

Database System Internals Operator Algorithms (part 2)

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Announcements

- Quiz 1 released on Gradescope morning of Feb. 10th, due by 11am on Feb 11th.
 - Topics are concepts from lab 1, usually "if you changed your lab 1 implementation in this way, describe the result"
 - Example quiz on website

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Block-Memory Refinement

```

for each group of M-1 pages r in R do
  for each page of tuples s in S do
    for all pairs of tuples t1 in r, t2 in s
      if t1 and t2 join then output (t1, t2)

```

What is the Cost?

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Block Memory Refinement

M=3

Disk

Patient		Insurance	
1	2	2	4
3	4	6	6
9	6	4	3
8	5	1	3
		2	8
		8	9


 Input buffer for Patient

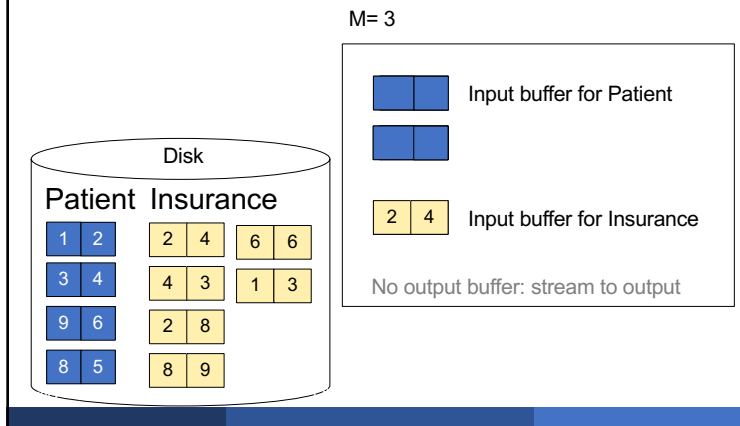

 Input buffer for Insurance

No output buffer: stream to output

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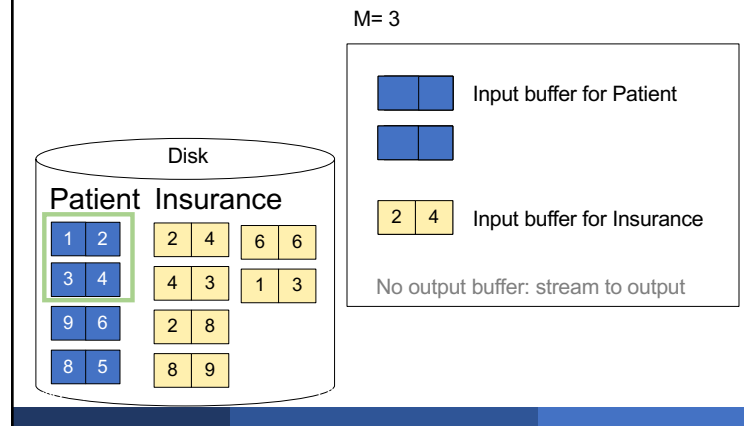
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Block Memory Refinement



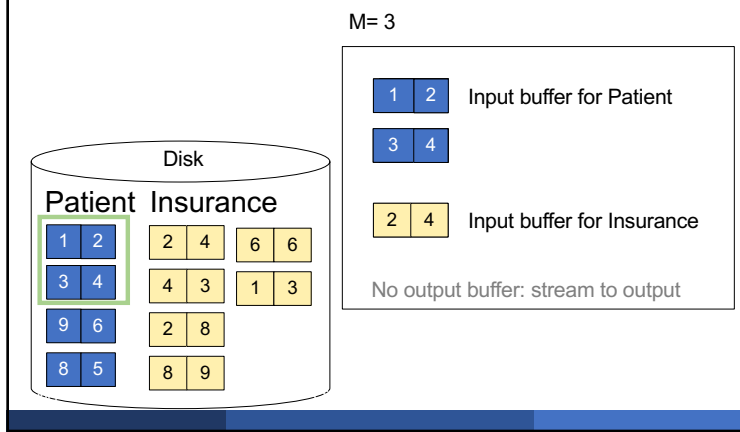
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Block Memory Refinement



