CSE 444: Database Internals

Lecture 8

Operator Algorithms (part 2)

Announcements

- Lab 2 released – new branch “lab2” contains instructions
- Lab 2 / part 1 due on Thursday
  - We will not run any tests – So bugs are OK
- Homework 2 due on Friday
- Paper review for master’s due on Friday

Outline

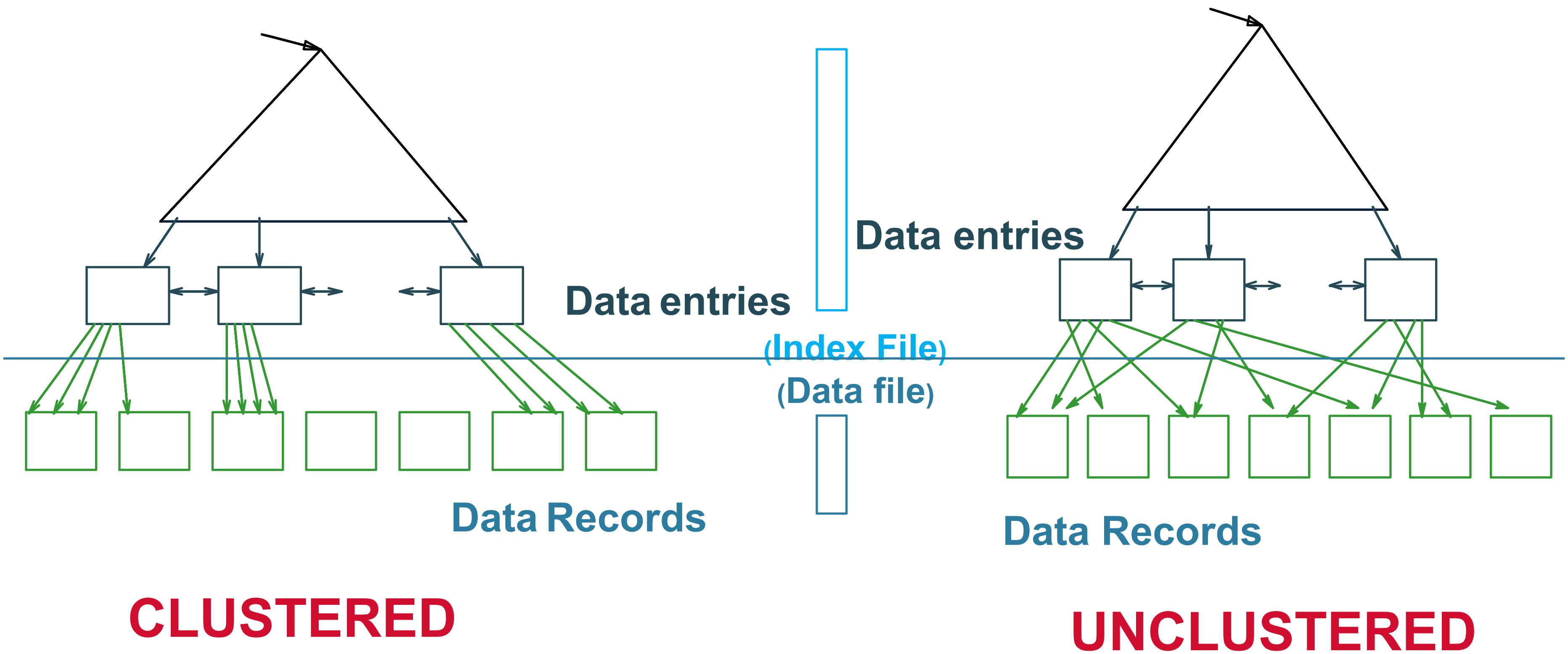
- **Join operator algorithms**
  - One-pass algorithms (Sec. 15.2 and 15.3)
  - Index-based algorithms (Sec 15.6)
  - Two-pass algorithms (Sec 15.4 and 15.5)

Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- B(R)= size of R in blocks
- T(R) = number of tuples in R
- V(R, a) = # of distinct values of attribute a

B+ Trees

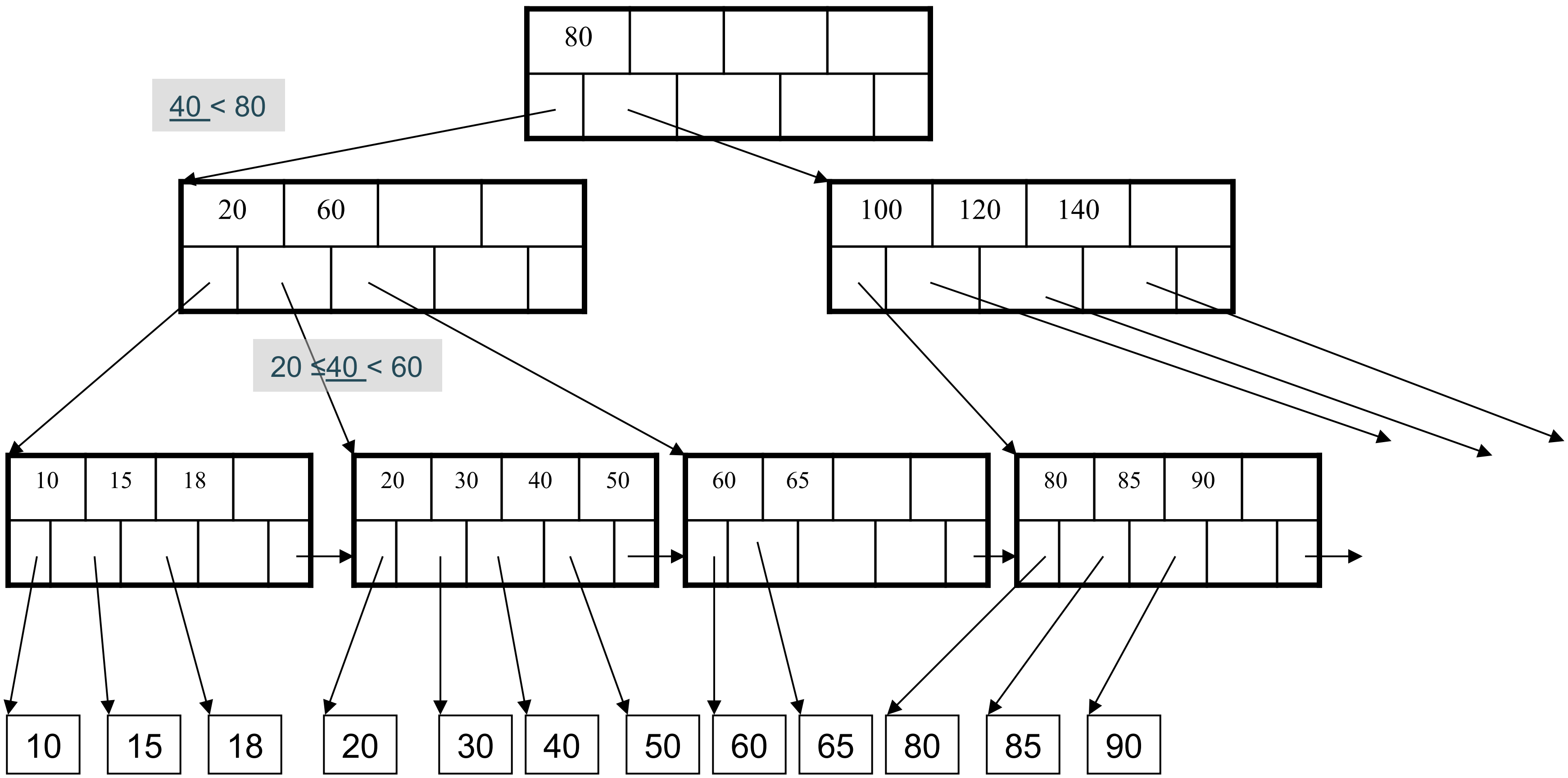


Note: can also store data records directly as data entries

B+ Tree Example

d = 2

Find the key 40



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What is the cost in each case?

