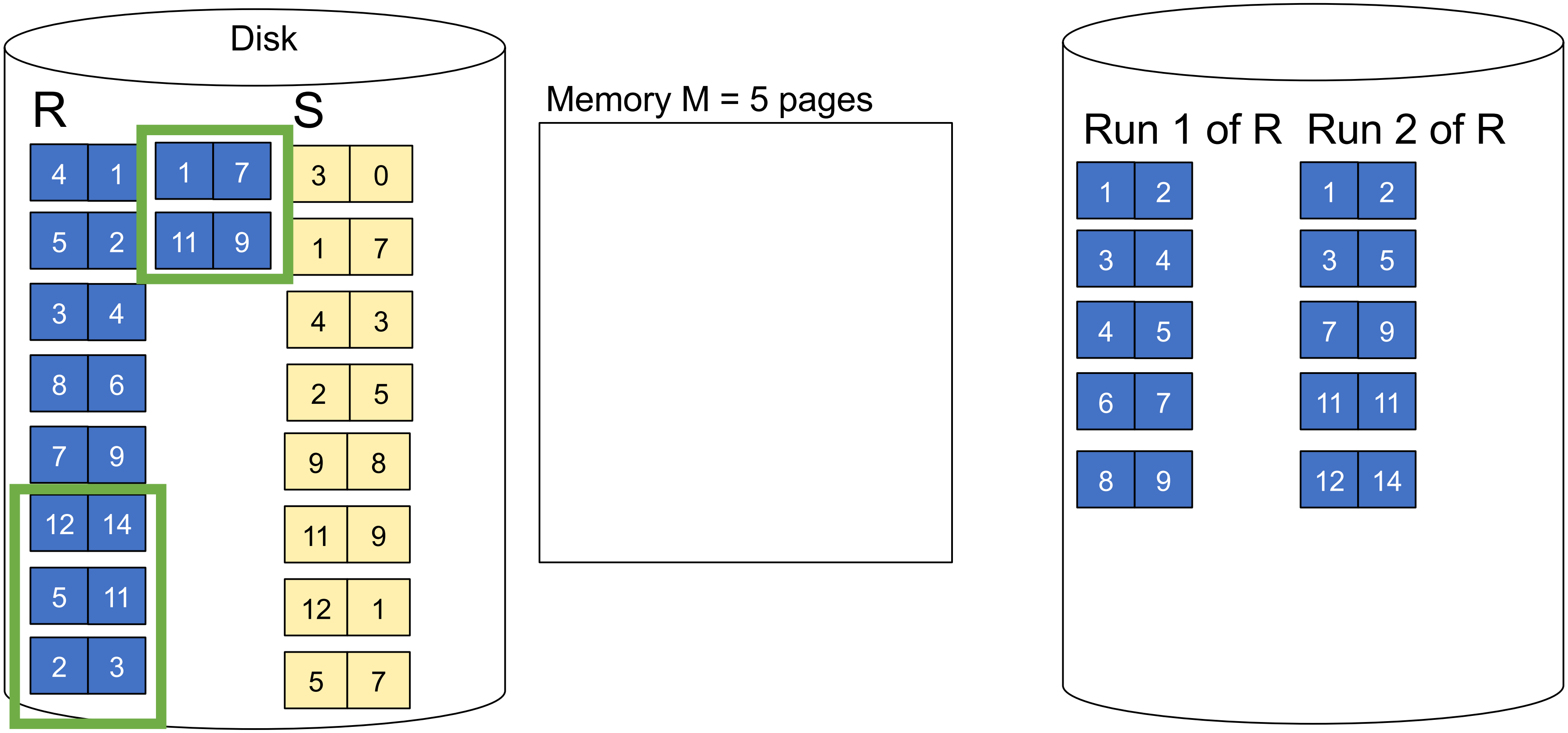
Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk



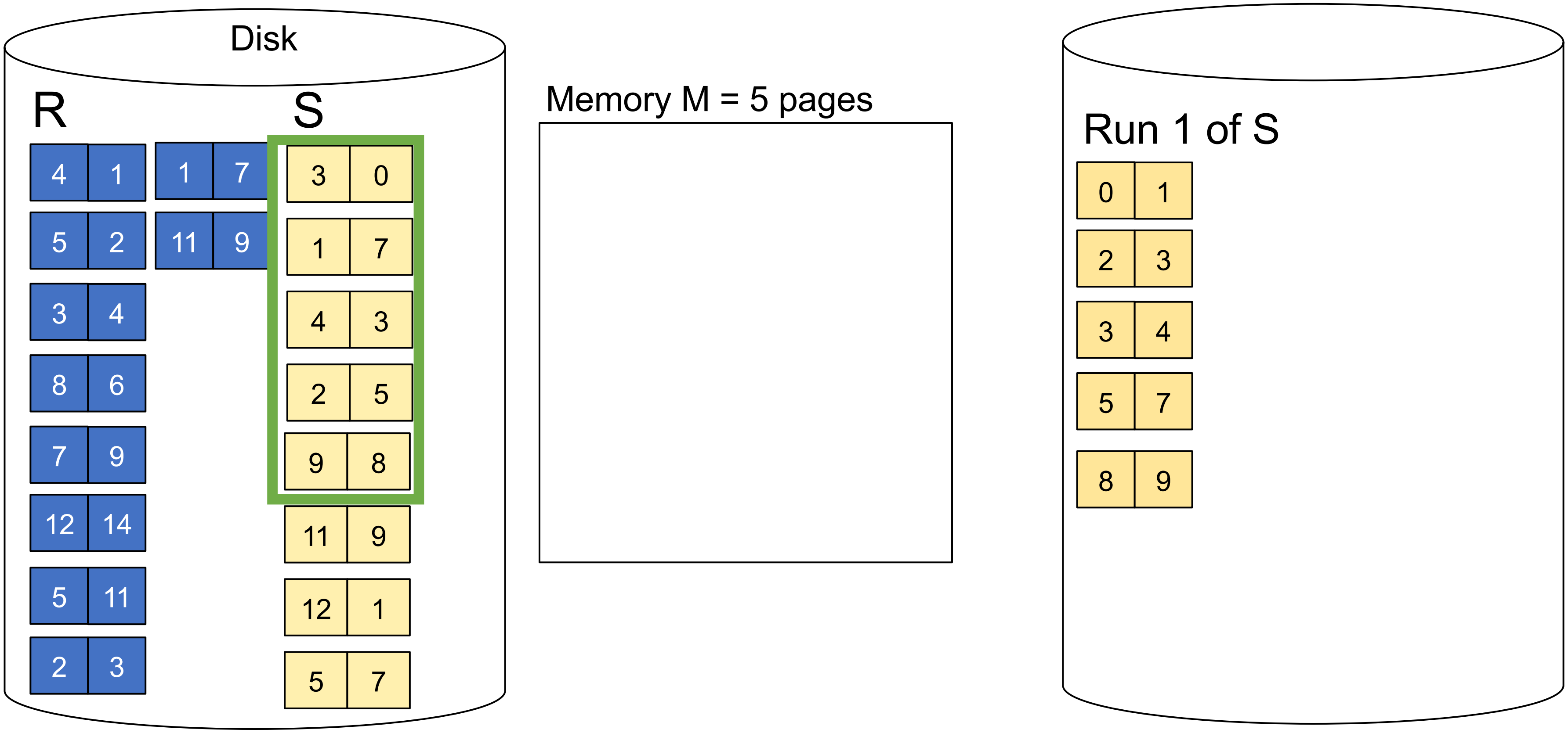
Merge-Join Example

Step 1: Repeat for next M pages until all R is processed



Merge-Join Example

Step 1: Do the same with S



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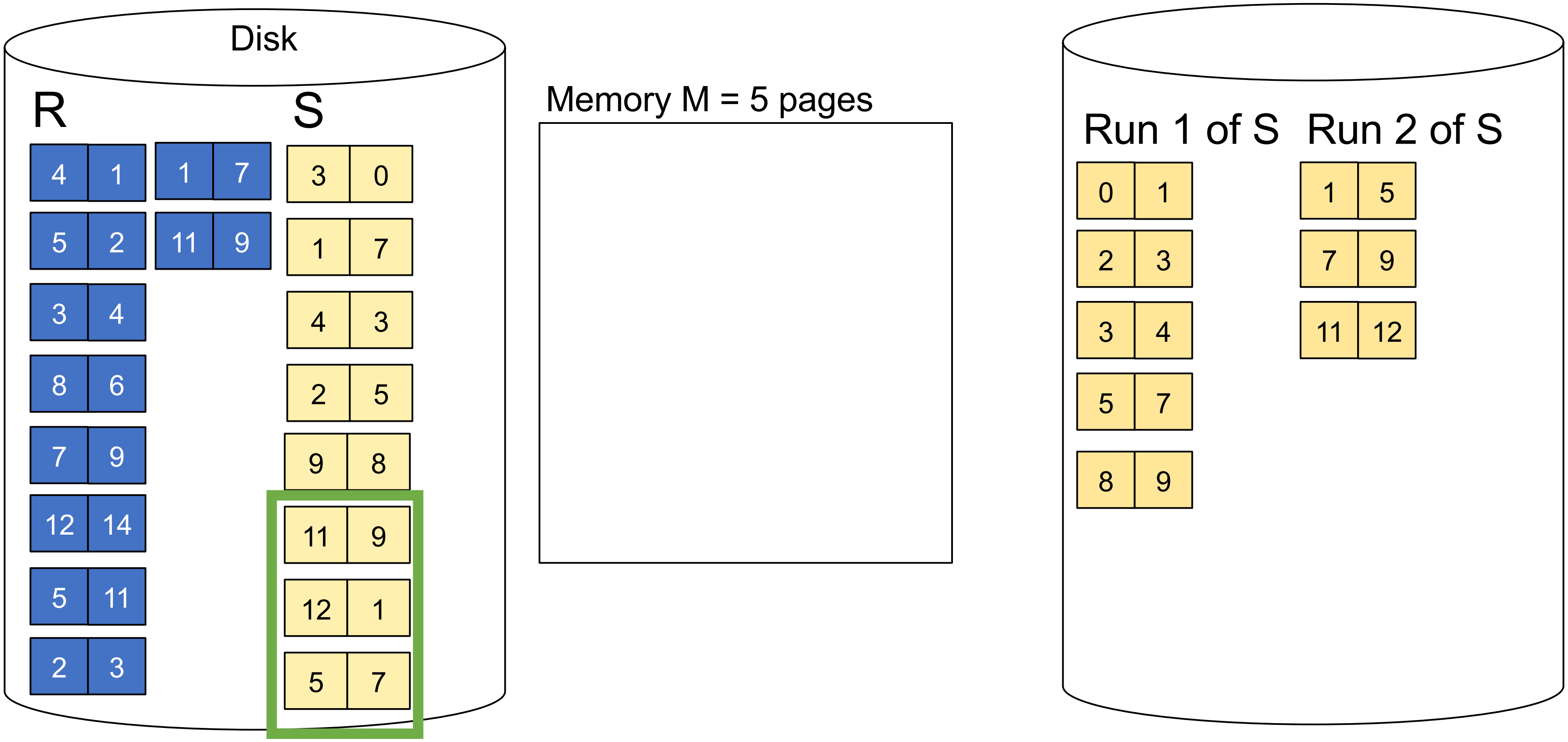
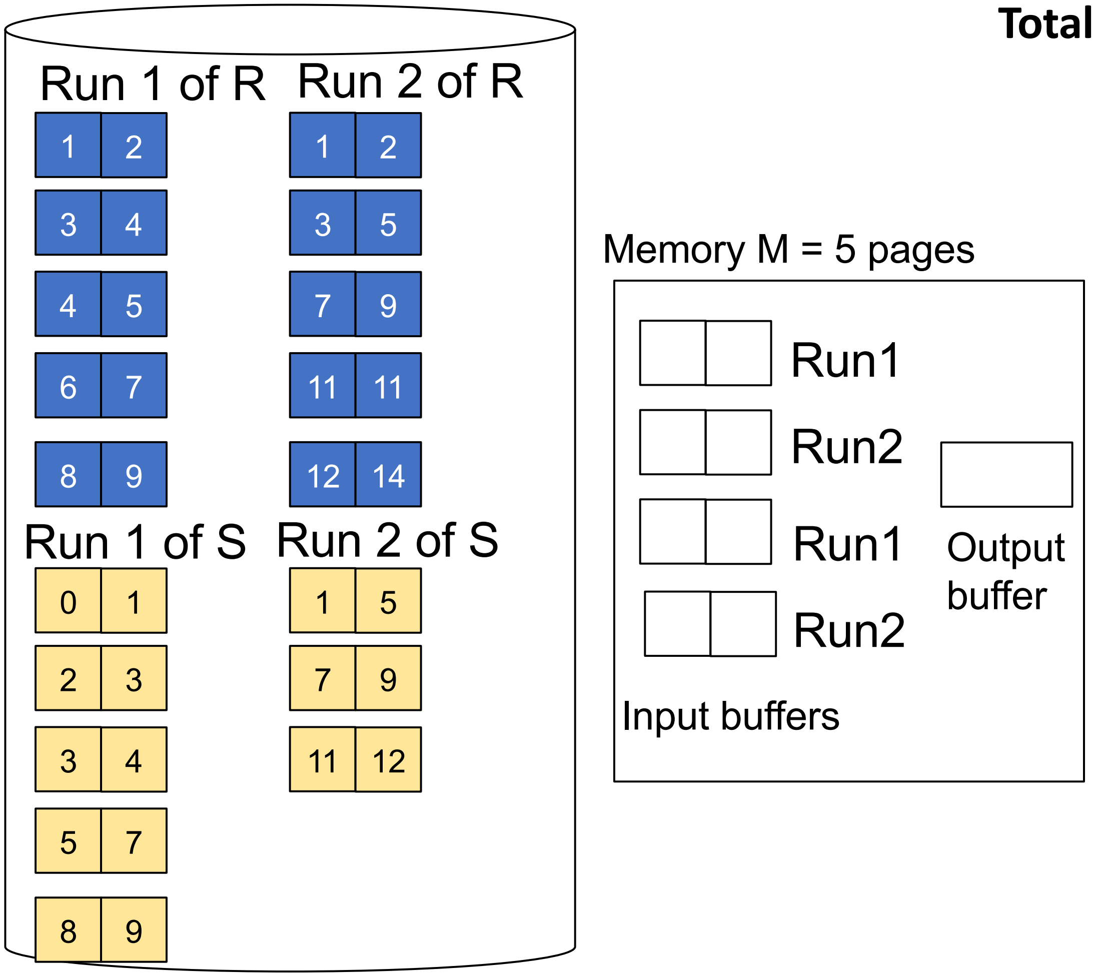


Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: 3B(R) + 3B(S)