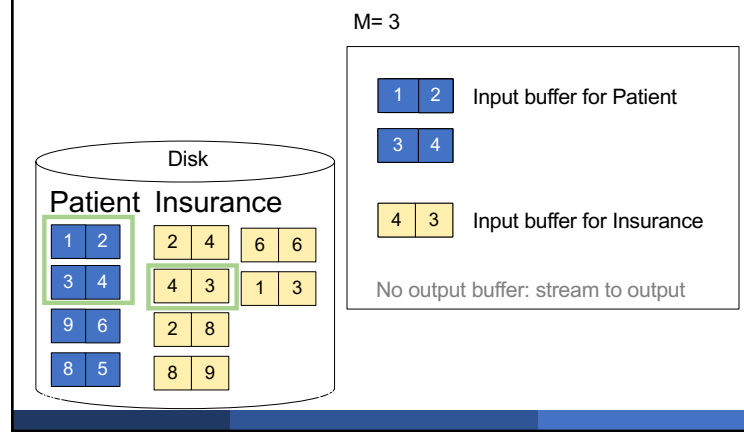
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Block Memory Refinement



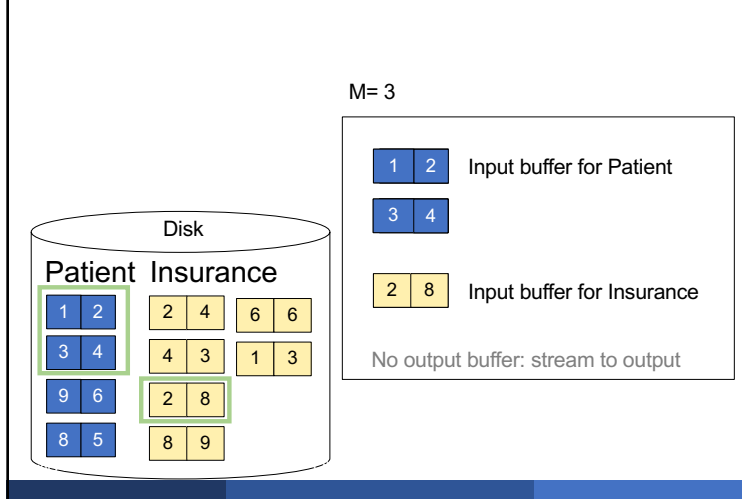
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Block Memory Refinement



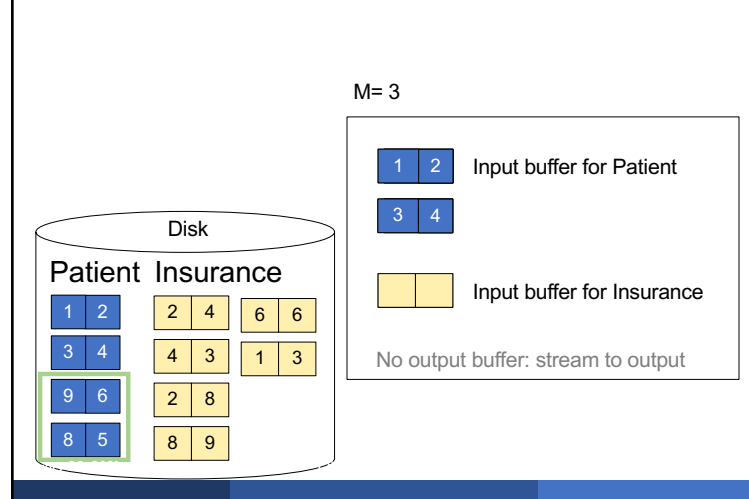
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Block Memory Refinement