- Clustered index on a:
- Unclustered index on a:

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What is the cost in each case?

- Clustered index on a: B(R)/V(R,a)
- Unclustered index on a: T(R)/V(R,a)

Note: we ignore I/O cost for index pages

Index Based Selection

- Example:

$B(R) = 2000$
$T(R) = 100{,}000$
$V(R, a) = 20$

cost of $\sigma_{a=v}(R) = ?$

- Table scan:
- Index based selection:

Index Based Selection

- Example:

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  $T(R) = 100{,}000$
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Index Based Selection

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cost of $\sigma_{a=v}(R) = ?$

- Table scan: $B(R) = 2{,}000$ I/Os
- Index based selection:
  - If index is clustered: $B(R)/V(R,a) = 100$ I/Os
  - If index is unclustered:

Index Based Selection

- Example: $B(R) = 2000$, $T(R) = 100{,}000$, $V(R, a) = 20$ — cost of $\sigma_{a=v}(R) = ?$
- Table scan: B(R) = 2,000 I/Os
- Index based selection:
  – If index is clustered: B(R)/V(R,a) = 100 I/Os
  – If index is unclustered: T(R)/V(R,a) = 5,000 I/Os

Index Based Selection

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Index Based Selection

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  – If index is clustered: $B(R)/V(R,a) = 100$ I/Os
  – If index is unclustered: $T(R)/V(R,a) = 5{,}000$ I/Os

Lesson: Don't build unclustered indexes when V(R,a) is small !

Index Nested Loop Join

$R \bowtie S$

- Assume S has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S

- Previous nested loop join: cost
  - $B(R) + T(R)*B(S)$
- Index Nested Loop Join Cost:
  - If index on S is clustered: $B(R) + T(R)B(S)/V(S,a)$
  - If index on S is unclustered: $B(R) + T(R)T(S)/V(S,a)$

Outline

- **Join operator algorithms**
  - One-pass algorithms (Sec. 15.2 and 15.3)
  - Index-based algorithms (Sec 15.6)
  - Two-pass algorithms (Sec 15.4 and 15.5)

Two-Pass Algorithms

- What if data does not fit in memory?
- Need to process it in multiple passes

- Two key techniques
  - Sorting
  - Hashing

Basic Terminology

- A run in a sequence is an increasing subsequence

- What are the runs?

  2, 4, 99, 103, 88, 77, 3, 79, 100, 2, 50

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External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, send to disk, repeat

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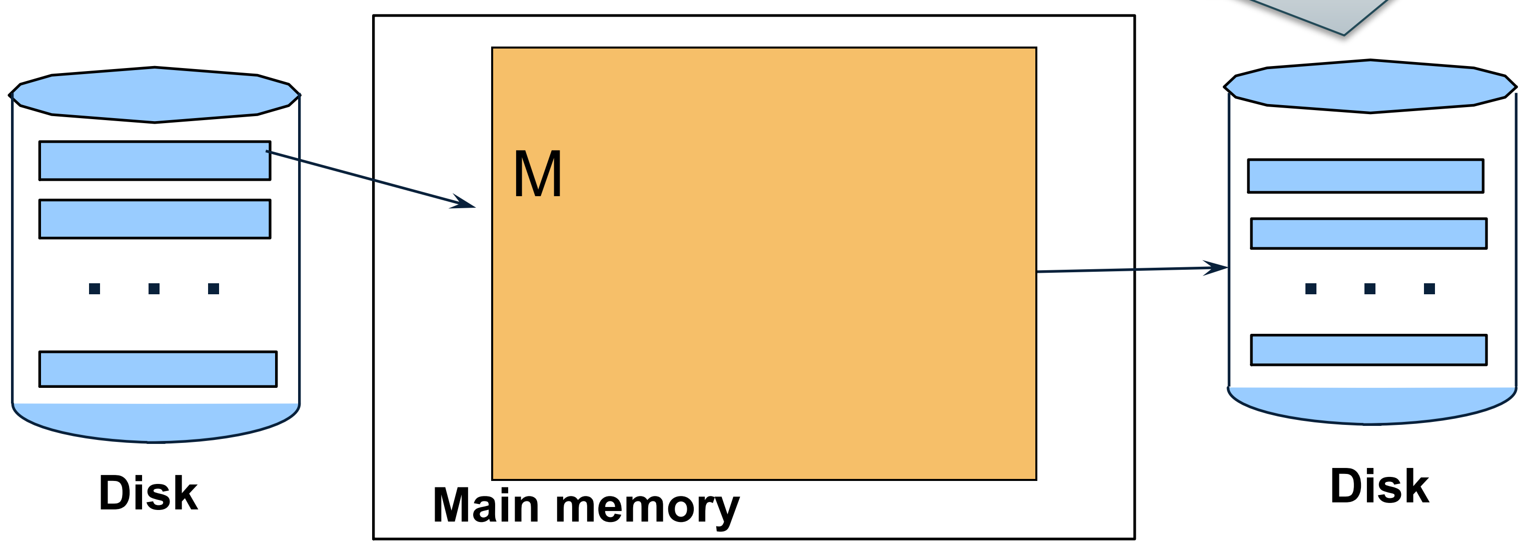

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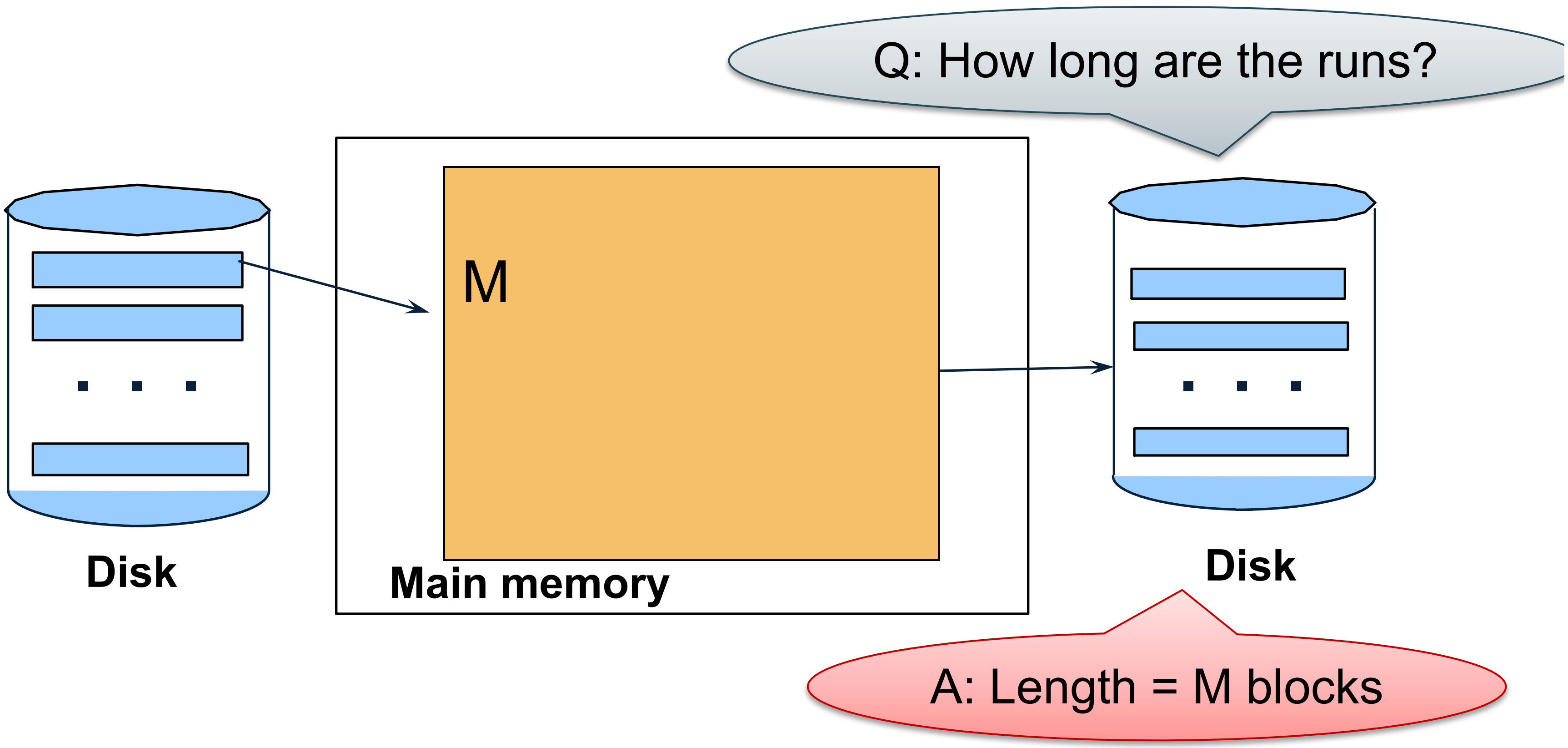