Run 1 of R

1 2

3 4

4 5

6 7

8 9

Run 2 of R

1 2

3 5

7 9

11 11

12 14

Run 1 of S

0 1

2 3

3 4

5 7

8 9

Run 2 of S

1 5

7 9

11 12

Memory M = 5 pages

Run1

Run2

Run1

Run2

Output buffer

Input buffers

Step 2: Join while merging
Output tuples

Merge-Join Example

Step 2: Join while merging sorted runs

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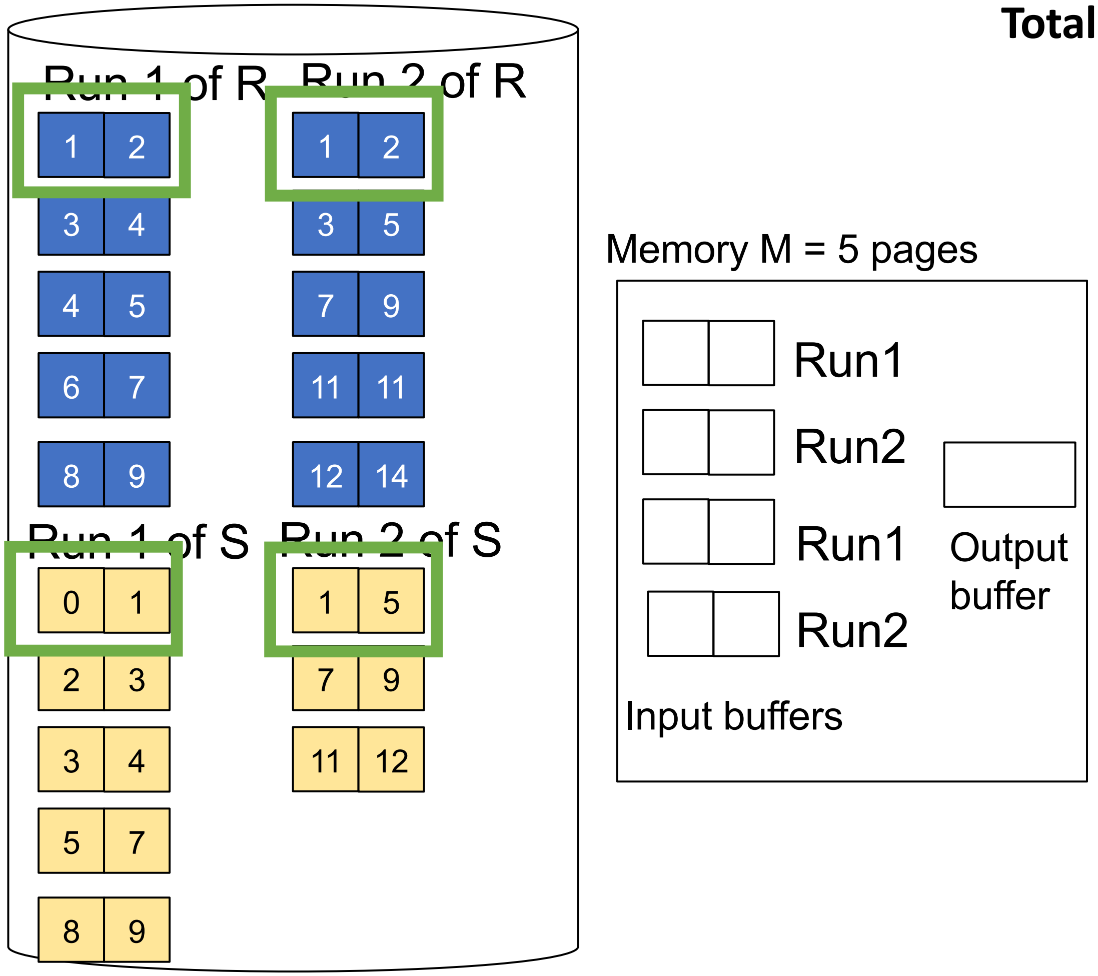

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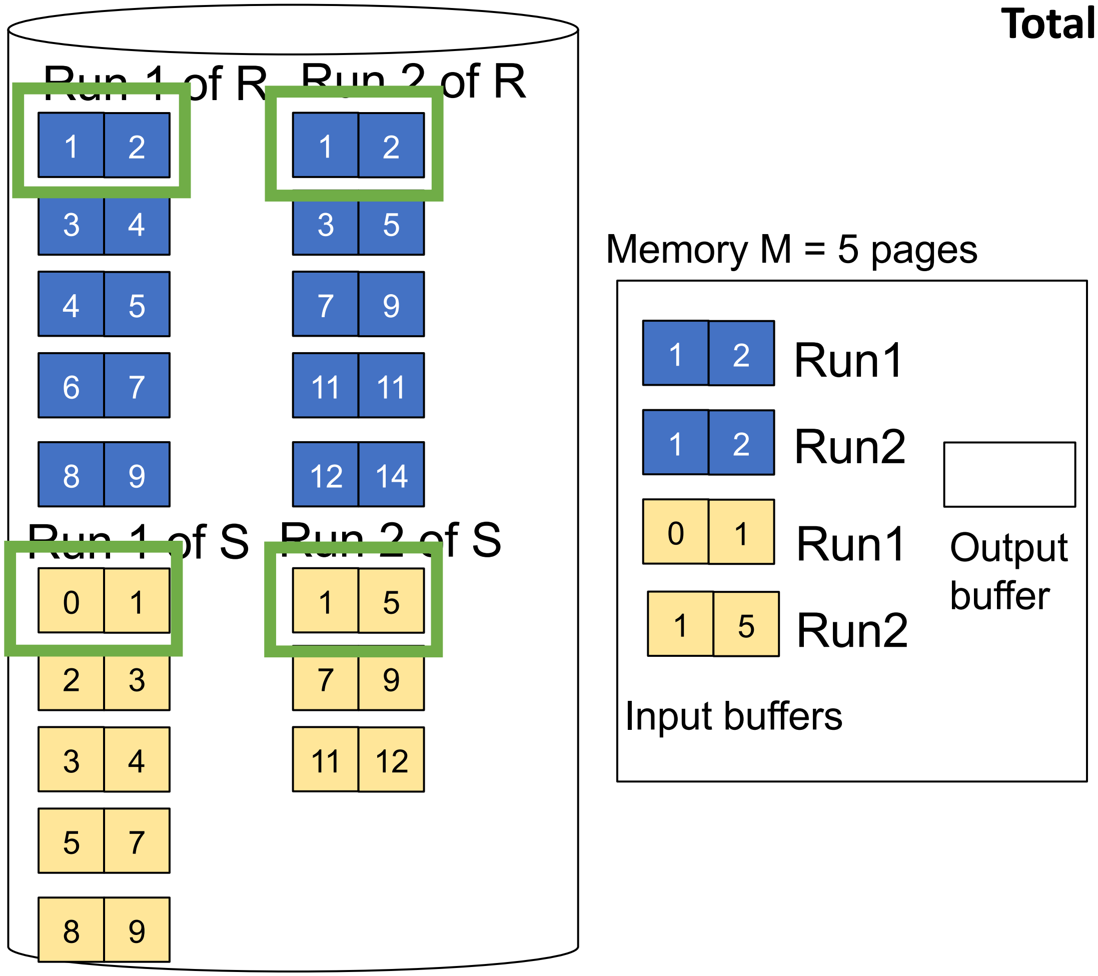

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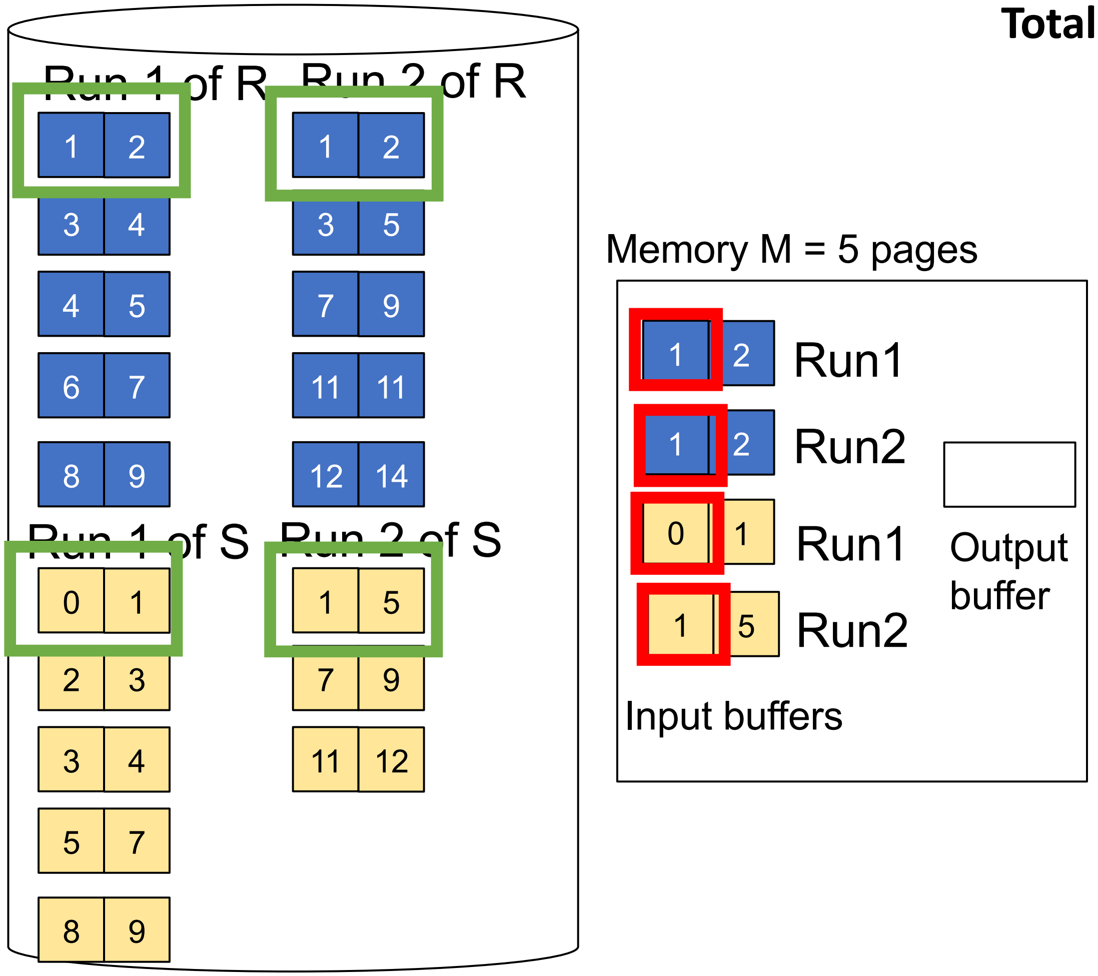

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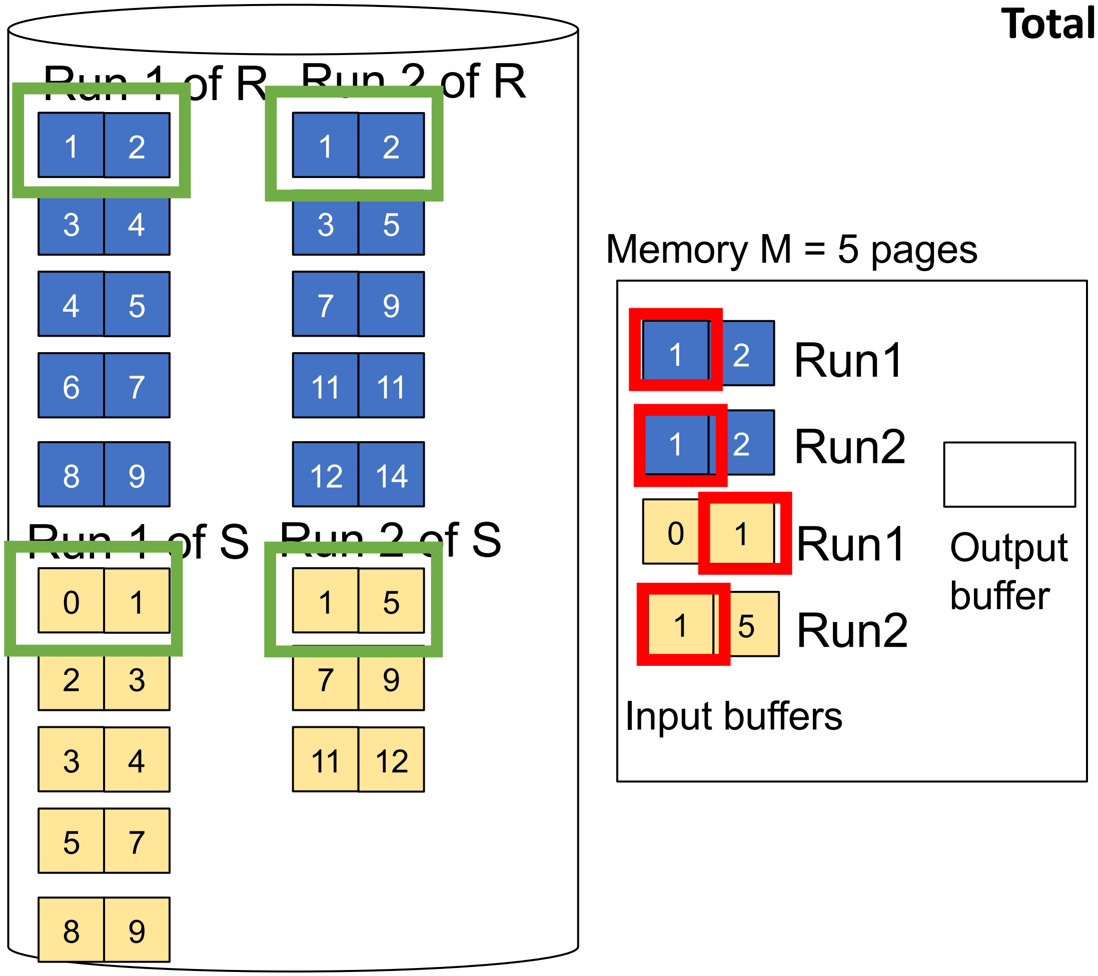