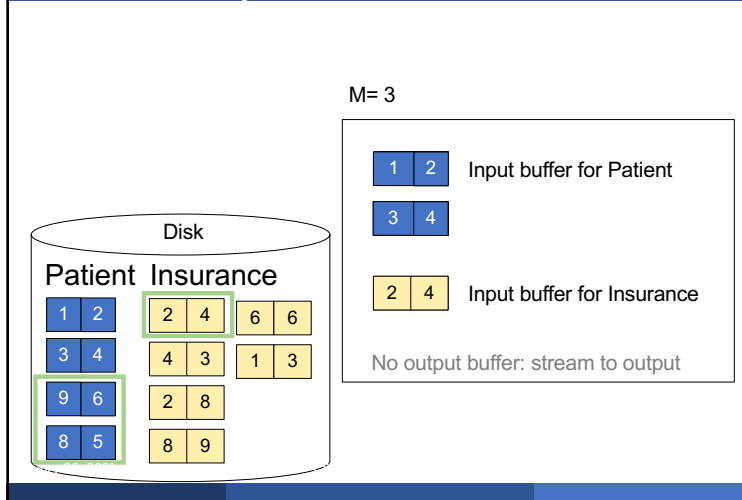
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Block Memory Refinement



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Block Memory Refinement



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Block Memory Refinement

```

for each group of M-1 pages r in R do
  for each page of tuples s in S do
    for all pairs of tuples t1 in r, t2 in s
      if t1 and t2 join then output (t1, t2)
  
```

What is the Cost?

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Block Memory Refinement

```

for each group of M-1 pages r in R do
  for each page of tuples s in S do
    for all pairs of tuples t1 in r, t2 in s
      if t1 and t2 join then output (t1, t2)
  
```

- Cost: $B(R) + B(R)B(S)/(M-1)$

What is the Cost?

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Sort-Merge Join

Sort-merge join: $R \bowtie S$

- Scan R and sort in main memory
- Scan S and sort in main memory
- Merge R and S
- Cost: $B(R) + B(S)$
- One pass algorithm when $B(S) + B(R) \leq M$
- Typically, this is NOT a one pass algorithm,
 - We'll see the multi-pass version next lecture

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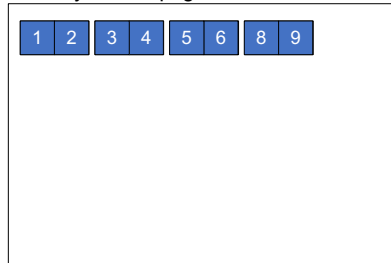
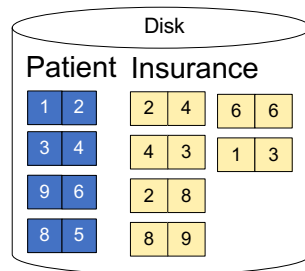
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Sort-Merge Join Example

Step 1: Scan Patient and **sort** in memory

Memory M = 21 pages



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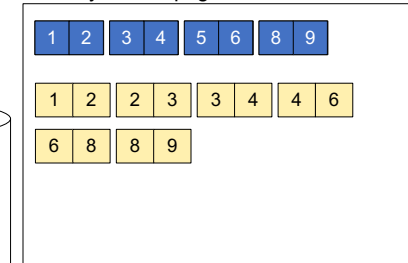
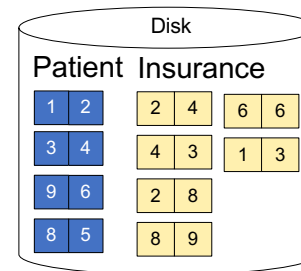
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Sort-Merge Join Example

