

## Database System Internals

## Operator Algorithms (part 2)

Paul G. Allen School of Computer Science and Engineering University of Washington, Seattle

## Block-Memory Refinement

for each group of $M-1$ pages $r$ in $R$ do for each page of tuples $s$ in $S$ do for all pairs of tuples $t_{1}$ in $r, t_{2}$ in $s$
if $t_{1}$ and $t_{2}$ join then output $\left(t_{1}, t_{2}\right)$

What is the Cost?

## Block Memory Refinement

$$
M=3
$$

Disk
Patient Insurance

| 1 | 2 |
| :--- | :--- |
| 3 | 4 |
| 9 | 6 |
| 8 | 5 |



Input buffer for Patient


Input buffer for Insurance

## Block Memory Refinement

$$
M=3
$$



Input buffer for Patient

$\square$ Input buffer for Insurance

## Block Memory Refinement

$$
M=3
$$



Input buffer for Patient


| 2 | 4 | Input buffer for Insurance |
| :--- | :--- | :--- |

## Block Memory Refinement



$$
M=3
$$

| 1 | 2 |
| :--- | :--- |
|  | Input buffer for Patient |
| 3 | 4 |


| 2 | 4 |
| :--- | :--- |
| Input buffer for Insurance |  |

## Block Memory Refinement



$$
M=3
$$



## Block Memory Refinement



$$
M=3
$$



| 2 | 8 | Input buffer for Insurance |
| :--- | :--- | :--- |

## Block Memory Refinement

$$
M=3
$$

Disk
Patient Insurance

| 1 | 2 |
| :--- | :--- |
| 3 | 4 |
| 9 | 6 |
| 8 | 5 |


| 2 | 4 | 6 | 6 |
| :---: | :---: | :---: | :---: |
| 4 | 3 | 1 | 3 |
| 2 | 8 |  |  |
| 8 | 9 |  |  |


| 1 | 2 | Input buffer for Patient |
| :--- | :--- | :--- |

```
3 4
```



Input buffer for Insurance

## Block Memory Refinement

$$
M=3
$$

Disk
Patient Insurance

| 1 | 2 | 2 | 4 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 4 | 4 | 3 | 1 | 3 |
| 9 | 6 | 2 | 8 |  |  |
| 8 | 5 | 8 | 9 |  |  |


| 1 | 2 | Input buffer for Patient |
| :--- | :--- | :--- |

$\square$
$3 \quad 4$

| 2 | 4 |
| :--- | :--- |

No output buffer: stream to output

## Block Memory Refinement

for each group of $\mathrm{M}-1$ pages $r$ in $R$ do for each page of tuples $s$ in $S$ do for all pairs of tuples $t_{1}$ in $r, t_{2}$ in $s$
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## Block Memory Refinement

for each group of $\mathrm{M}-1$ pages r in R do
for each page of tuples $s$ in $S$ do
for all pairs of tuples $t_{1}$ in $r, t_{2}$ in $s$
if $t_{1}$ and $t_{2}$ join then output $\left(t_{1}, t_{2}\right)$

- Cost: $B(R)+B(R) B(S) /(M-1)$

What is the Cost?

## Sort-Merge Join

Sort-merge join: $R \bowtie S$

- Scan R and sort in main memory
- Scan S and sort in main memory
- Merge R and S
- Cost: B(R) + B(S)
- One pass algorithm when $B(S)+B(R)<=M$
- Typically, this is NOT a one pass algorithm,
- We'll see the multi-pass version next lecture


## Sort-Merge Join Example

Step 1: Scan Patient and sort in memory
Memory $\mathrm{M}=21$ pages

Disk
Patient Insurance


| 2 | 4 |
| :--- | :--- |
| 4 | 3 |
| 2 | 8 |
| 8 | 9 |

## Sort-Merge Join Example

Step 2: Scan Insurance and sort in memory
Memory M = 21 pages

| Disk |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Patient Insurance |  |  |  |  |  |
| 1 | 2 | 2 | 4 | 6 | 6 |
| 3 | 4 | 4 | 3 | 1 | 3 |
| 9 | 6 | 2 | 8 |  |  |
| 8 | 5 | 8 | 9 |  |  |