Run 1 of R: 1 2 | 3 4 | 4 5 | 6 7 | 8 9

Run 2 of R: 1 2 | 3 5 | 7 9 | 11 11 | 12 14

Run 1 of S: 0 1 | 2 3 | 3 4 | 5 7 | 8 9

Run 2 of S: 1 5 | 7 9 | 11 12

Memory M = 5 pages

1 2 Run1

1 2 Run2

0 1 Run1

1 5 Run2

Input buffers

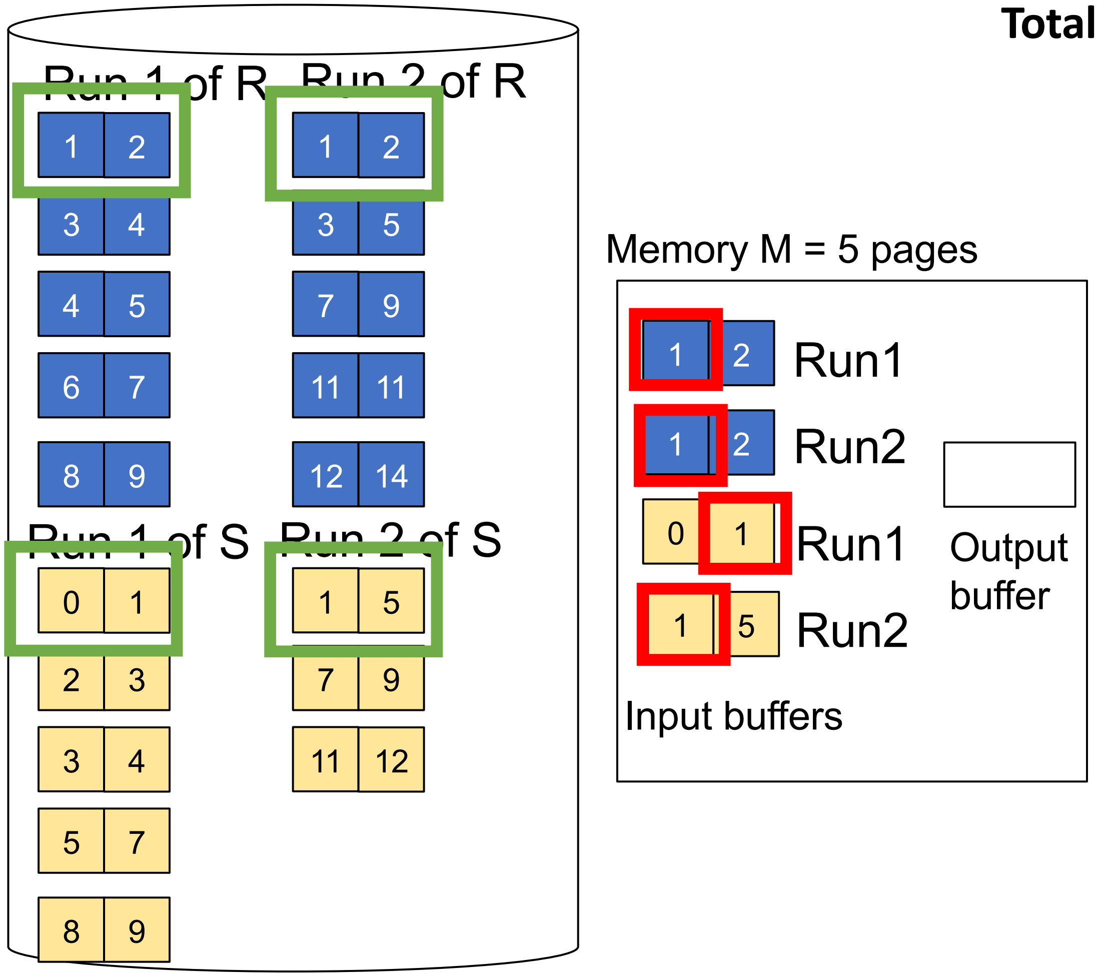
Output buffer

Step 2: Join while merging
Output tuples

Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: 3B(R) + 3B(S)



Step 2: Join while merging

Output tuples

(1,1)

(1,1)

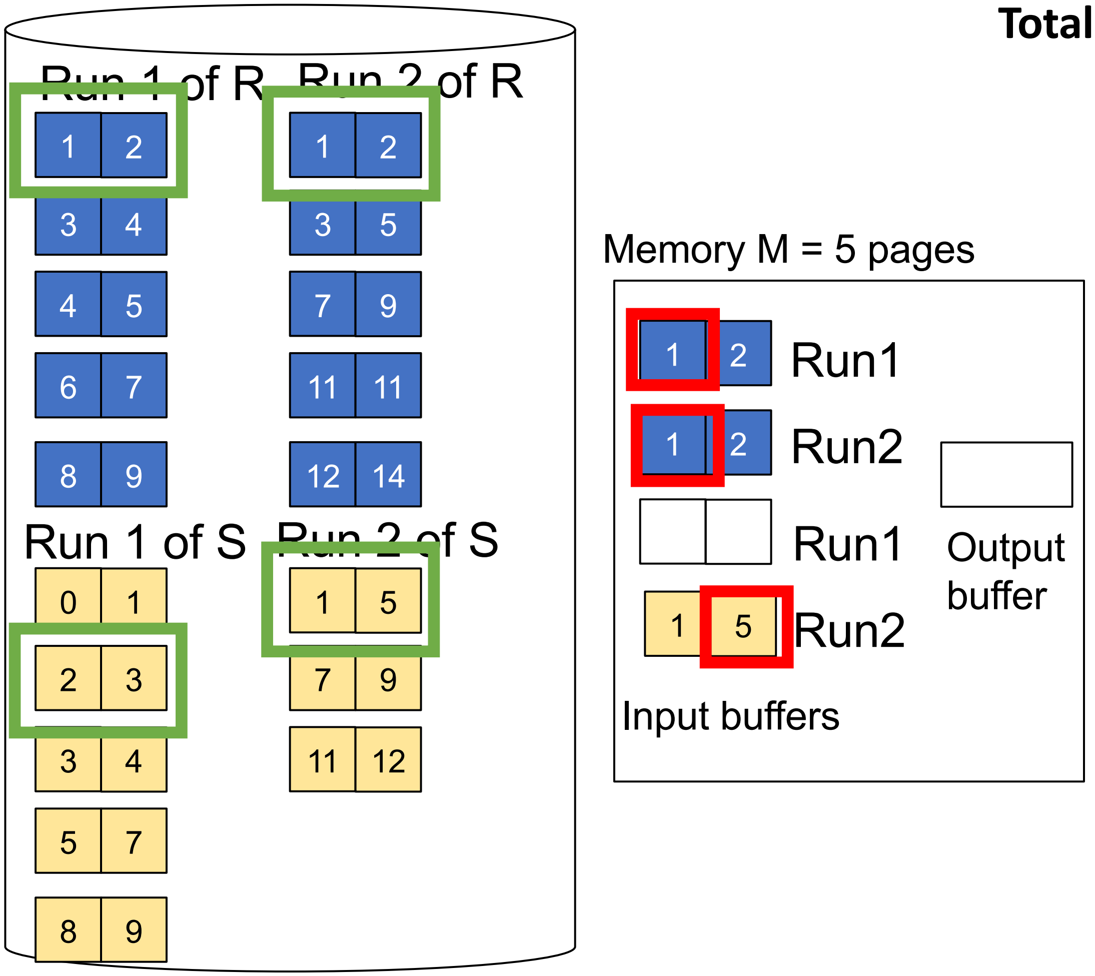
(1,1)

(1,1)

Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: 3B(R) + 3B(S)



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Output tuples

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(1,1)

(1,1)

Merge-Join Example

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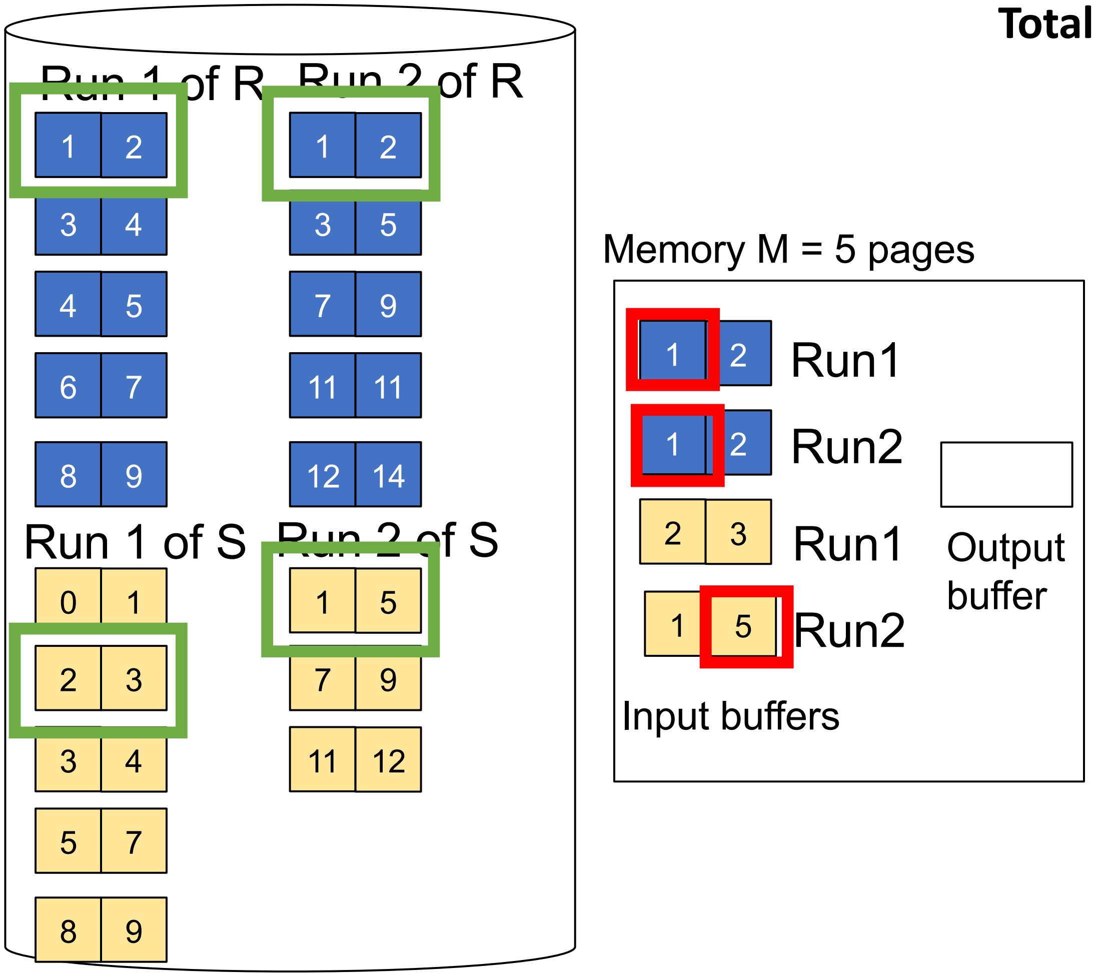

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(1,1)

(1,1)

Merge-Join Example

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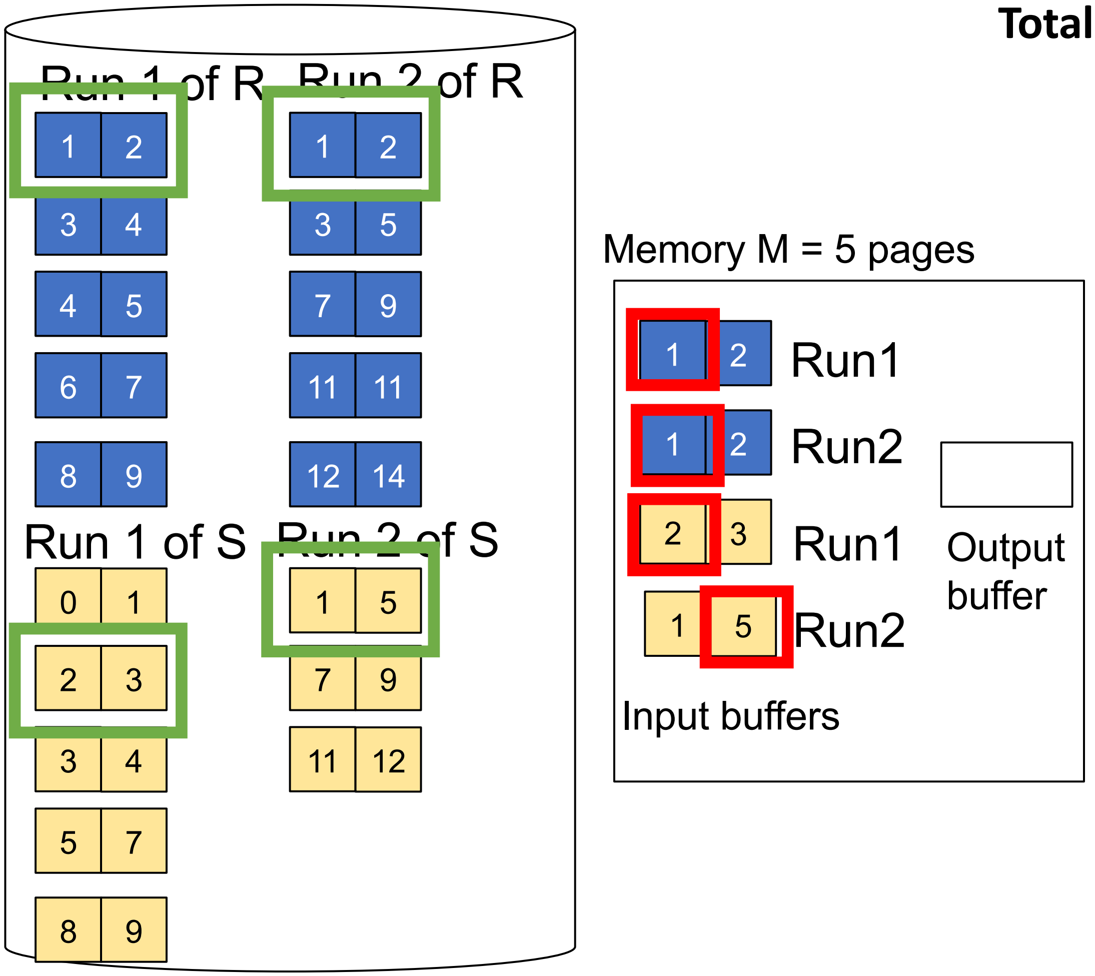

Step 2: Join while merging

Output tuples

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(1,1)

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(1,1)