Step 2: Scan Insurance and **sort** in memory

Memory M = 21 pages



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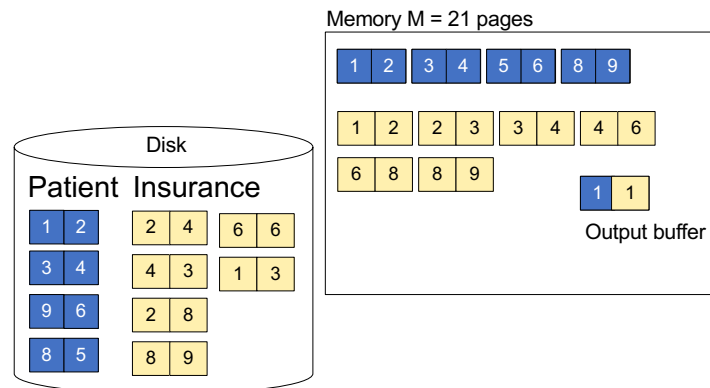
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Sort-Merge Join Example

Step 3: **Merge** Patient and Insurance



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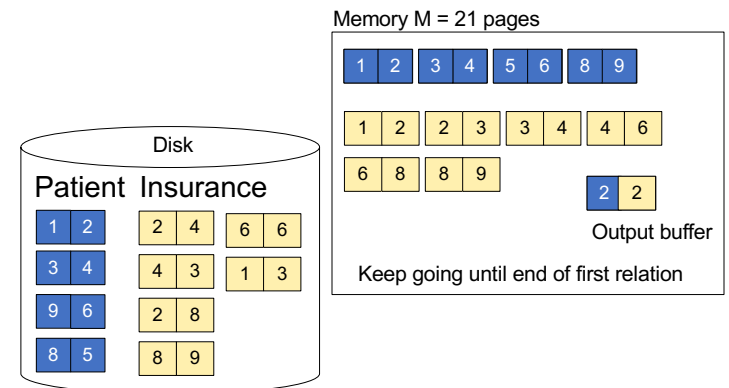
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Sort-Merge Join Example

Step 3: **Merge** Patient and Insurance



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Outline

Join operator algorithms

- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)

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Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)$ = size of R in blocks
- $T(R)$ = number of tuples in R
- $V(R, a)$ = # of distinct values of attribute a

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Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)$ = size of R in blocks
- $T(R)$ = number of tuples in R
- $V(R, a)$ = # of distinct values of attribute a

What is the cost in each case?

- Clustered index on a:
- Unclustered index on a:

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Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)$ = size of R in blocks
- $T(R)$ = number of tuples in R
- $V(R, a)$ = # of distinct values of attribute a

What is the cost in each case?

- Clustered index on a: $B(R)/V(R, a)$
- Unclustered index on a:

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Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)$ = size of R in blocks
- $T(R)$ = number of tuples in R
- $V(R, a)$ = # of distinct values of attribute a

What is the cost in each case?

- Clustered index on a: $B(R)/V(R, a)$
- Unclustered index on a: $T(R)/V(R, a)$

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Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)$ = size of R in blocks
- $T(R)$ = number of tuples in R
- $V(R, a)$ = # of distinct values of attribute a

What is the cost in each case?

- Clustered index on a: $B(R)/V(R, a)$
- Unclustered index on a: $T(R)/V(R, a)$

Note: we ignore I/O cost for index pages

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Index Based Selection

▪ **Example:**

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

▪ **Table scan:**

▪ **Index based selection:**

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Index Based Selection

▪ **Example:**

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

▪ **Table scan:** $B(R) = 2,000$ I/Os

▪ **Index based selection:**

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Index Based Selection

▪ **Example:**

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

▪ **Table scan:** $B(R) = 2,000$ I/Os

▪ **Index based selection:**

- If index is clustered:
- If index is unclustered:

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Index Based Selection

▪ **Example:**

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

▪ **Table scan:** $B(R) = 2,000$ I/Os

▪ **Index based selection:**

- If index is clustered: $B(R)/V(R,a) = 100$ I/Os
- If index is unclustered:

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Index Based Selection

Example:

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

Table scan: $B(R) = 2,000$ I/Os

Index based selection:

- If index is clustered: $B(R)/V(R,a) = 100$ I/Os
- If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os

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Index Based Selection

Example:

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

Table scan: $B(R) = 2,000$ I/Os!