## Sort-Merge Join Example

Step 3: Merge Patient and Insurance
Memory M = 21 pages


| 1 | 2 | 3 | 4 | 5 | 6 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |$|$| 1 | 2 | 2 | 3 | 3 | 4 | $\left.\left\lvert\, \begin{array}{ll}4 & 6 \\ \hline 6 & 8 \\ \hline & 8\end{array}\right.\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Sort-Merge Join Example

## Step 3: Merge Patient and Insurance

Memory M = 21 pages


| 1 2 3 4 5 6 8 9 <br> 1 3       <br> 1 2 2 3 3 4 4 6 <br> 6 8 8 9     |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Outline

- Join operator algorithms
- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)


## Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)=$ size of $R$ in blocks
- $T(R)=$ number of tuples in $R$
- $\mathrm{V}(\mathrm{R}, \mathrm{a})=$ \# of distinct values of attribute a


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What is the cost in each case?

- Clustered index on a:
- Unclustered index on a:


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What is the cost in each case?

- Clustered index on a: $\quad B(R) / V(R, a)$
- Unclustered index on a:


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- $B(R)=$ size of $R$ in blocks
- $T(R)=$ number of tuples in $R$
- $\mathrm{V}(\mathrm{R}, \mathrm{a})=$ \# of distinct values of attribute a

What is the cost in each case?

- Clustered index on $a$ : $\quad B(R) / V(R, a)$
- Unclustered index on $a$ : $T(R) / V(R, a)$

Note: we ignore I/O cost for index pages

## Index Based Selection

- Example:

$$
\begin{aligned}
& B(R)=2000 \\
& T(R)=100,000 \\
& V(R, a)=20
\end{aligned}
$$

```
cost of }\mp@subsup{\sigma}{a=v}{}(R)=\mathrm{ ?
```

- Table scan:
- Index based selection:


## Index Based Selection



- Index based selection:


## Index Based Selection

- Example: $\begin{aligned} & \begin{array}{l}B(R)=2000 \\ T(R)=100,000 \\ \mathrm{~V}(R, a)=20\end{array} \\ & \text { - Table scan: } \mathrm{B}(\mathrm{R})=2,000 \mathrm{I} / \mathrm{Os}\end{aligned}$
- Index based selection:
- If index is clustered:
- If index is unclustered:


## Index Based Selection

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- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100$ I/Os
- If index is unclustered:


## Index Based Selection



- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100$ I/Os
- If index is unclustered: $T(R) / V(R, a)=5,000 \mathrm{I} / \mathrm{Os}$


## Index Based Selection

- Example: $\begin{aligned} & \begin{array}{l}\mathrm{B}(\mathrm{R})=2000 \\ \mathrm{~T}(\mathrm{R})=100,000 \\ \mathrm{~V}(\mathrm{R}, \mathrm{a})=20\end{array} \\ & \text { - Table scan: } \mathrm{B}(\mathrm{R})=2,000 \mathrm{I} / \mathrm{Os}\end{aligned}$
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$$
\operatorname{cost} \text { of } \sigma_{a=v}(\mathrm{R})=\text { ? }
$$

- Table scan: $B(R)=2,000$ I/Os
- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100 I / O s$
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Lesson: Don't build unclustered indexes when $\mathrm{V}(\mathrm{R}, \mathrm{a})$ is small!

## Index Based Selection



- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100 I / O s$
- If index is unclustered: $T(R) / V(R, a)=5,000 \mathrm{I} / \mathrm{Os}$

Lesson: Don't build unclustered indexes when $\mathrm{V}(\mathrm{R}, \mathrm{a})$ is small!