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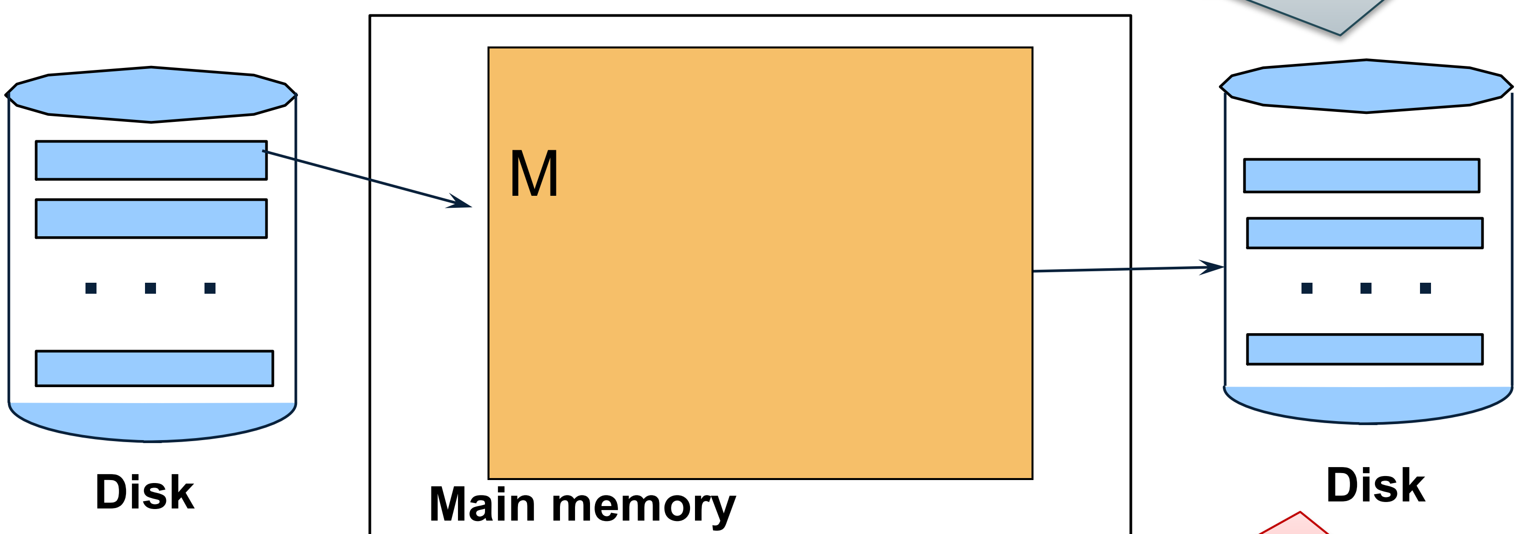


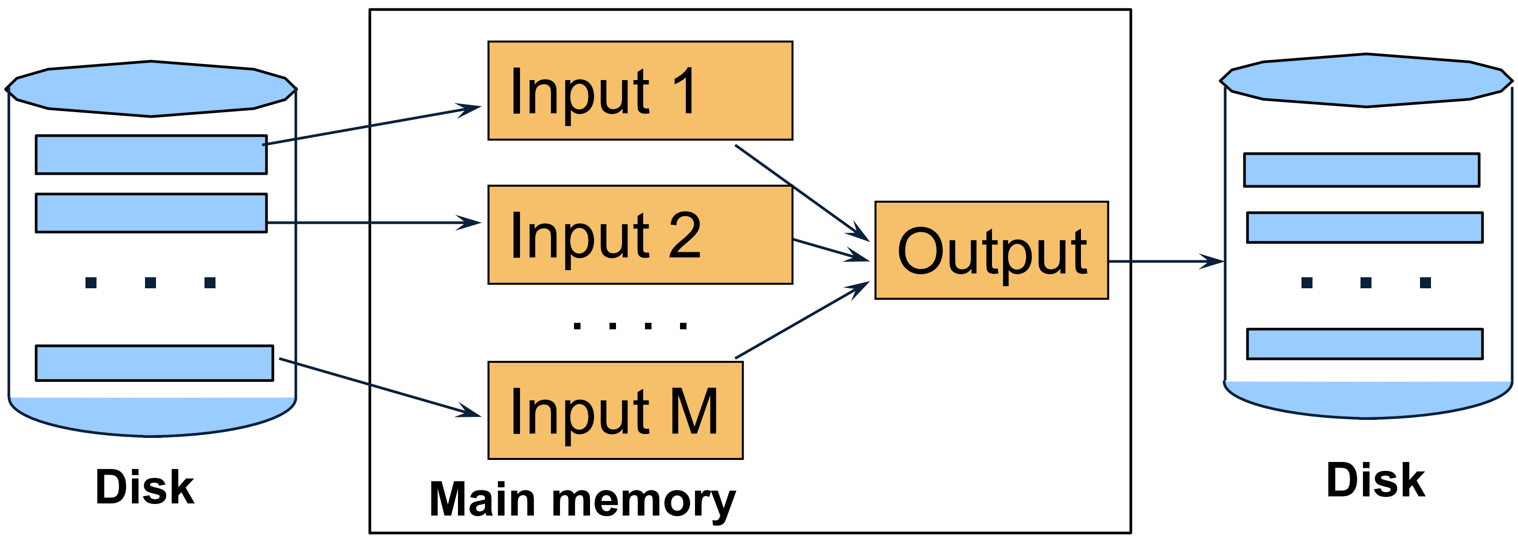
A: Length = M blocks

Sidenote: Can increase to length 2M using “replacement selection” (details in book)

External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge M – 1 runs into a new run
- Result: runs of length M (M – 1) ≈ M^2



Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0

Example

- Merging three runs to produce a longer run:

0, **14, 33, 88, 92, 192, 322**
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0, ?

Example

- Merging three runs to produce a longer run:

0, **14, 33, 88, 92, 192, 322**
2, 4, 7, 43, 78, 103, 523
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Output:
0, 1, ?