Index based selection:

- If index is clustered: $B(R)/V(R,a) = 100$ I/Os
- If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os!

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Index Based Selection

Example:

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

Table scan: $B(R) = 2,000$ I/Os!

Index based selection:

- If index is clustered: $B(R)/V(R,a) = 100$ I/Os
- If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os!

Lesson: Don't build unclustered indexes when $V(R,a)$ is small !

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Index Based Selection

Example:

$B(R) = 2000$
 $T(R) = 100,000$
 $V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

Table scan: $B(R) = 2,000$ I/Os

Index based selection:

- If index is clustered: $B(R)/V(R,a) = 100$ I/Os
- If index is unclustered: $T(R)/V(R,a) = 5,000$ I/Os

Lesson: Don't build unclustered indexes when $V(R,a)$ is small !

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Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S
- Previous nested loop join: cost
 - $B(R) + T(R) * B(S)$
- **Index Nested Loop Join Cost:**
 - If index on S is clustered: $B(R) + T(R)B(S)/V(S,a)$
 - If index on S is unclustered: $B(R) + T(R)T(S)/V(S,a)$

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Outline

▪ Join operator algorithms

- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)

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Two-Pass Algorithms

- Fastest algorithm seen so far is one-pass hash join
What if data does not fit in memory?
- Need to process it in multiple passes
- Two key techniques
 - **Sorting**
 - **Hashing**

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Basic Terminology

- A run in a sequence is an increasing subsequence
- What are the runs?
2, 4, 99, 103, 88, 77, 3, 79, 100, 2, 50

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Basic Terminology

- A run in a sequence is an increasing subsequence
- What are the runs?

2, 4, 99, 103, 88, 77, 3, 79, 100, 2, 50

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External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

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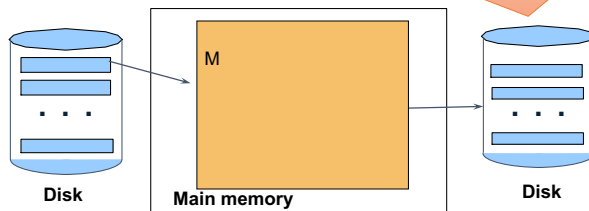
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External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

Q: How long are the runs?



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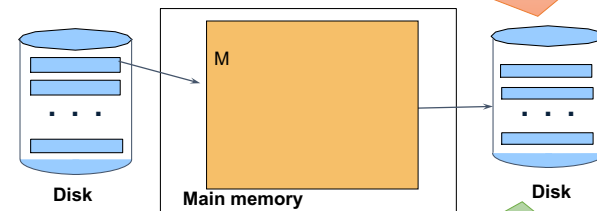
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External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

Q: How long are the runs?



A: Length = M blocks

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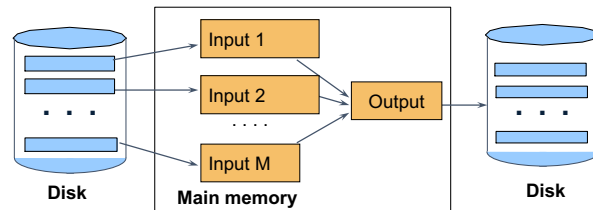
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Phase two: merge M runs into a bigger run

- Merge M – 1 runs into a new run
- Result: runs of length M (M – 1) \approx M²



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Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0

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Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0, ?

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Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0, 1, ?