Merge-Join Example

Step 2: Join while merging sorted runs

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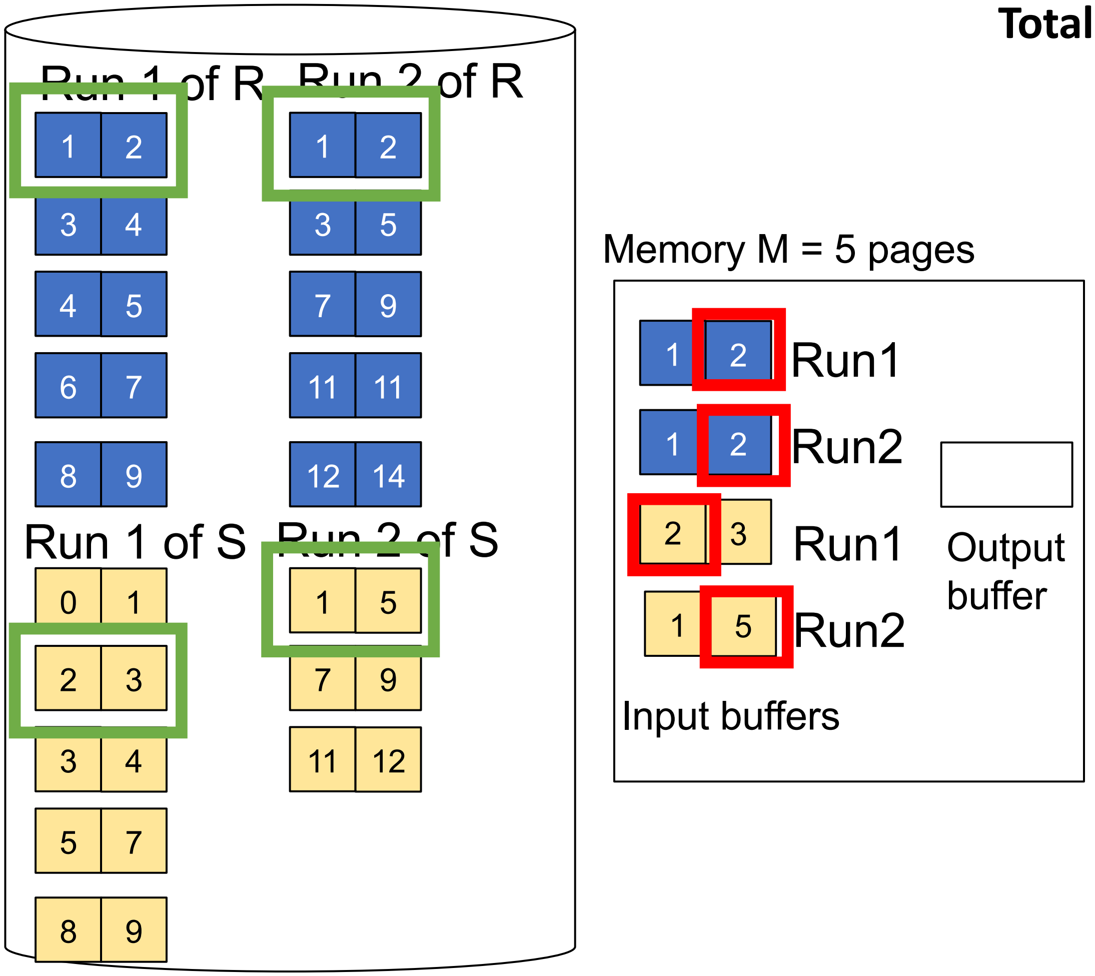


Step 2: Join while merging

Output tuples

(1,1)

(1,1)

(1,1)

(1,1)

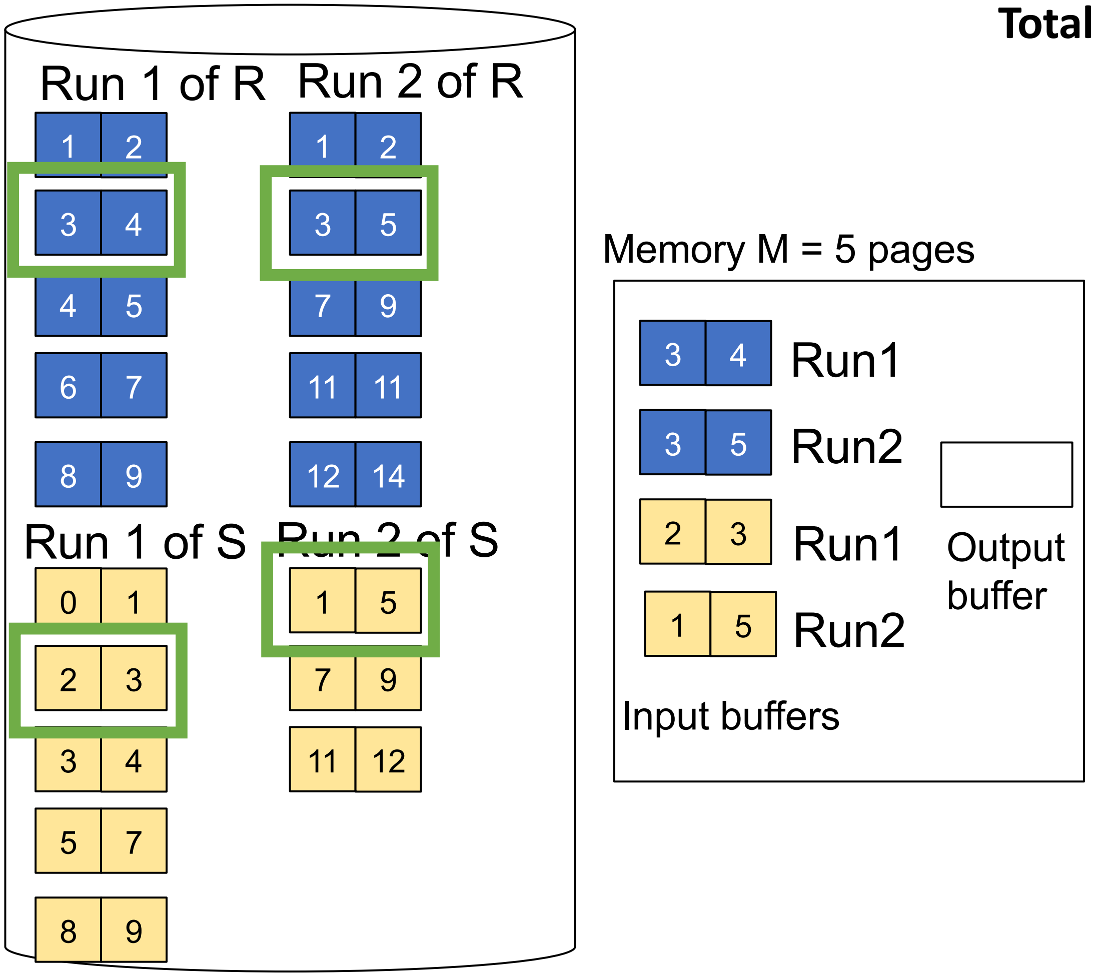
(2,2)

(2,2)

Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: 3B(R) + 3B(S)



Step 2: Join while merging

Output tuples

(1,1)

(1,1)

(1,1)

(1,1)

(2,2)

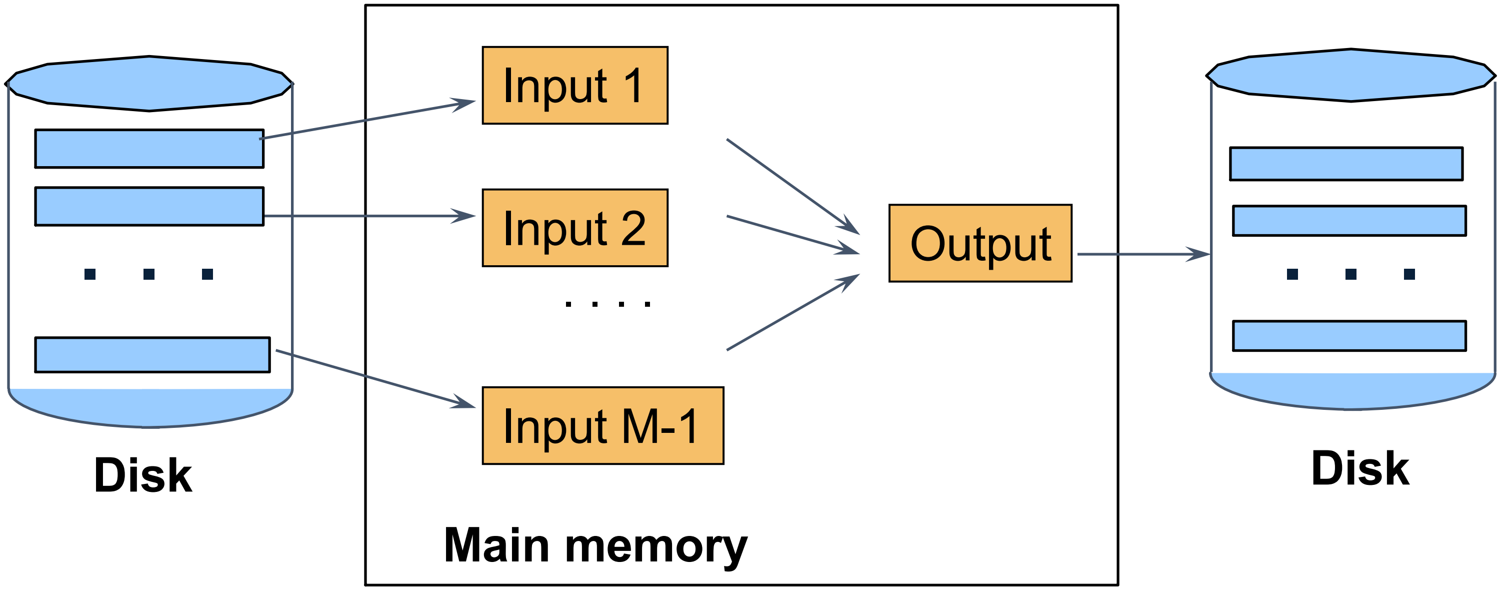
(2,2)

(3,3)

(3,3)

…

Merge-Join



M_1 = B(R)/M runs for R

M_2 = B(S)/M runs for S

Merge-join M_1 + M_2 runs;

need $M_1 + M_2 <= M$ to process all runs

i.e. $B(R) + B(S) <= M^2$

Summary of External Join Algorithms

- Block Nested Loop: B(S) + B(R)*B(S)/(M-1)
- Index Join:
  - Clustered: B(R) + T(R)B(S)/V(S,a)
  - Unclustered: B(R) + T(R)T(S)/V(S,a)
- Merge Join: 3B(R)+3B(S)
  - B(R)+B(S) <= M^2
- Partitioned Hash Join: (coming up next)

Partitioned Hash Algorithms

- Partition R it into k buckets on disk:
  R_1, R_2, R_3, …, R_k

Partitioned Hash Algorithms

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  $B(R_i) = B(R)/k$, for all i

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  $B(R_i) \leq M$

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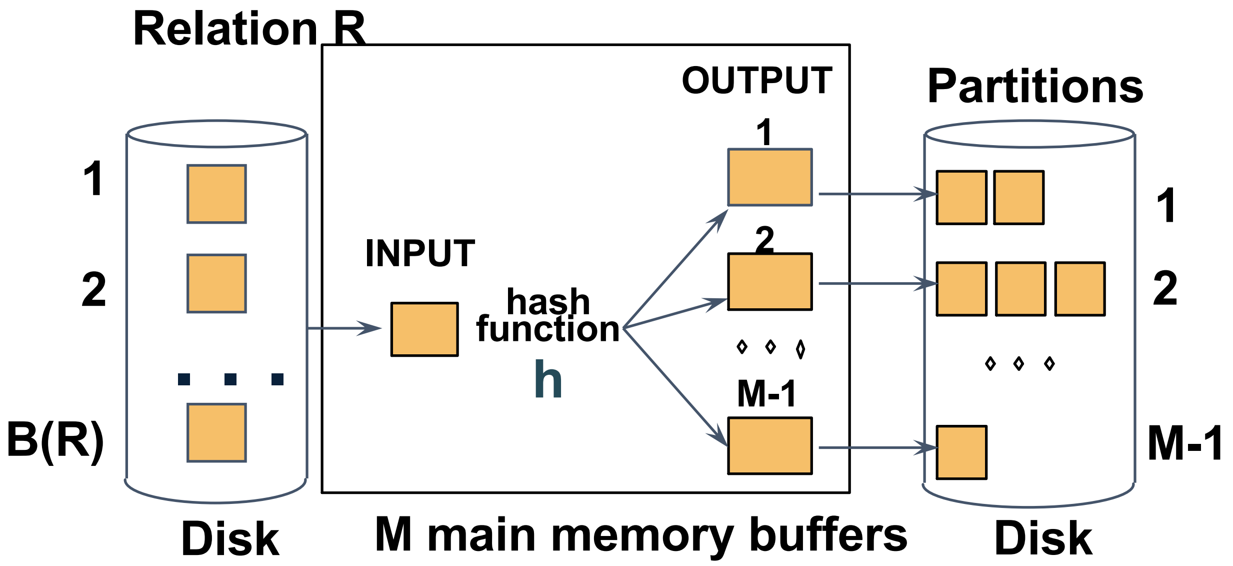
- Goal: each R_i should fit in main memory:
  $B(R_i) \leq M$

How do we choose k?