Example

- Merging three runs to produce a longer run:

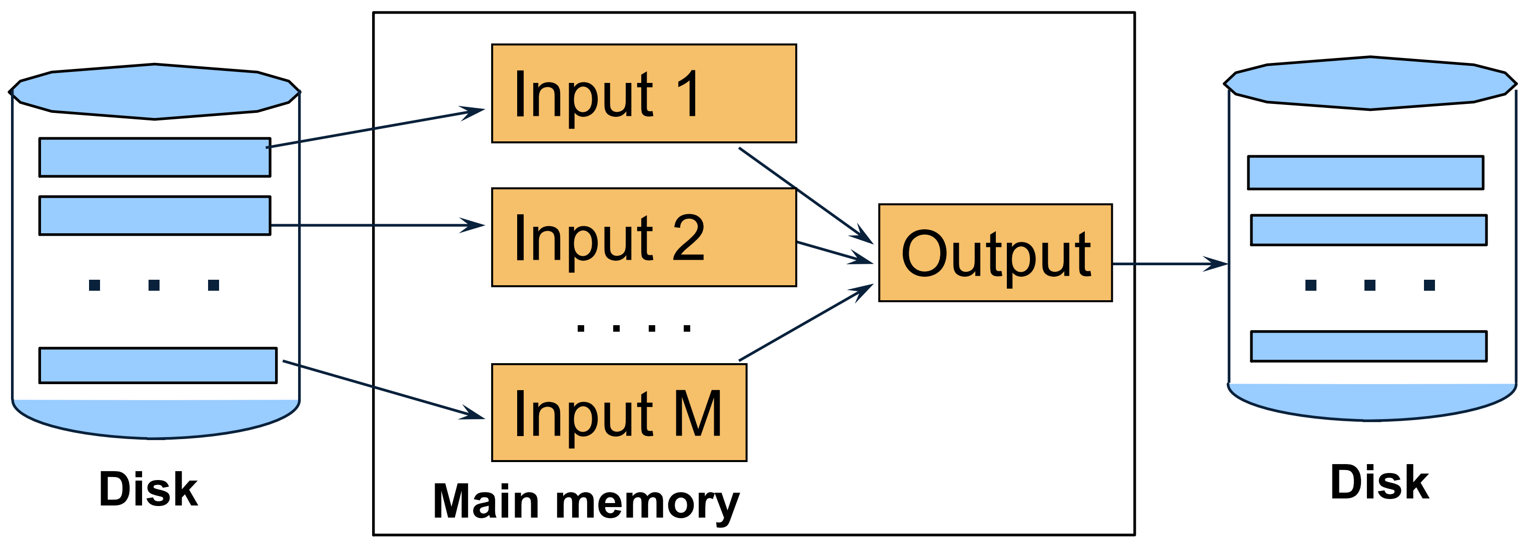
0, **14, 33, 88, 92, 192, 322**
2, 4, 7, **43, 78, 103, 523**
1, 6, **9, 12, 33, 52, 88, 320**

Output:
0, 1, 2, 4, 6, 7, ?

External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge M – 1 runs into a new run
- Result: runs of length M (M – 1) $\approx M^2$



If approx. $B \le M^2$ then we are done

Cost of External Merge Sort

- Assumption: $B(R) <= M^2$
- Read+write+read = 3B(R)

Discussion

- What does B(R) <= M^2 mean?
- How large can R be?

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- Example:
  - Page size = 32KB
  - Memory size 32GB: M = 10^6-pages

Discussion

- What does $B(R) <= M^2$ mean?
- How large can R be?

- Example:
  - Page size = 32KB
  - Memory size 32GB: $M = 10^6$ pages

- R can be as large as 10^{12} pages
  - 32×10^{15} Bytes = 32 PB

Merge-Join

Join R $\bowtie$ S

- How?....

Merge-Join

Join R $\bowtie$ S

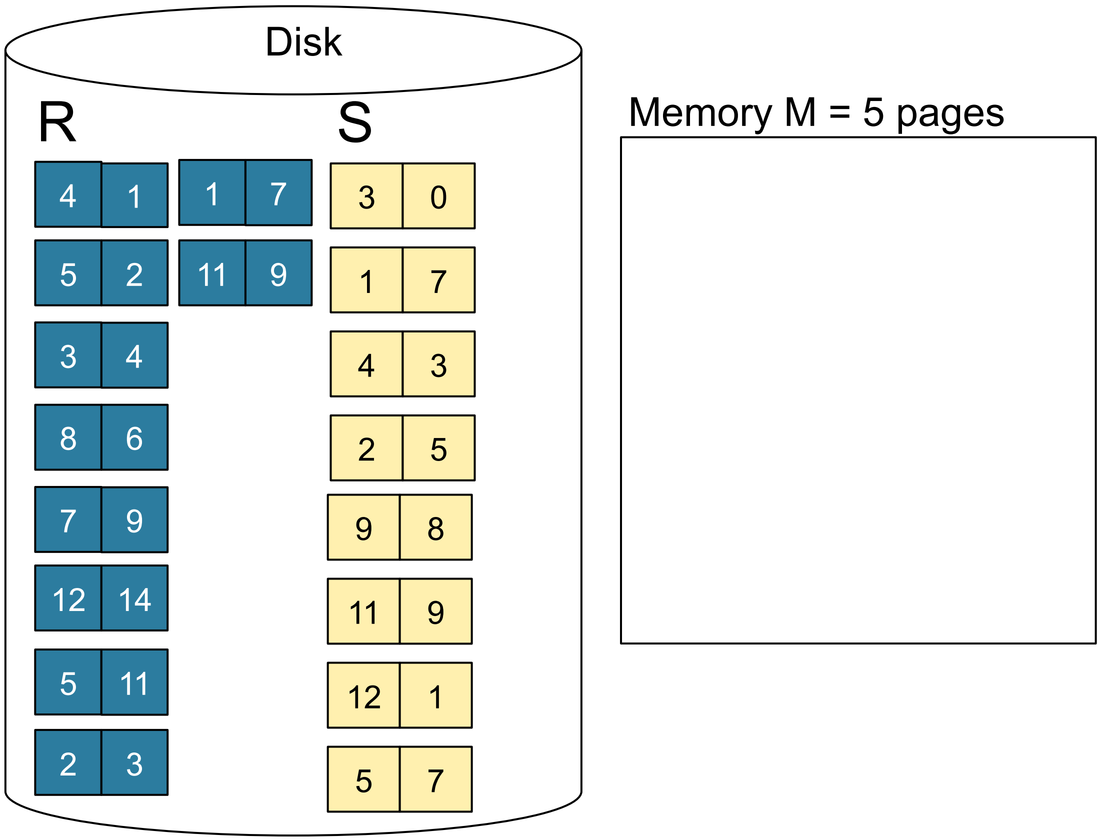
- Step 1a: generate initial runs for R
- Step 1b: generate initial runs for S
- Step 2: merge and join
  - Either merge first and then join
  - Or merge & join at the same time