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Example

- Merging three runs to produce a longer run:

0, **14**, 33, 88, 92, 192, 322
 2, 4, 7, **43**, 78, 103, 523
 1, 6, **9**, 12, 33, 52, 88, 320

Output:

0, **1**, 2, 4, 6, 7, **?**

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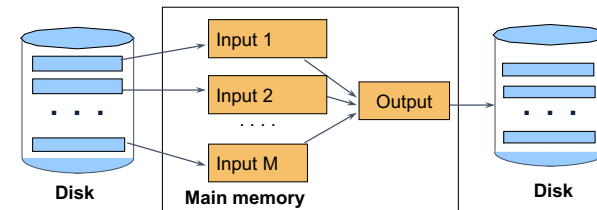
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External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge $M - 1$ runs into a new run
- Result: runs of length $M(M - 1) \approx M^2$



If approx. $B \leq M^2$ then we are done

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Cost of External Merge Sort

- Assumption: $B(R) \leq M^2$
- Read+write+read = $3B(R)$

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Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?

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Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?
- Example:
 - Page size = 32KB
 - Memory size 32GB: $M = 10^6$ -pages

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Discussion

- What does $B(R) \leq M^2$ mean?
- How large can R be?
- Example:
 - Page size = 32KB
 - Memory size 32GB: $M = 10^6$ pages
- R can be as large as 10^{12} pages
 - 32×10^{15} Bytes = 32 PB

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Merge-Join

- Join $R \bowtie S$
- How?....

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Merge-Join

- Join $R \bowtie S$
- Step 1a: generate initial runs for R
 - Step 1b: generate initial runs for S
 - Step 2: merge and join
 - Either merge first and then join
 - Or merge & join at the same time

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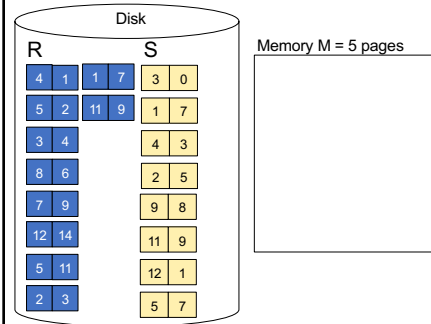
Merge-Join Example

Setup: Want to join R and S

Relation R has 10 pages with 2 tuples per page

Relation S has 8 pages with 2 tuples per page

Values shown are values of join attribute for each given tuple



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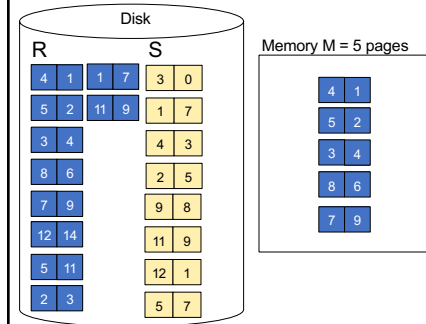
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Merge-Join Example

Step 1: Read M pages of R and sort in memory



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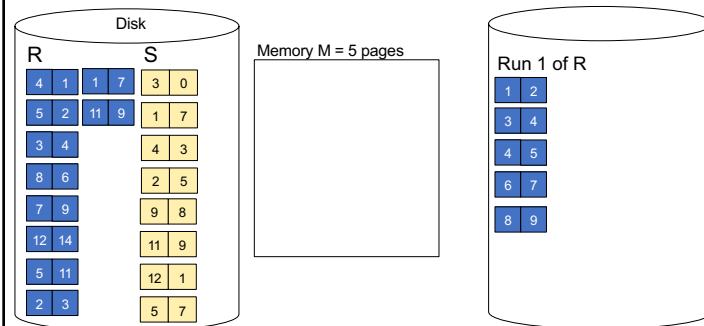
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Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk



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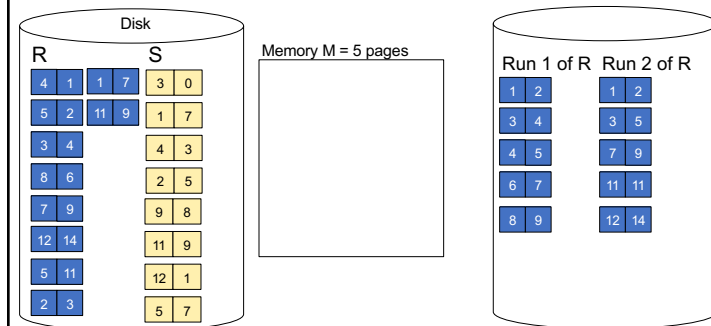
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Merge-Join Example