Partitioned Hash Algorithms

- We choose k = M-1 Each bucket has size approx. B(R)/(M-1) ≈ B(R)/M



Assumption: B(R)/M ≤ M, i.e. $B(R) \leq M^2$

Partitioned Hash Join (Grace-Join)

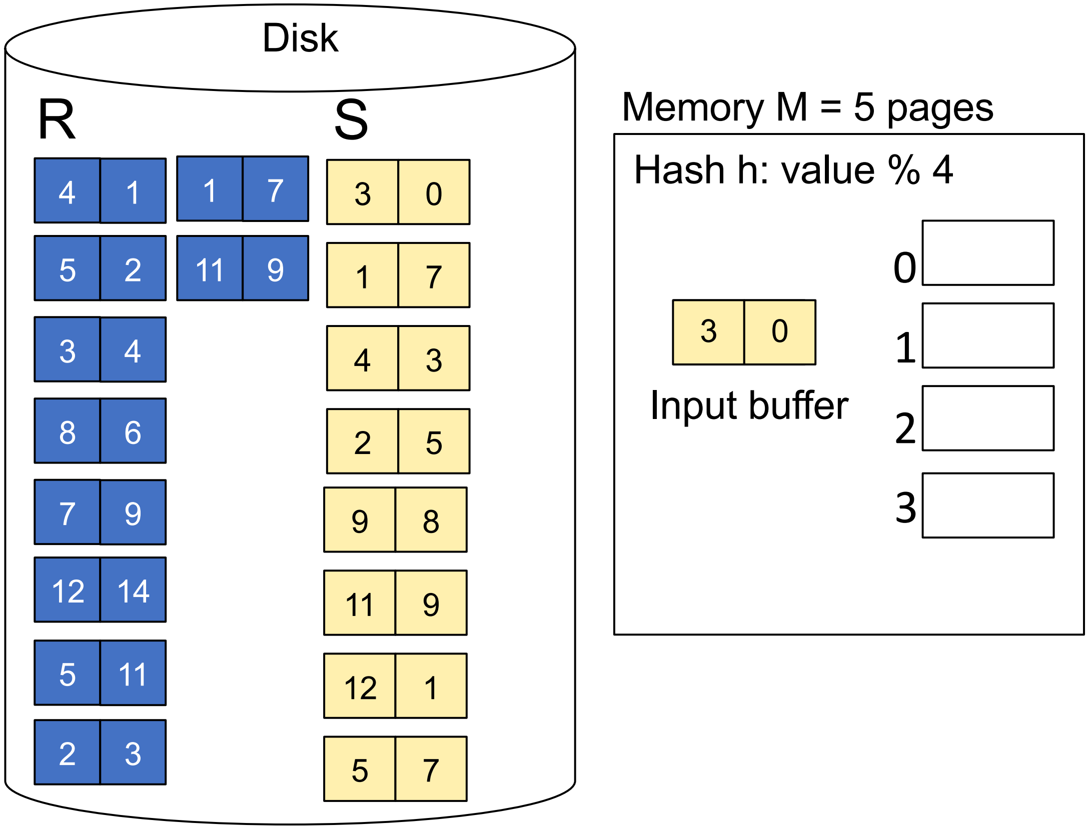
R ⋈ S

- Step 1:
  - Hash S into M-1 buckets
  - Send all buckets to disk
- Step 2
  - Hash R into M-1 buckets
  - Send all buckets to disk
- Step 3
  - Join every pair of buckets

Note: partitioned hash-join is sometimes called *grace-join*

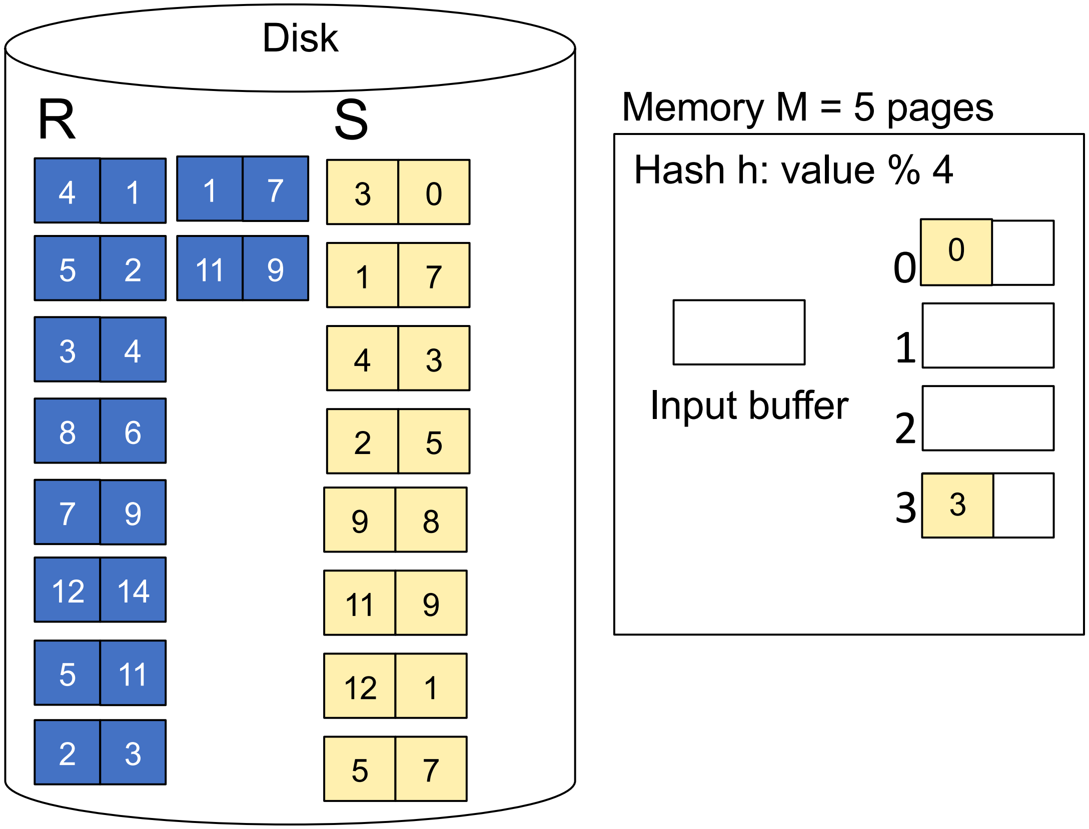
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into M-1 (=4 buckets)



Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



Partitioned Hash-Join Example

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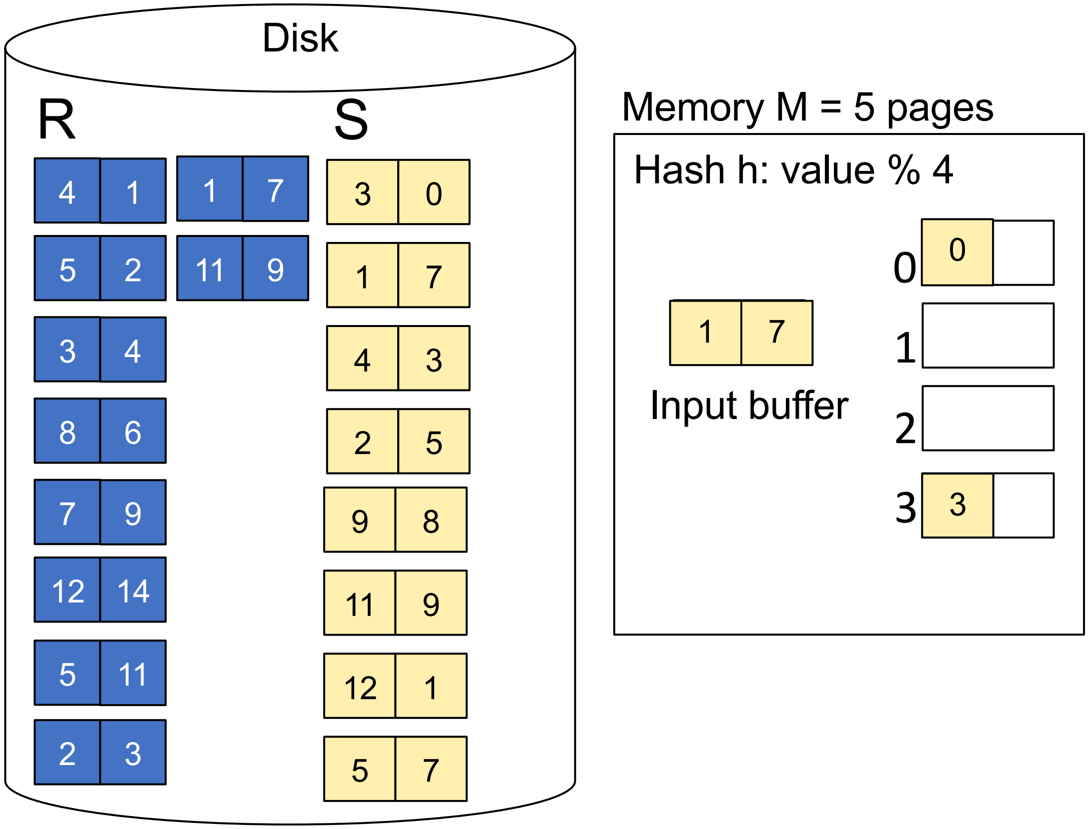
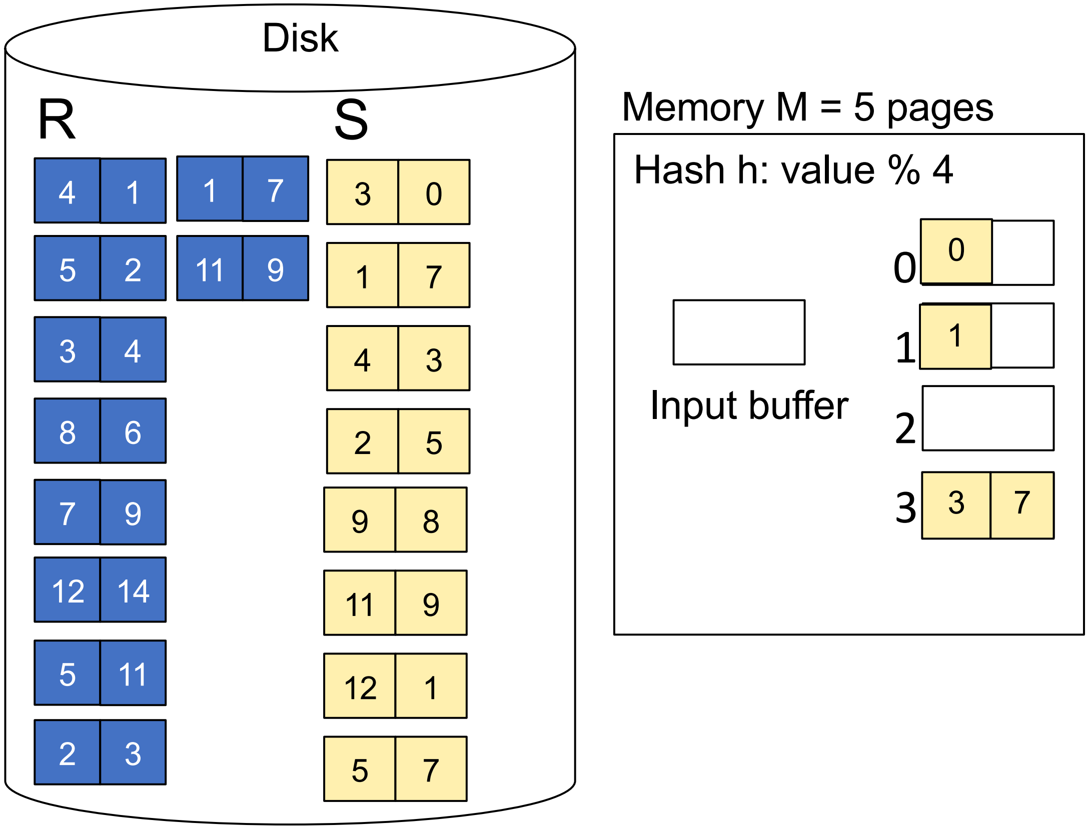


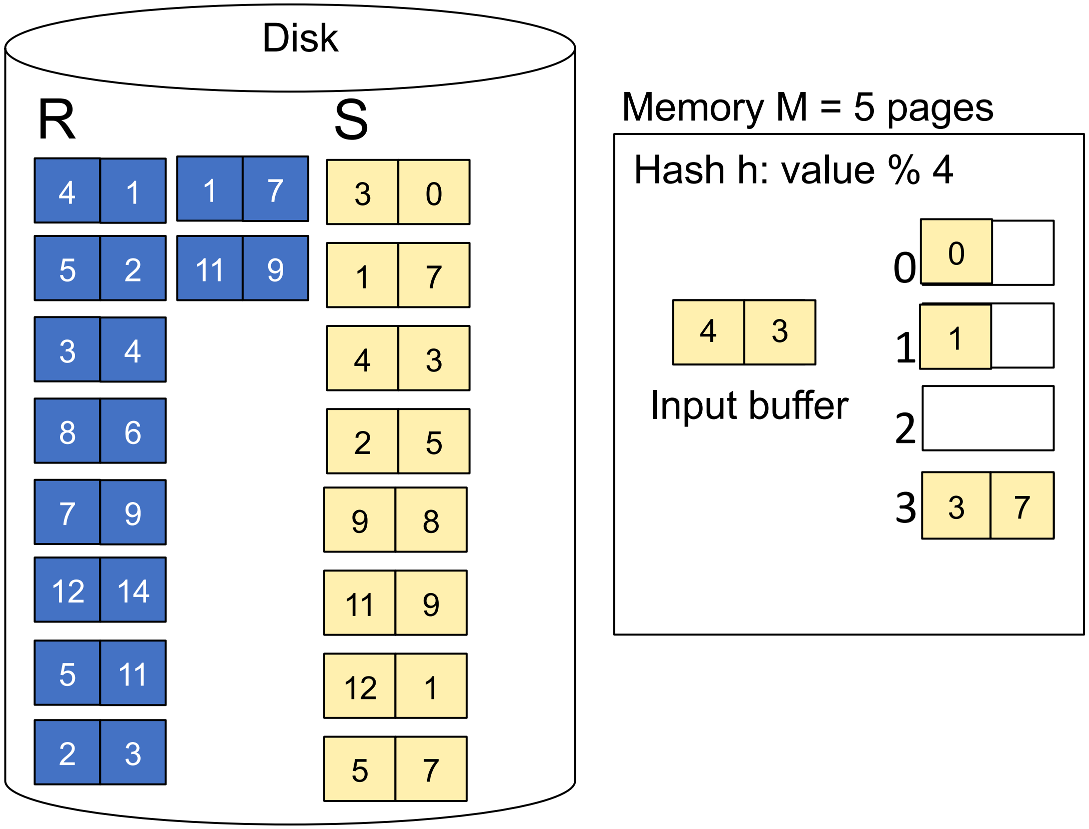
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



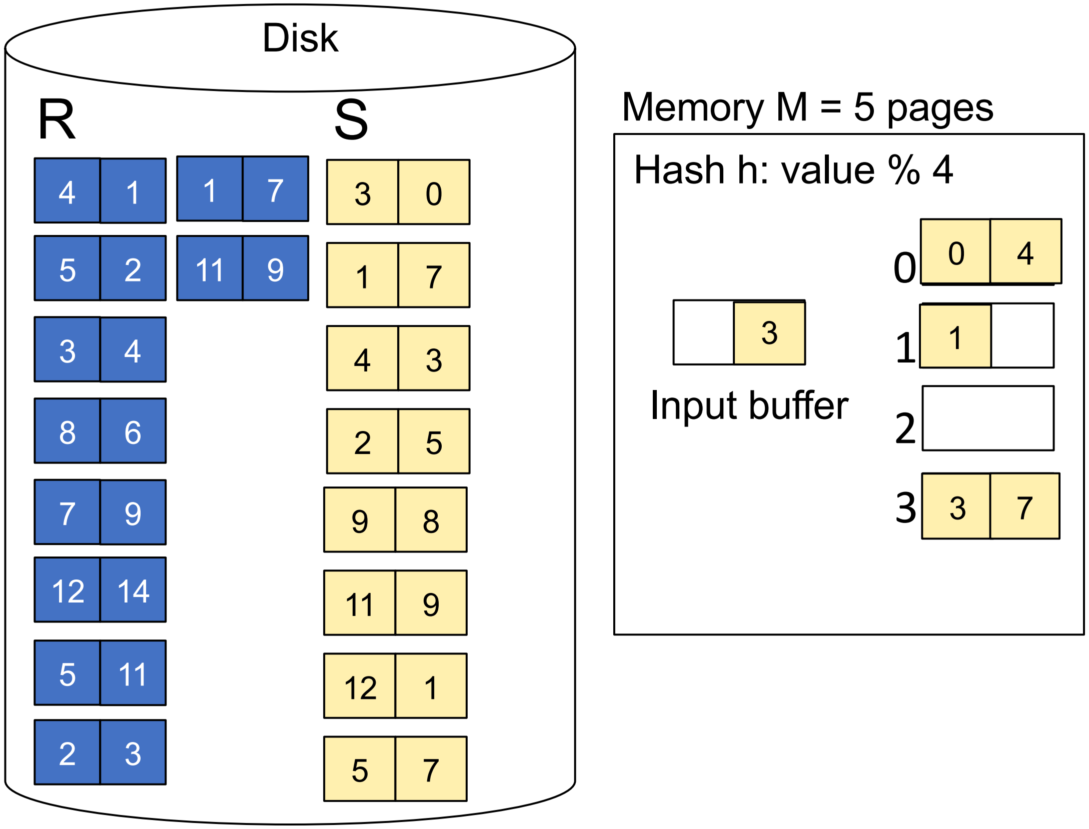
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
When a bucket fills up, flush it to disk