Merge-Join Example

Setup: Want to join R and S

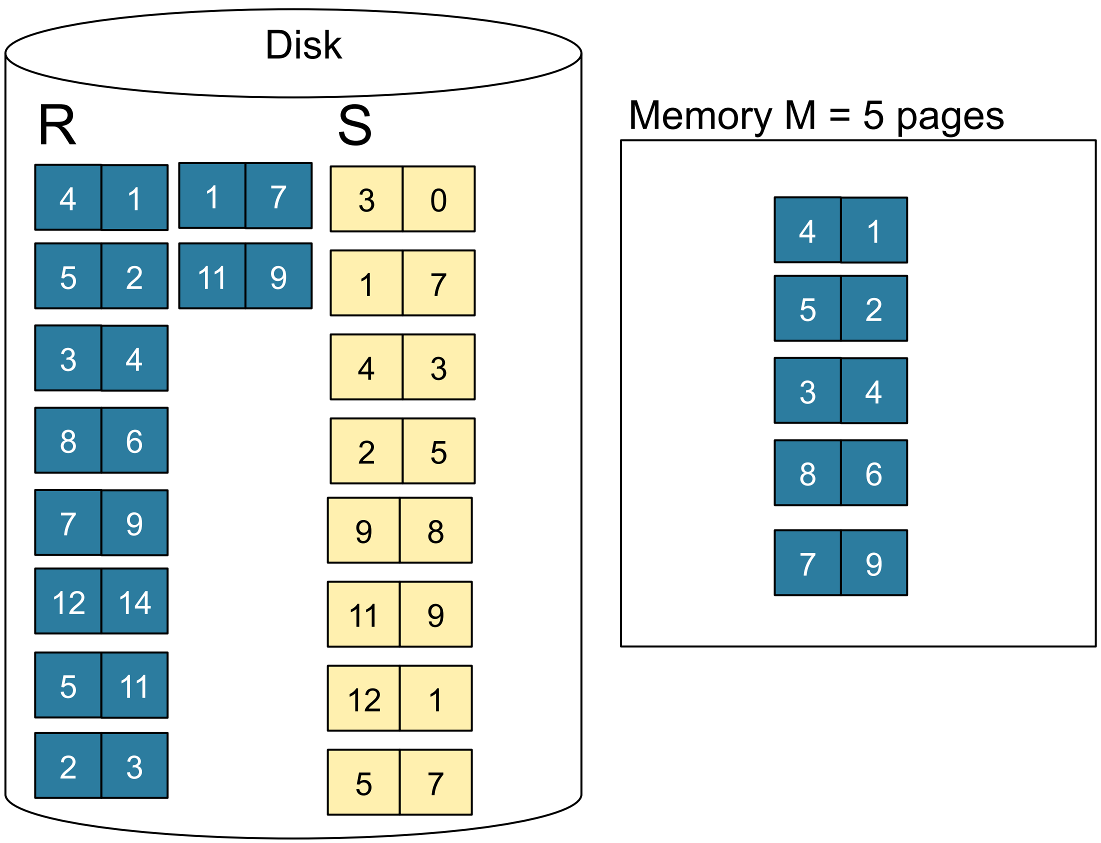
- Relation R has 10 pages with 2 tuples per page
- Relation S has 8 pages with 2 tuples per page