Step 1: Repeat for next M pages until all R is processed



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Merge-Join Example

Step 1: Do the same with S

Disk

R	S
4 1	1 7
5 2	11 9
3 4	
8 6	
7 9	
12 14	
5 11	
2 3	

Memory M = 5 pages

Run 1 of S Run 2 of S

0 1	1 5
2 3	7 9
3 4	11 12
5 7	
8 9	

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Run 1 of R Run 2 of R

1 2	1 2
3 4	3 5
4 5	7 9
6 7	11 11
8 9	12 14

Run 1 of S Run 2 of S

0 1	1 5
2 3	7 9
3 4	11 12
5 7	
8 9	

Memory M = 5 pages

Step 2: Join while merging
Output tuples

1 2
1 2
0 1
1 5

Input buffers

Run1

Run2

Output buffer

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Run 1 of R Run 2 of R

1 2	1 2
3 4	3 5
4 5	7 9
6 7	11 11
8 9	12 14

Run 1 of S Run 2 of S

0 1	1 5
2 3	7 9
3 4	11 12
5 7	
8 9	

Memory M = 5 pages

Step 2: Join while merging
Output tuples

1 2
1 2
0 1
1 5

Input buffers

Run1

Run2

Output buffer

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Run 1 of R Run 2 of R

1 2	1 2
3 4	3 5
4 5	7 9
6 7	11 11
8 9	12 14

Run 1 of S Run 2 of S

0 1	1 5
2 3	7 9
3 4	11 12
5 7	
8 9	

Memory M = 5 pages

Step 2: Join while merging
Output tuples

1 2
1 2
0 1
1 5

Input buffers

Run1

Run2

Output buffer

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages

Run 1 of R
1 2
3 4
4 5
6 7
8 9

Run 2 of R
1 2
3 5
7 9
11 11
12 14

Run 1 of S
0 1
2 3
3 4
5 7
8 9

Run 2 of S
1 5
7 9
11 12

Input buffers

1 2	Run1
1 2	Run2
2 3	Run1
1 5	Run2

Output buffer

Step 2: Join while merging
Output tuples
(1,1)
(1,1)
(1,1)
(1,1)

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages

Run 1 of R
1 2
3 4
4 5
6 7
8 9

Run 2 of R
1 2
3 5
7 9
11 11
12 14

Run 1 of S
0 1
2 3
3 4
5 7
8 9

Run 2 of S
1 5
7 9
11 12

Input buffers

1 2	Run1
1 2	Run2
2 3	Run1
1 5	Run2

Output buffer

Step 2: Join while merging
Output tuples
(1,1)
(1,1)
(1,1)
(1,1)
(2,2)
(2,2)

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Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: $3B(R) + 3B(S)$

Memory $M = 5$ pages

Run 1 of R
1 2
3 4
4 5
6 7
8 9

Run 2 of R
1 2
3 5
7 9
11 11
12 14

Run 1 of S
0 1
2 3
3 4
5 7
8 9

Run 2 of S
1 5
7 9
11 12

Input buffers

3 4	Run1
3 5	Run2
2 3	Run1
1 5	Run2

Output buffer

Step 2: Join while merging
Output tuples
(1,1)
(1,1)
(1,1)
(1,1)
(2,2)
(2,2)
(3,3)
(3,3)
...

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Merge-Join

Disk → **Main memory** → **Disk**

Input 1
Input 2
...
Input M

Output

Disk

$M_1 = B(R)/M$ runs for R
 $M_2 = B(S)/M$ runs for S
 Merge-join $M_1 + M_2$ runs;
 need $M_1 + M_2 \leq M$ to process all runs
 i.e. $B(R) + B(S) \leq M^2$

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