Partitioned Hash-Join Example

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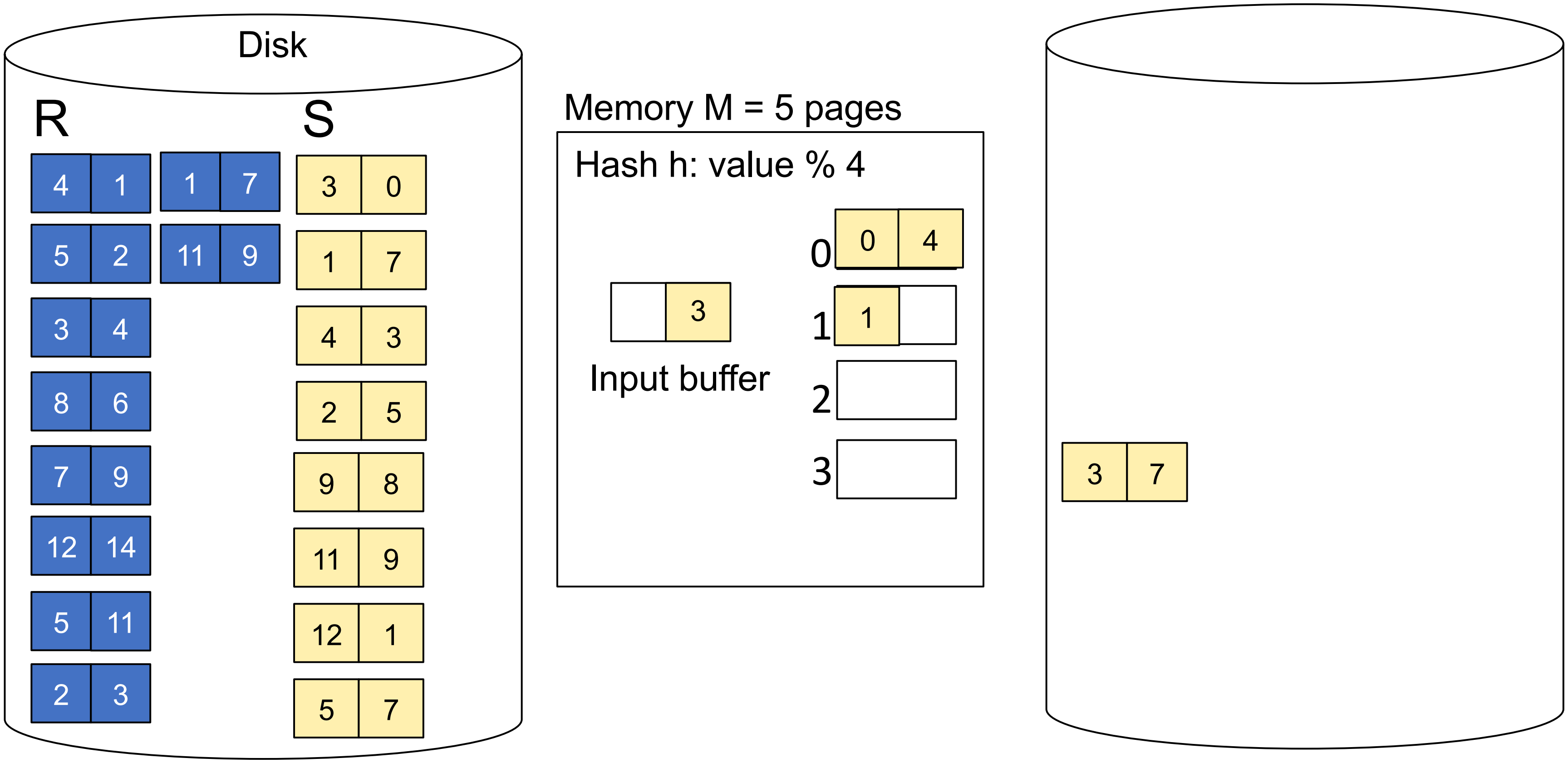
When a bucket fills up, flush it to disk



Partitioned Hash-Join Example

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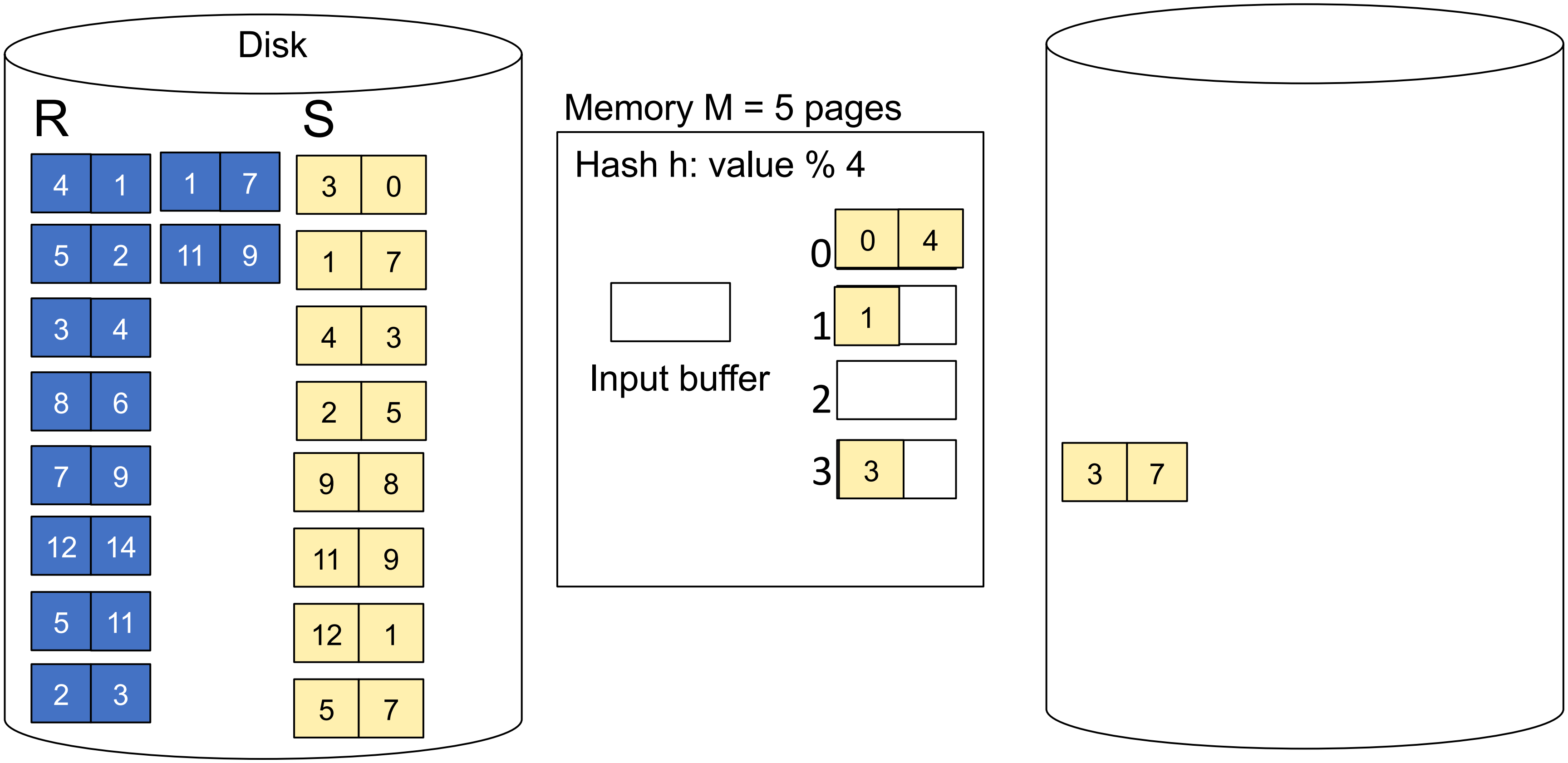
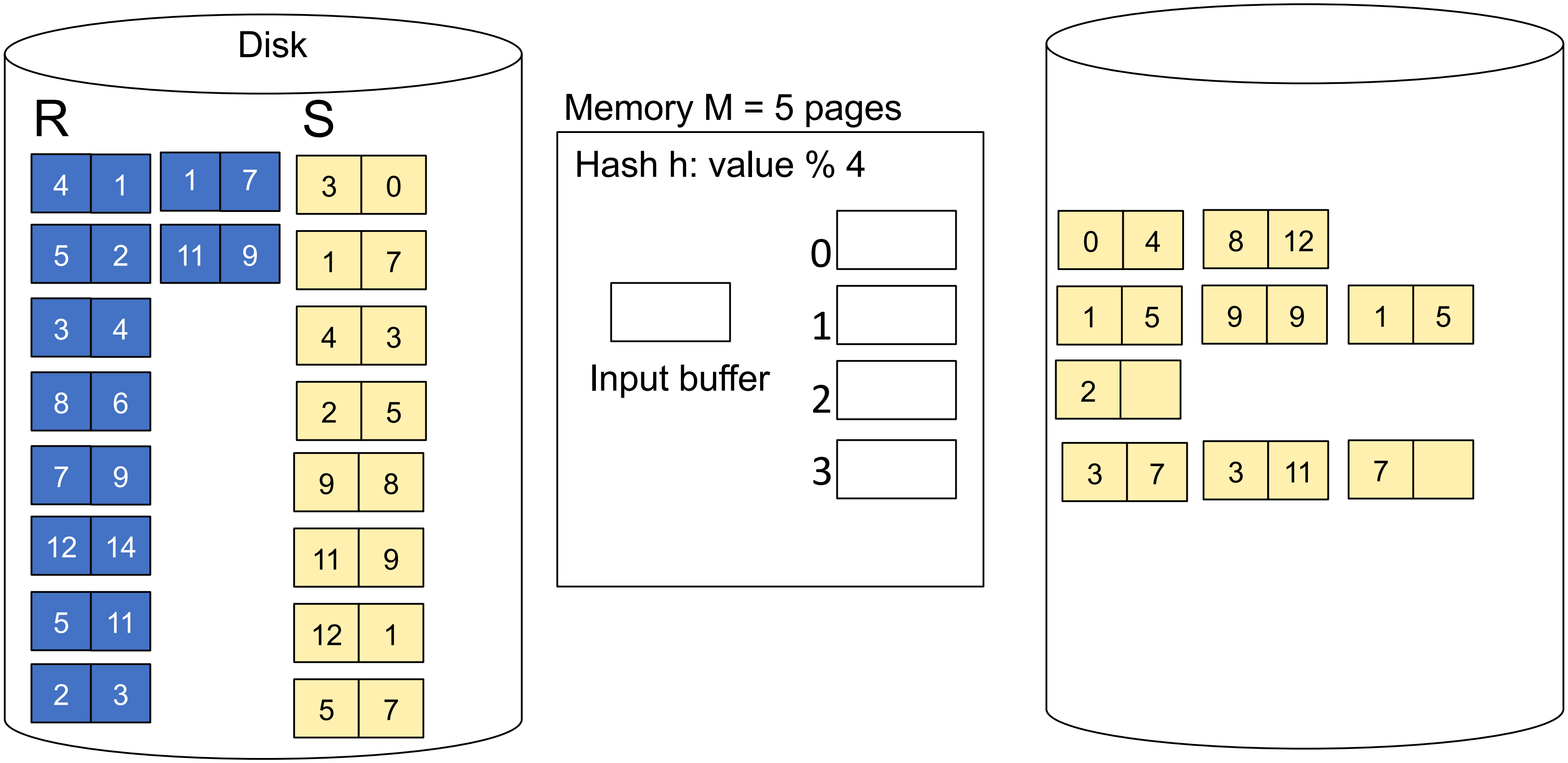


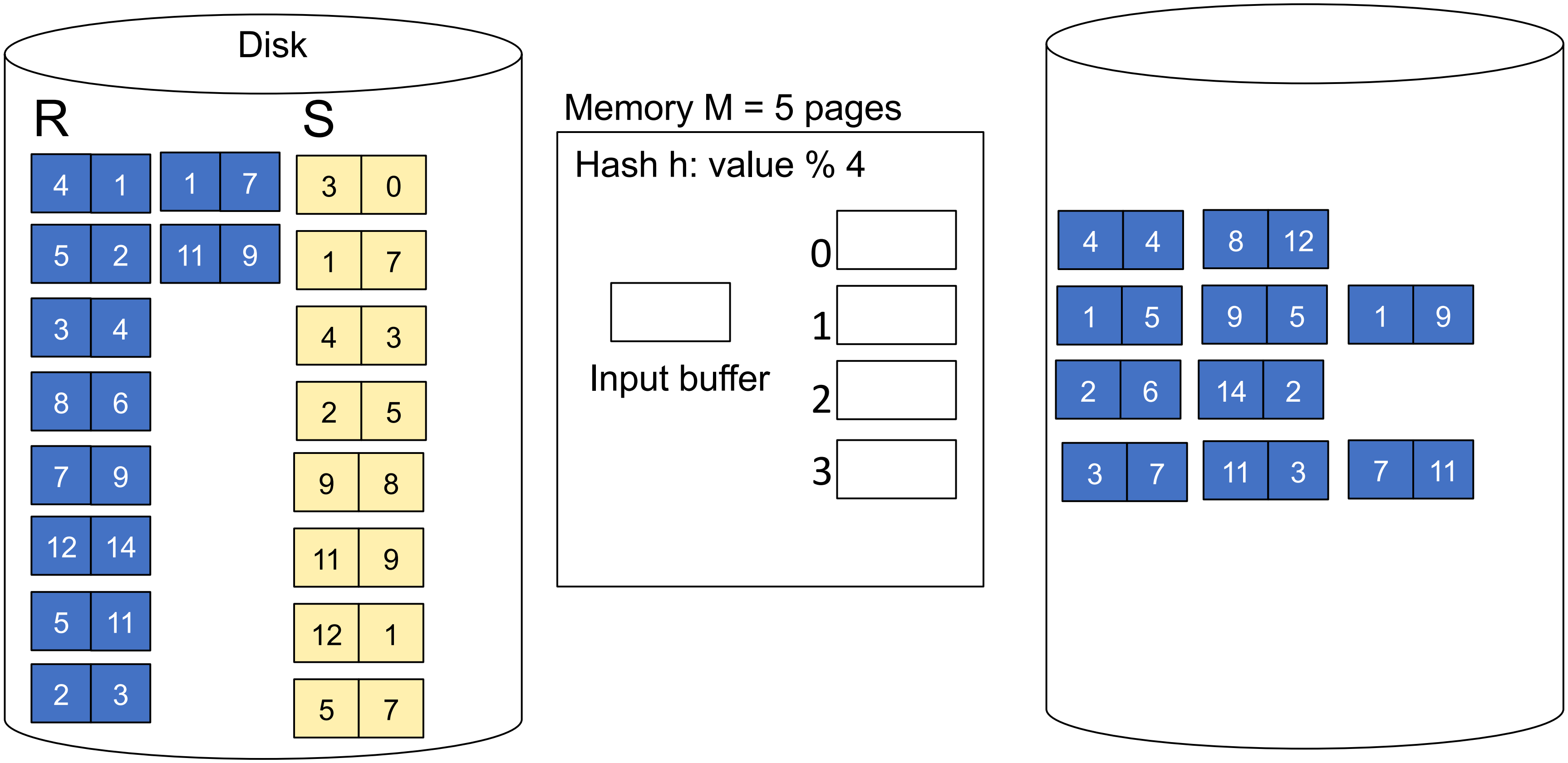
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets
At the end, we get relation S back on disk split into 4 buckets