Values shown are values of join attribute for each given tuple



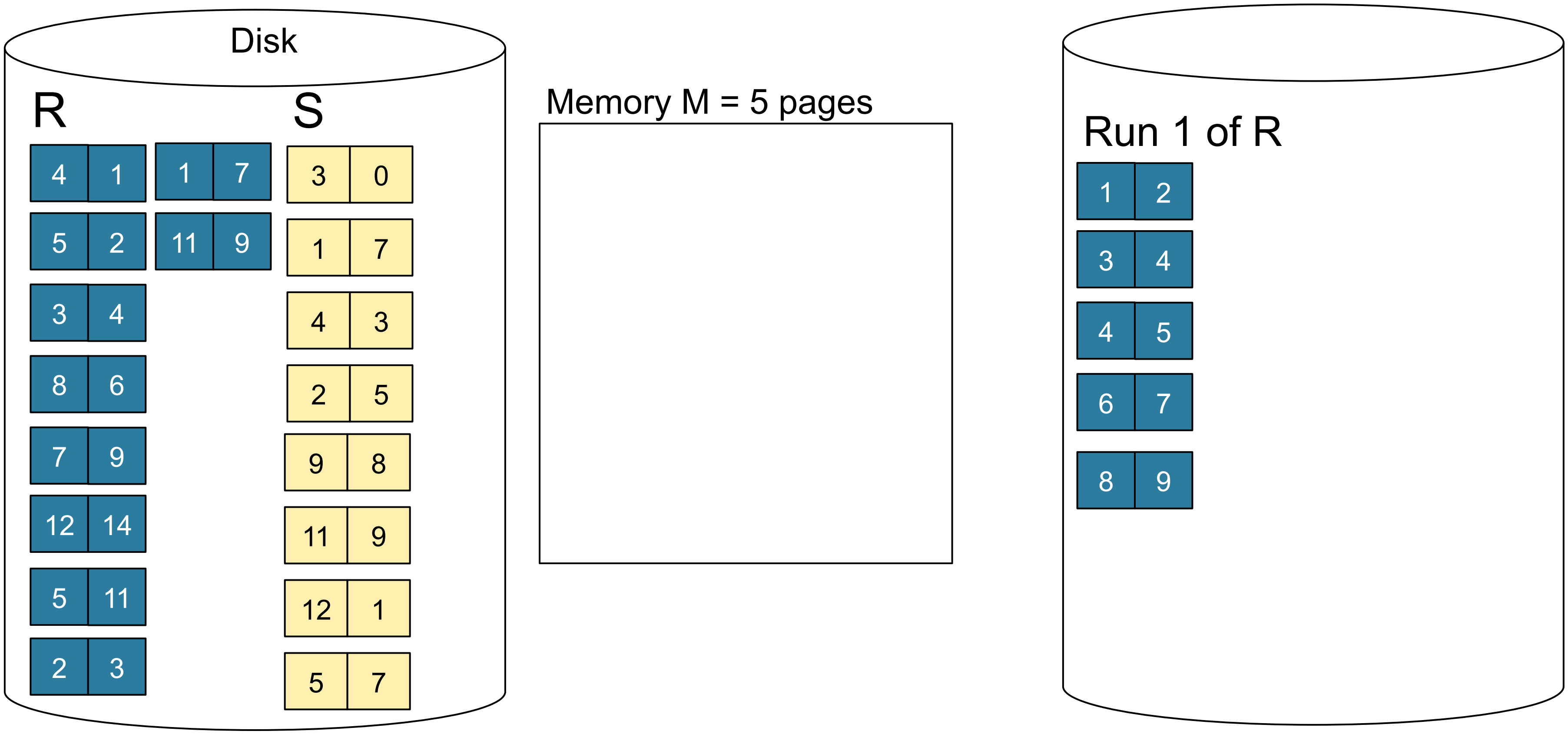
Merge-Join Example

Step 1: Read M pages of R and sort in memory



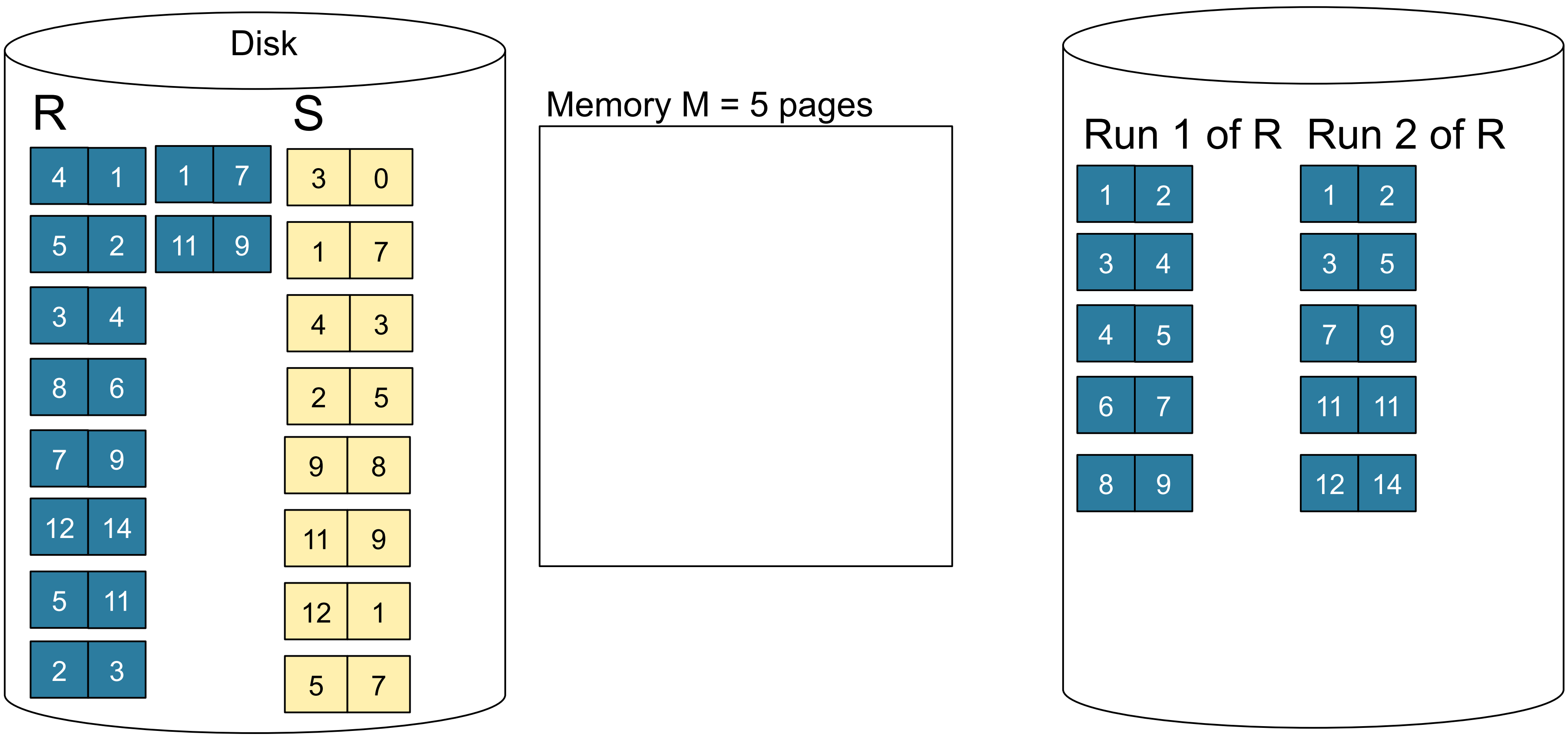
Merge-Join Example

Step 1: Read M pages of R and sort in memory, then write to disk



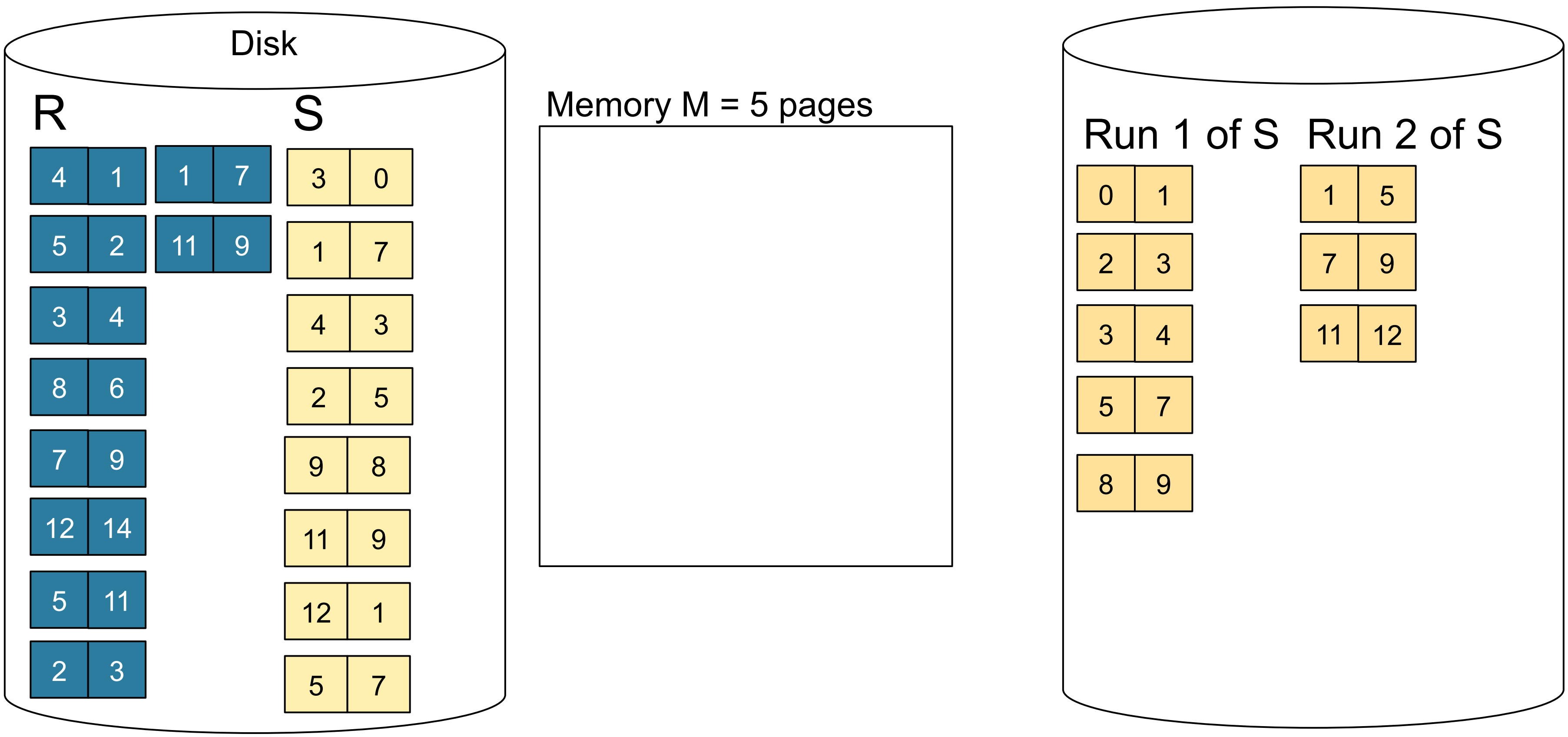
Merge-Join Example

Step 1: Repeat for next M pages until all R is processed



Merge-Join Example

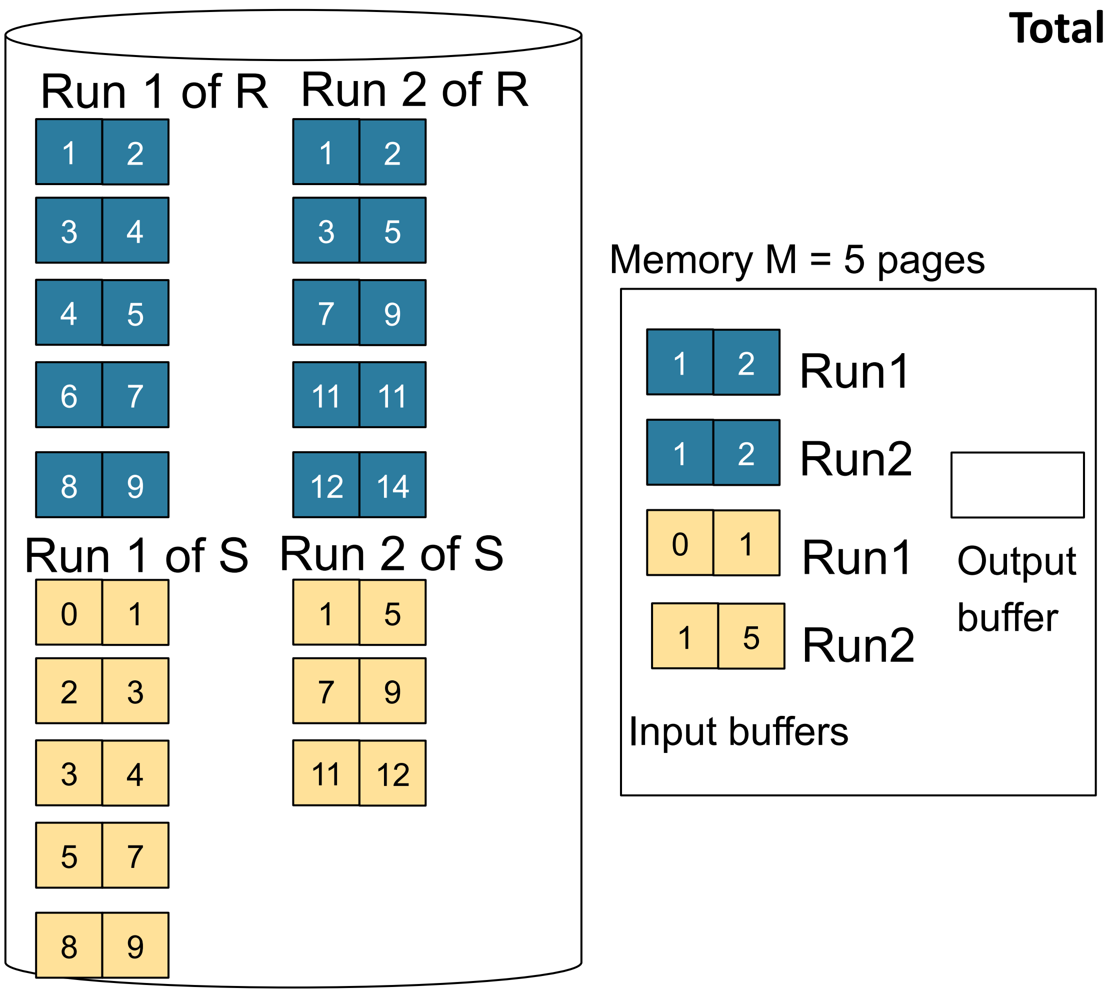
Step 1: Do the same with S