## Index Nested Loop Join

## $R \bowtie S$

- Assume $S$ has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S
- Previous nested loop join: cost
- $B(R)+T(R) * B(S)$
- Index Nested Loop Join Cost:
- If index on $S$ is clustered: $B(R)+T(R) B(S) / V(S, a)$
- If index on $S$ is unclustered: $B(R)+T(R) T(S) / V(S, a)$


## Outline

- Join operator algorithms
- One-pass algorithms (Sec. 15.2 and 15.3)
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- Two-pass algorithms (Sec 15.4 and 15.5)


## Two-Pass Algorithms

- Fastest algorithm seen so far is one-pass hash join What if data does not fit in memory?
- Need to process it in multiple passes
- Two key techniques
- Sorting
- Hashing


## Basic Terminology

- A run in a sequence is an increasing subsequence
- What are the runs?
$2,4,99,103,88,77,3,79,100,2,50$


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- What are the runs?
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## External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

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Q: How long are the runs?


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Phase one: load $M$ blocks in memory, sort, send to disk, repeat
Q: How long are the runs?


A: Length $=\mathrm{M}$ blocks

Phase two: merge $M$ runs into a bigger run

- Merge $\mathrm{M}-1$ runs into a new run
- Result: runs of length $M(M-1) \approx M^{2}$



## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320
Output:
0

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320
Output:
0,?

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320
Output:
0, 1, ?

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
$1,6, \mathbf{9}, \mathbf{1 2}, \mathbf{3 3}, \mathbf{5 2}, \mathbf{8 8}, 320$
Output:
$0,1,2,4,6,7$, ?

## External Merge-Sort: Step 2

Phase two: merge $M$ runs into a bigger run

- Merge $\mathrm{M}-1$ runs into a new run
- Result: runs of length $M(M-1) \approx M^{2}$


If approx. $\mathrm{B}<=\mathrm{M}^{2}$ then we are done

## Cost of External Merge Sort

- Assumption: $B(R)<=M^{2}$
- Read+write+read $=3 B(R)$


## Discussion

- What does $B(R)<=M^{2}$ mean?
- How large can $R$ be?


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- How large can $R$ be?
- Example:
- Page size $=32 \mathrm{~KB}$
- Memory size 32GB: $\mathrm{M}=10^{6}$-pages


## Discussion

- What does $B(R)<=M^{2}$ mean?
- How large can $R$ be?
- Example:
- Page size = 32KB
- Memory size 32GB: $\mathrm{M}=10^{6}$ pages
- $R$ can be as large as $10^{12}$ pages
- $32 \times 10^{15}$ Bytes $=32 \mathrm{~PB}$


# Merge-Join 

Join $R \bowtie S$

- How?....


## Merge-Join

Join $R \bowtie S$

- Step 1a: generate initial runs for $R$
- Step 1b: generate initial runs for $S$
- Step 2: merge and join
- Either merge first and then join
- Or merge \& join at the same time


## Merge-Join Example

## Setup: Want to join R and S

Relation R has 10 pages with 2 tuples per page
Relation $S$ has 8 pages with 2 tuples per page
Values shown are values of join attribute for each given tuple


Memory M = 5 pages


## Merge-Join Example

Step 1: Read $M$ pages of $R$ and sort in memory


## Merge-Join Example

Step 1: Read $M$ pages of $R$ and sort in memory, then write to disk


## Merge-Join Example

Step 1: Repeat for next $M$ pages until all $R$ is processed


## Merge-Join Example

Step 1: Do the same with S


Run 1 of $S$ Run 2 of $S$

| 0 | 1 |
| :---: | :---: |
| 2 | 3 |
| 3 | 4 |
| 5 | 7 |
| 8 | 9 |


| 1 | 5 |
| :---: | :---: |
| 7 | 9 |
| 11 | 12 |

## Merge-Join Example

Step 2: Join while merging sorted runs


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## Merge-Join


$M_{1}=B(R) / M$ runs for $R$
$M_{2}=B(S) / M$ runs for $S$

Merge-join $M_{1}+M_{2}$ runs;
need $M_{1}+M_{2}<=M$ to process all runs
i.e. $B(R)+B(S)<=M^{2}$