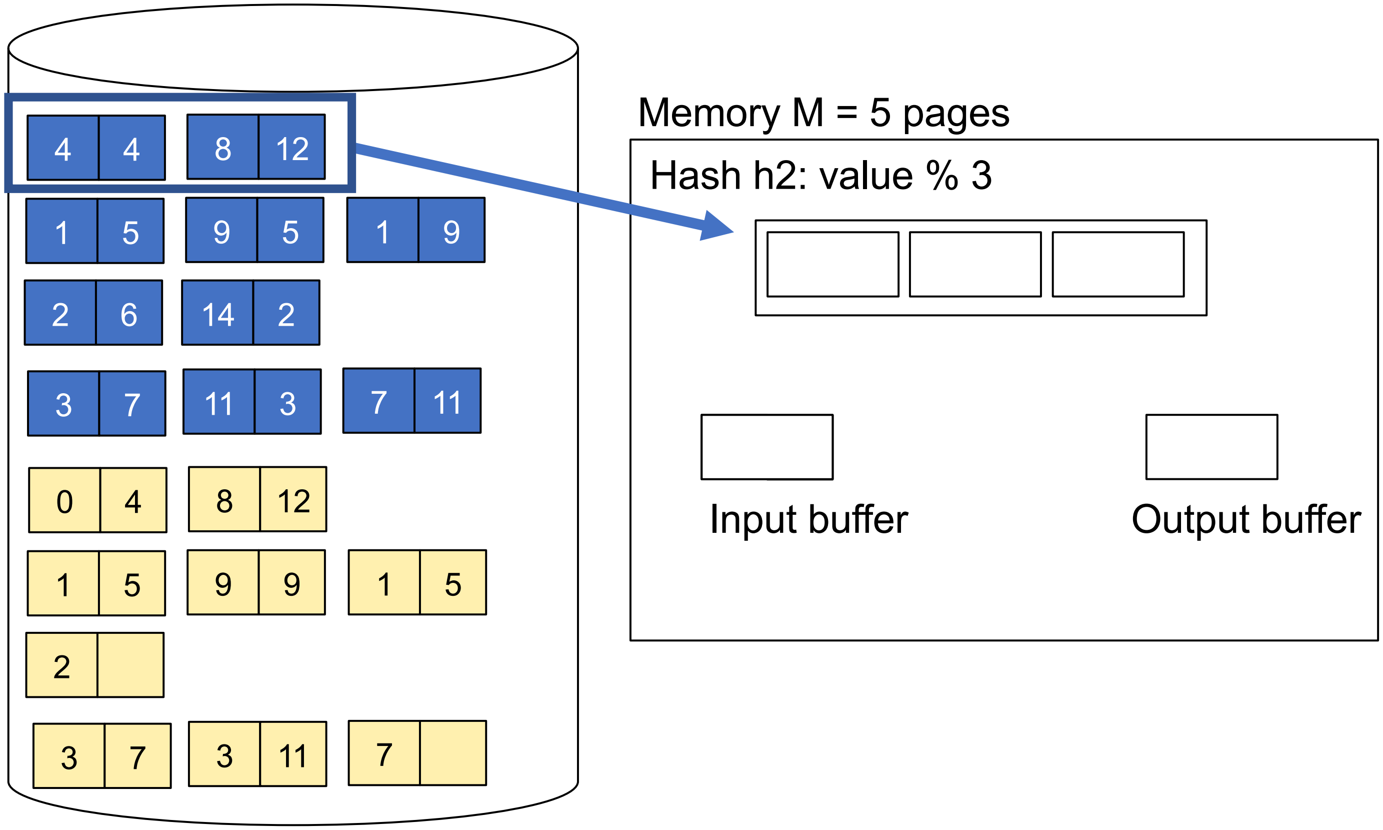
Partitioned Hash-Join Example

Step 2: Read relation R one page at a time and hash into same 4 buckets



Partitioned Hash-Join Example

Step 3: Read one partition of R and create hash table in memory using a ***different*** hash function



Partitioned Hash-Join Example

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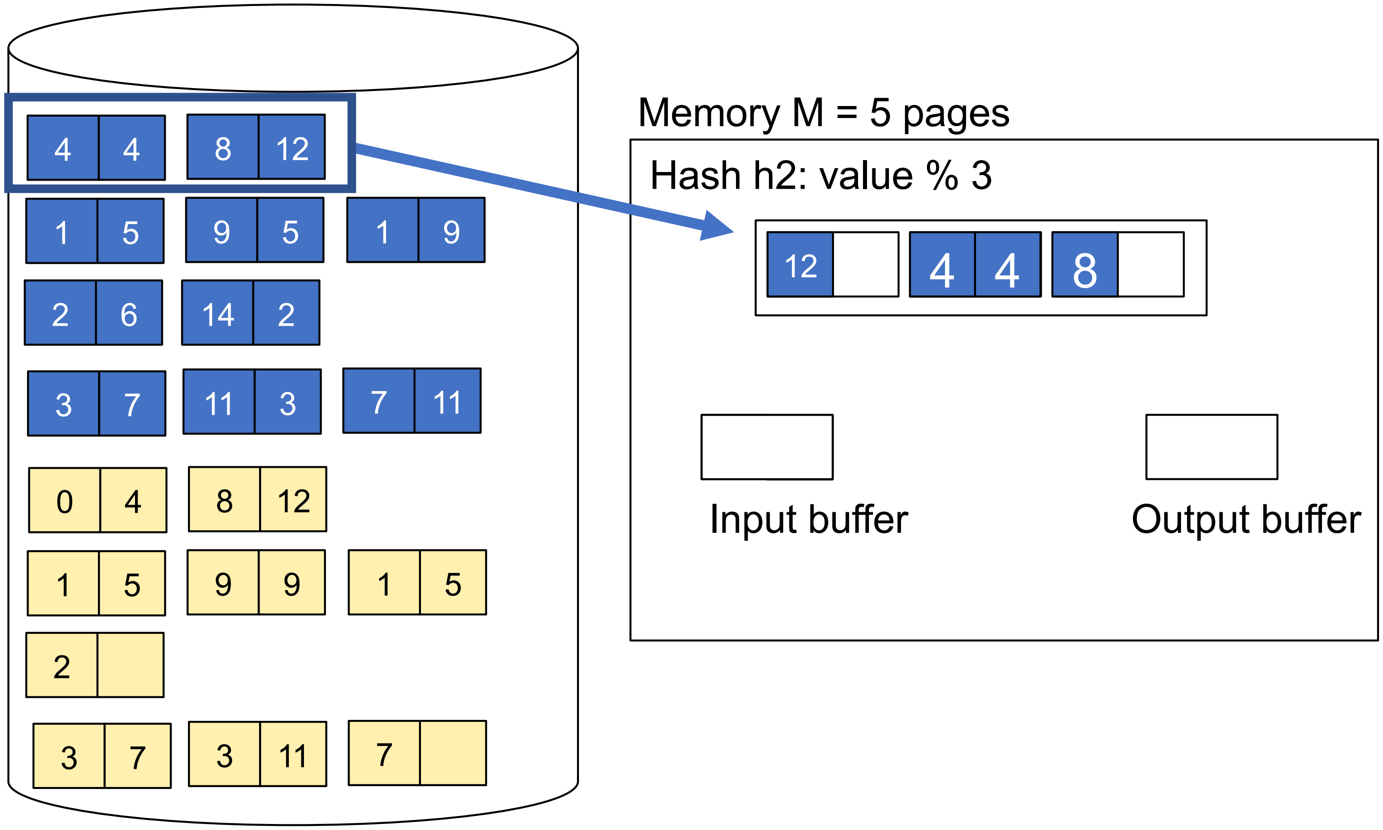

Partitioned Hash-Join Example

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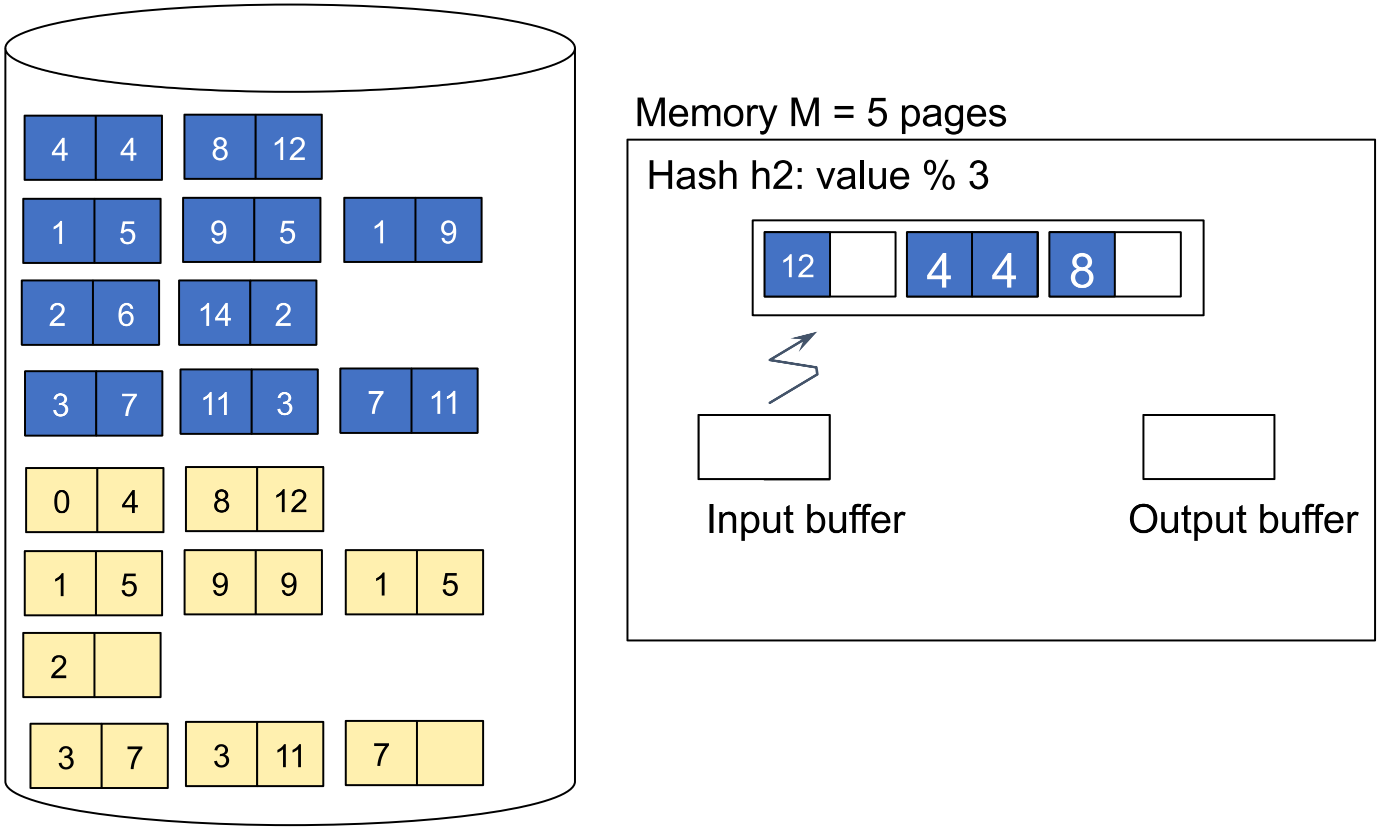
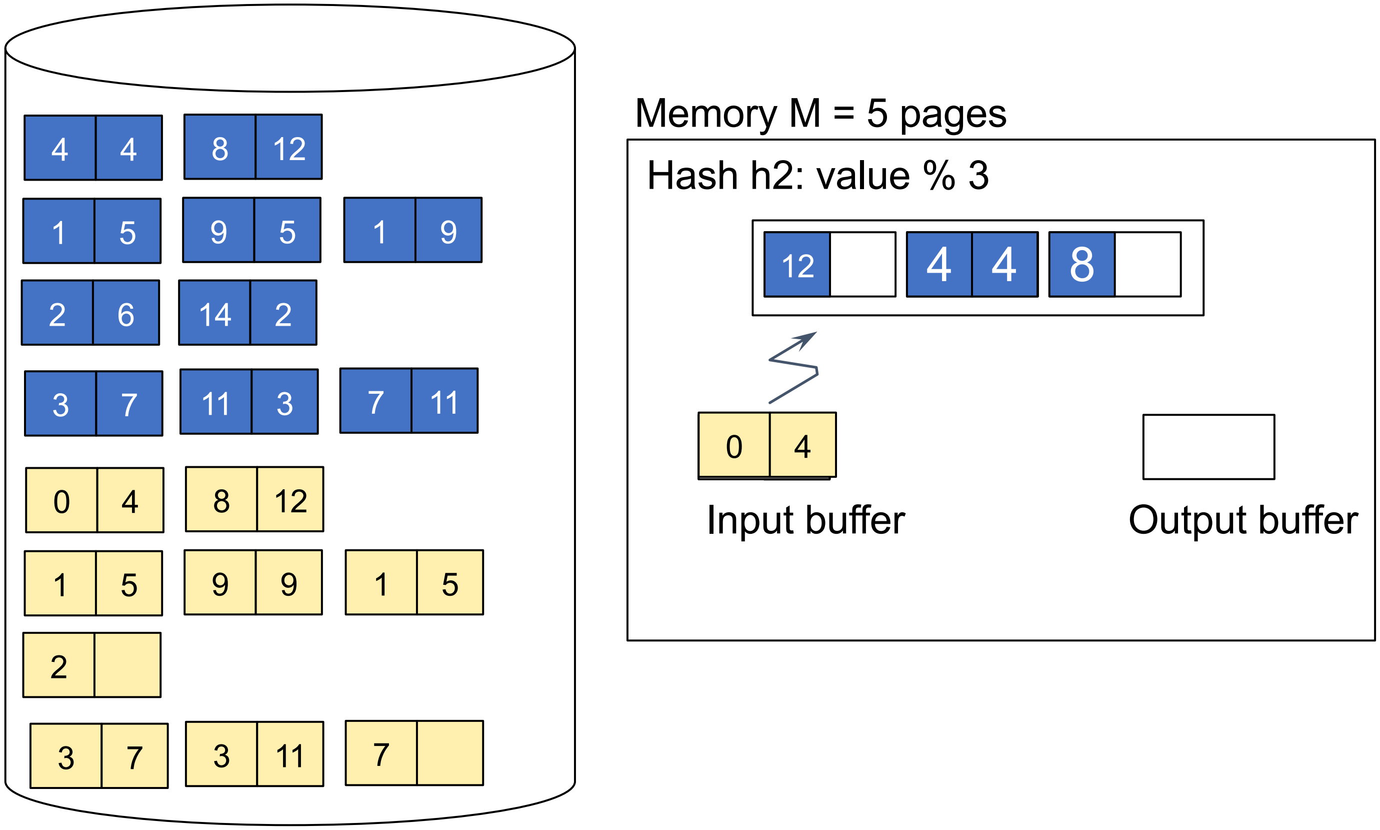