Merge-Join Example

Step 2: Join while merging sorted runs

Total cost: 3B(R) + 3B(S)

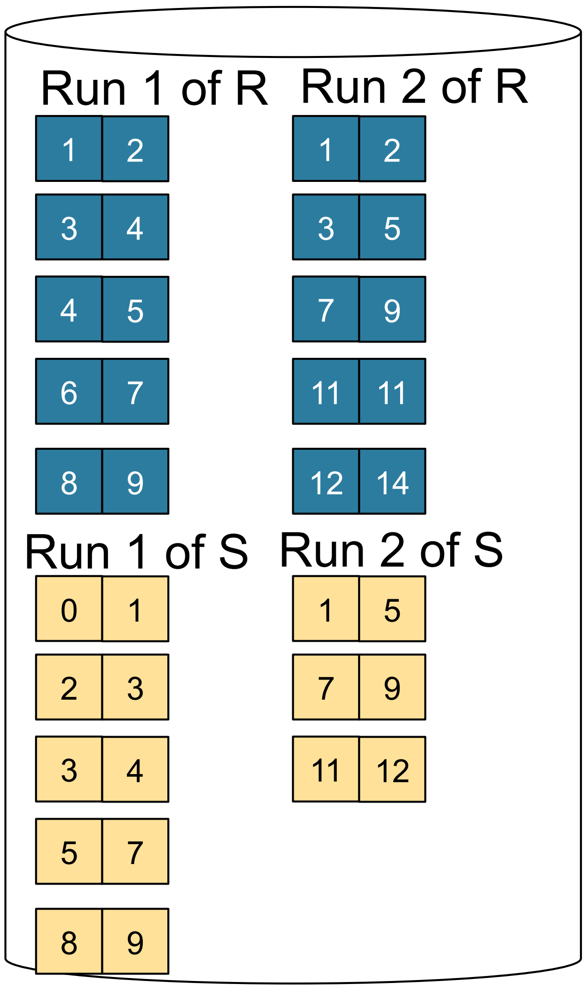


Step 4: Join while merging

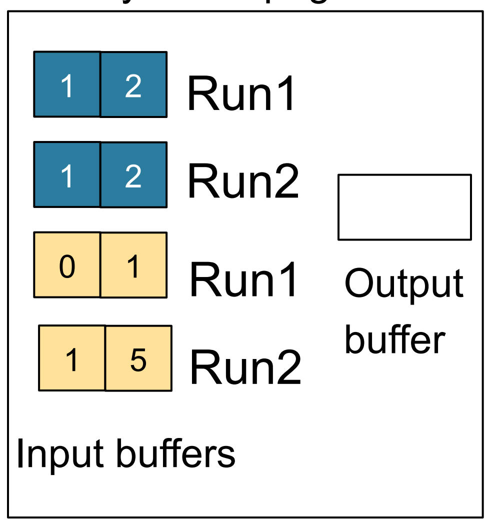
Output tuples

Merge-Join Example

Step 2: Join while merging sorted runs



Memory M = 5 pages



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Output tuples

- (1,1)
- (1,1)
- (1,1)
- (1,1)

Merge-Join Example

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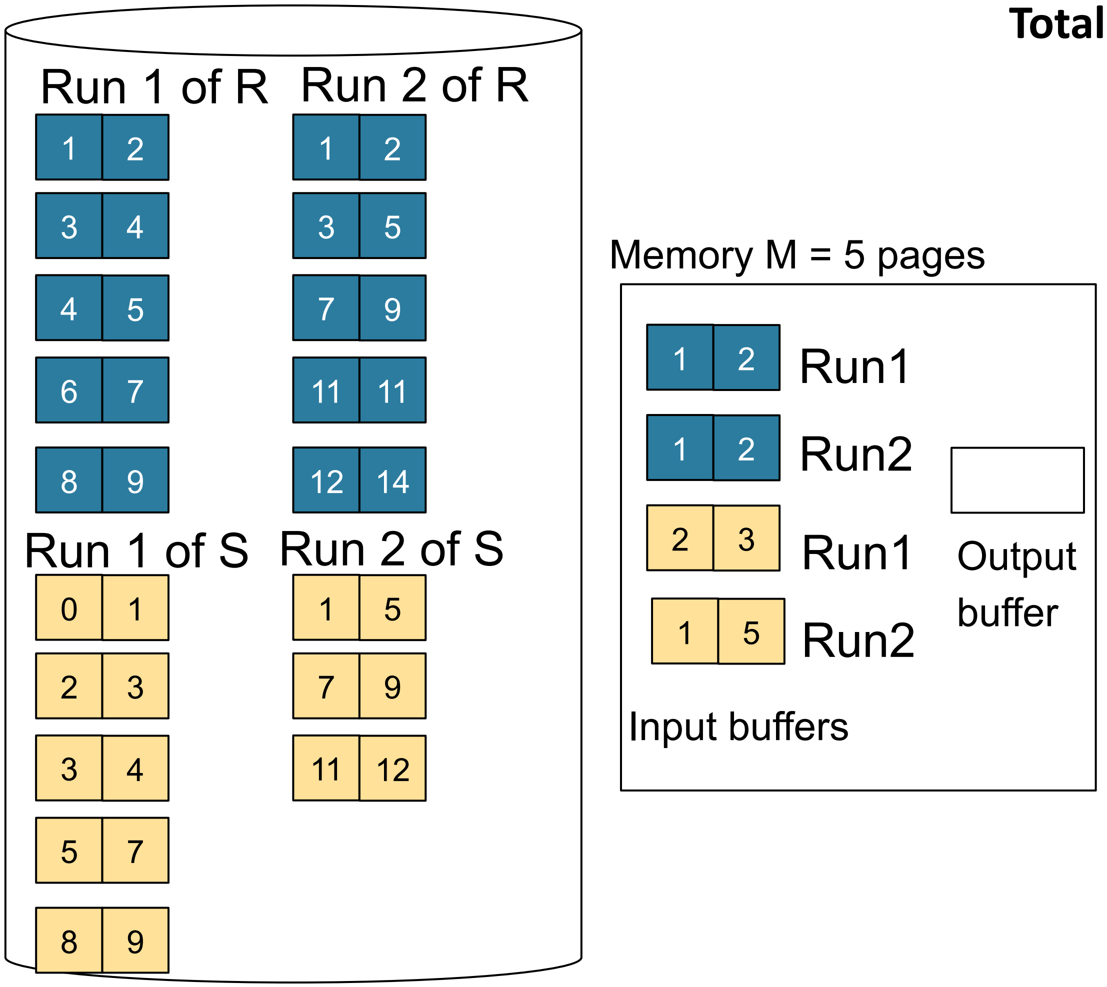