Partitioned Hash-Join Example

Step 4: Scan matching partition of S and probe the hash table

Step 5: Repeat for all the buckets

Total cost: 3B(R) + 3B(S)



Partitioned Hash-Join Example

Step 4: Scan matching partition of S and probe the hash table
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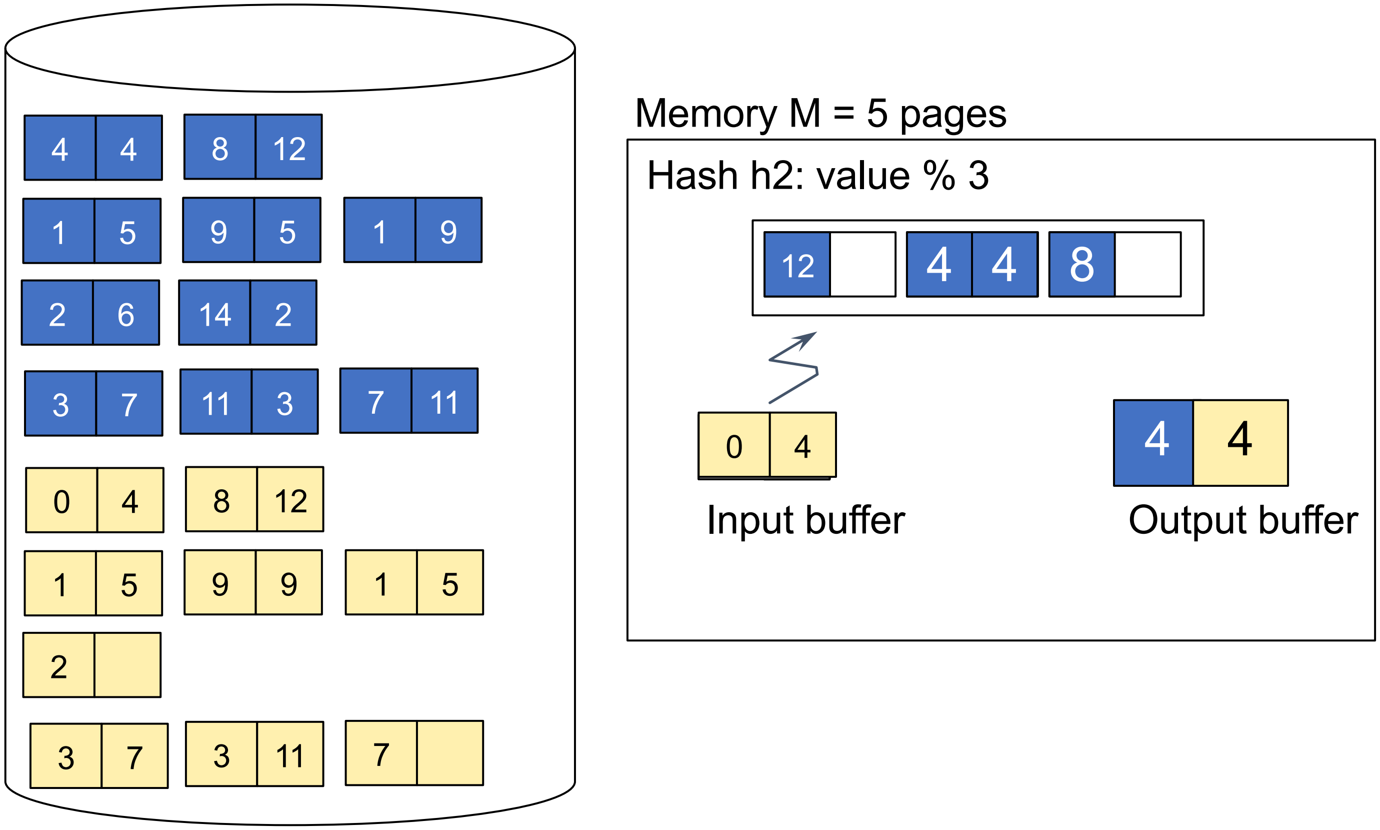
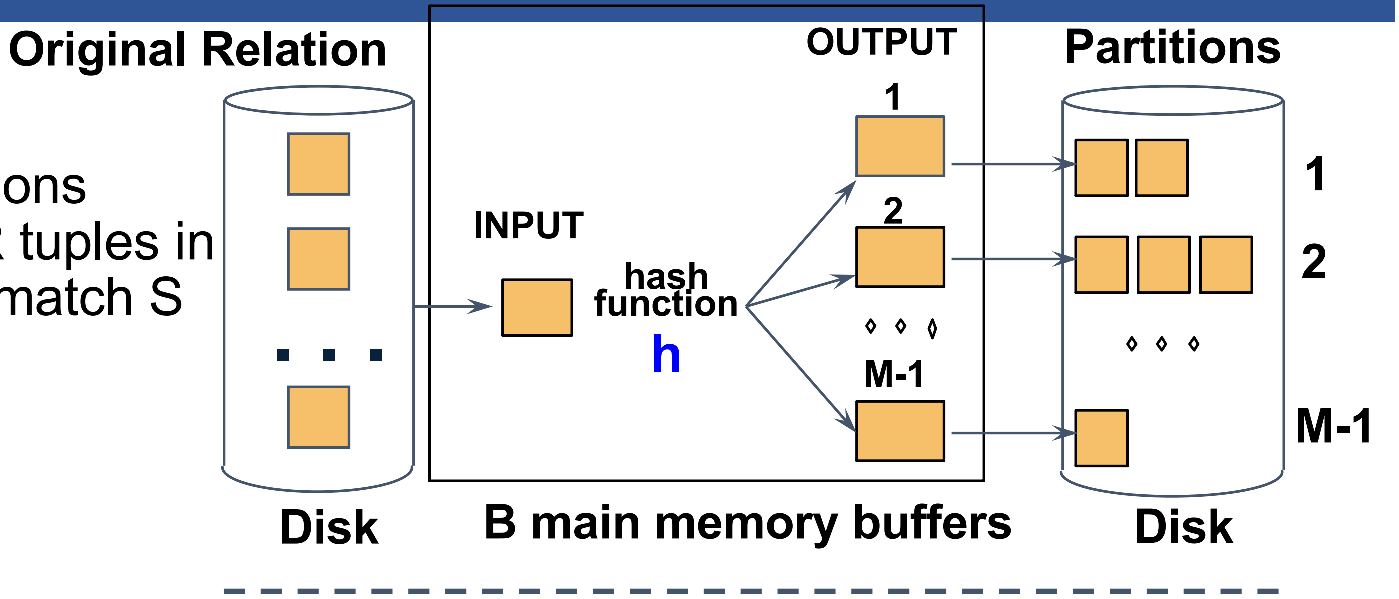
Total cost: 3B(R) + 3B(S)



Partitioned Hash-Join

- Partition both relations using hash fn **h**: R tuples in partition i will only match S tuples in partition i.



Partitioned Hash-Join

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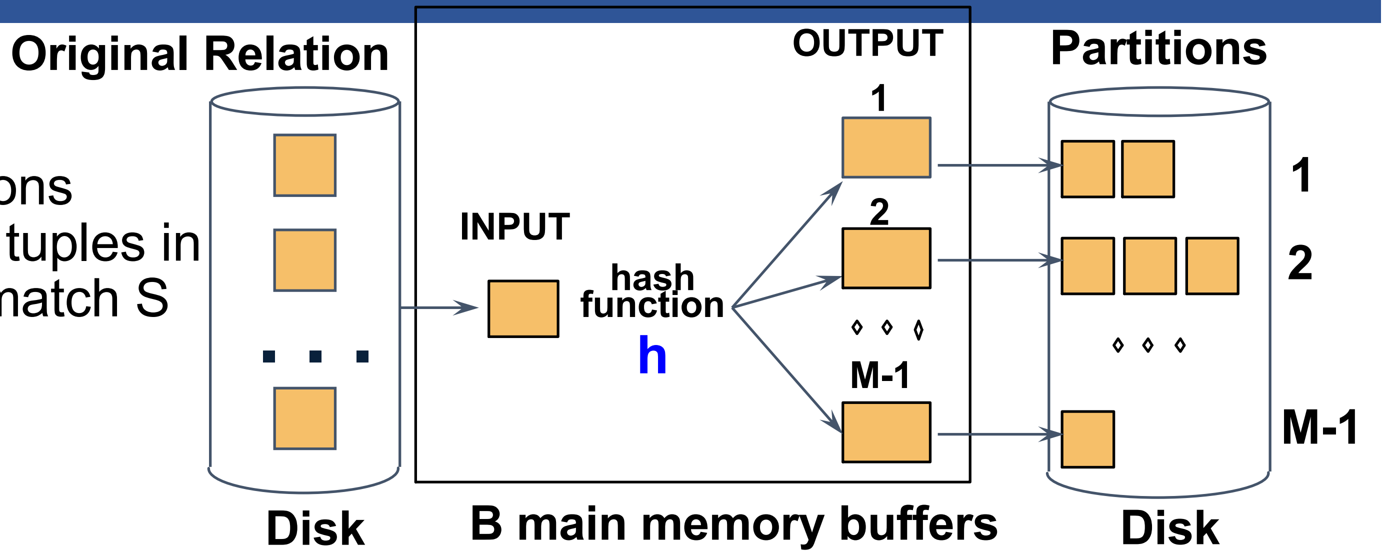
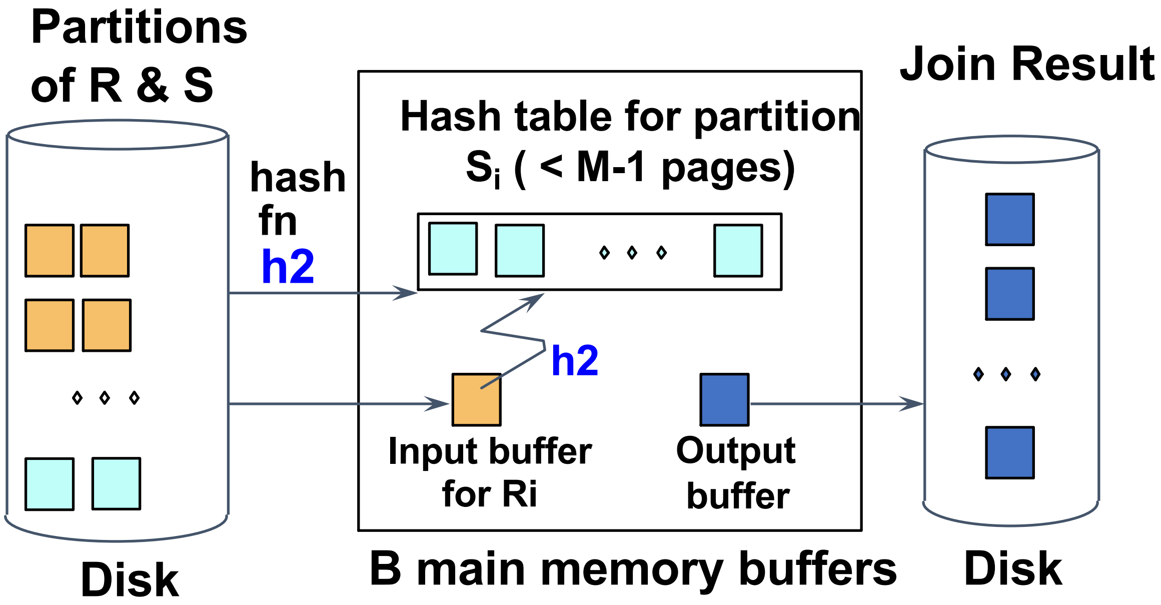


- Read in a partition of R, hash it using **h2 (<> h!)**. Scan matching partition of S, search for matches.



Partitioned Hash-Join

- Cost: 3B(R) + 3B(S)
- Assumption: min(B(R), B(S)) <= M^2

Hybrid Hash Join Algorithm (see book)

- Partition S into k buckets

  t buckets S_1 , …, S_t stay in memory

  k-t buckets S_{t+1}, …, S_k to disk
- Partition R into k buckets
  - First t buckets join immediately with S
  - Rest k-t buckets go to disk
- Finally, join k-t pairs of buckets:

  (R_{t+1},S_{t+1}), (R_{t+2},S_{t+2}), …, (R_k,S_k)

Summary of External Join Algorithms

- Block Nested Loop: B(S) + B(R)*B(S)/(M-1)

- Index Join:
  - Clustered: B(R) + T(R)B(S)/V(S,a)
  - Unclustered: B(R) + T(R)T(S)/V(S,a)

- Merge Join: 3B(R)+3B(S)
  - B(R)+B(S) <= M^2

- Partitioned Hash Join: 3B(R)+3B(S)
  - min(B(R), B(S)) <= M^2