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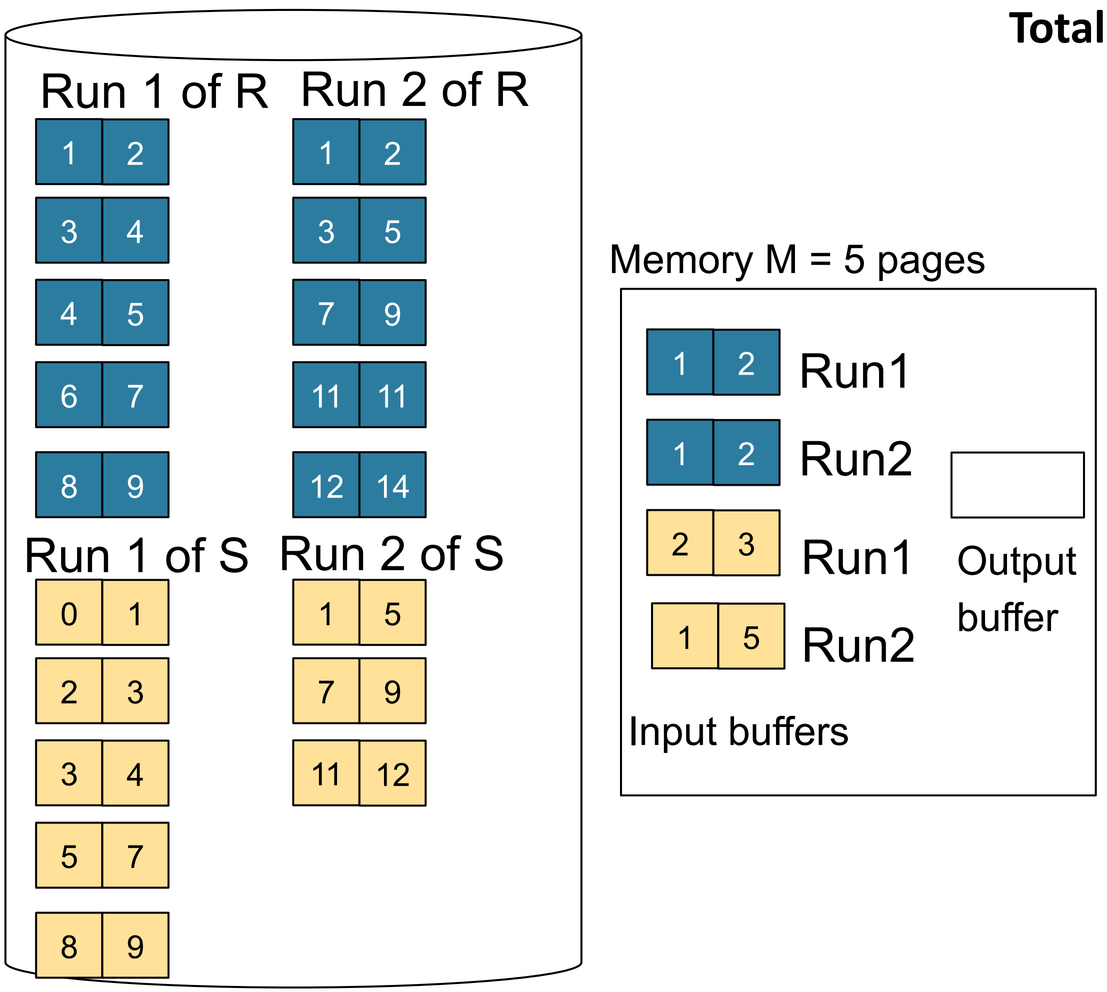

Step 4: Join while merging

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- (1,1)
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- (1,1)
- (2,2)
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Merge-Join Example

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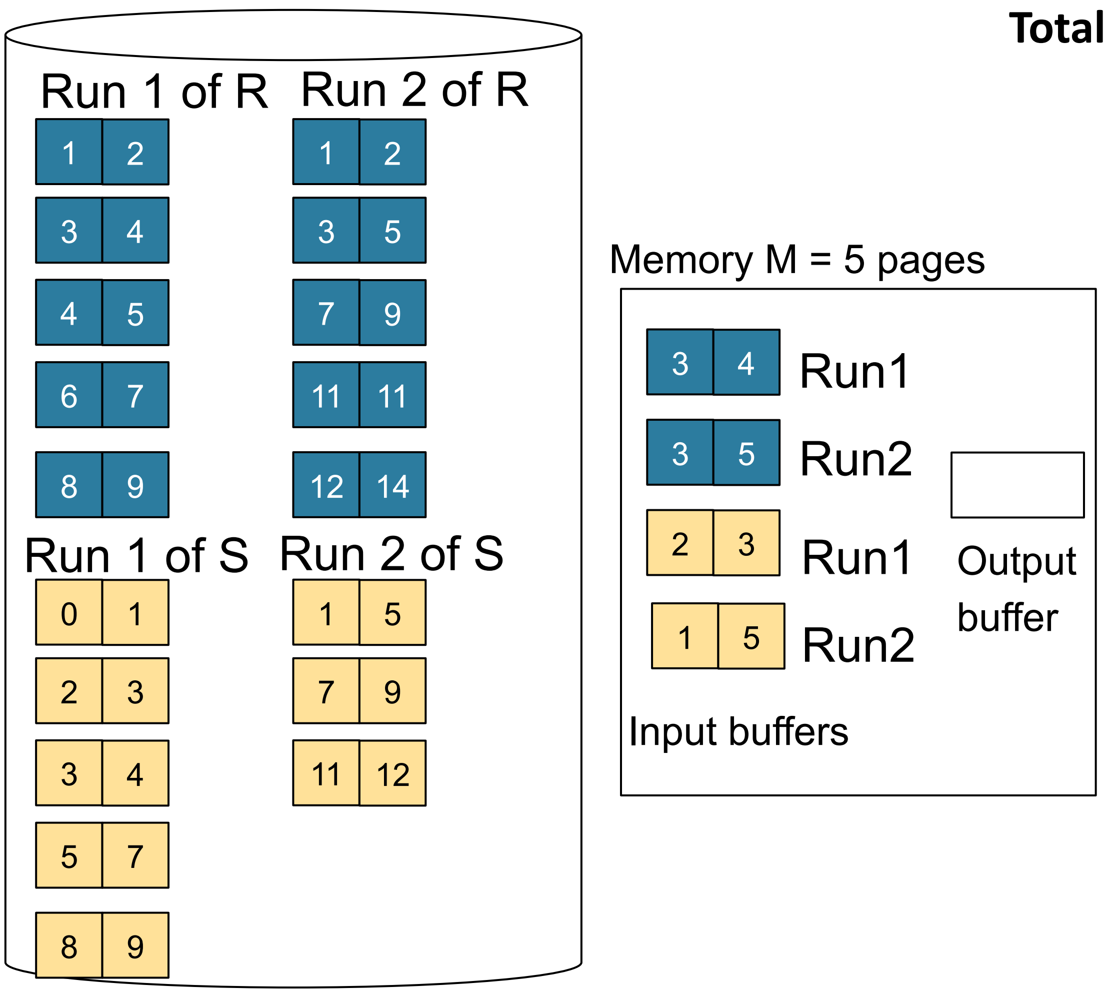

Step 4: Join while merging

Output tuples

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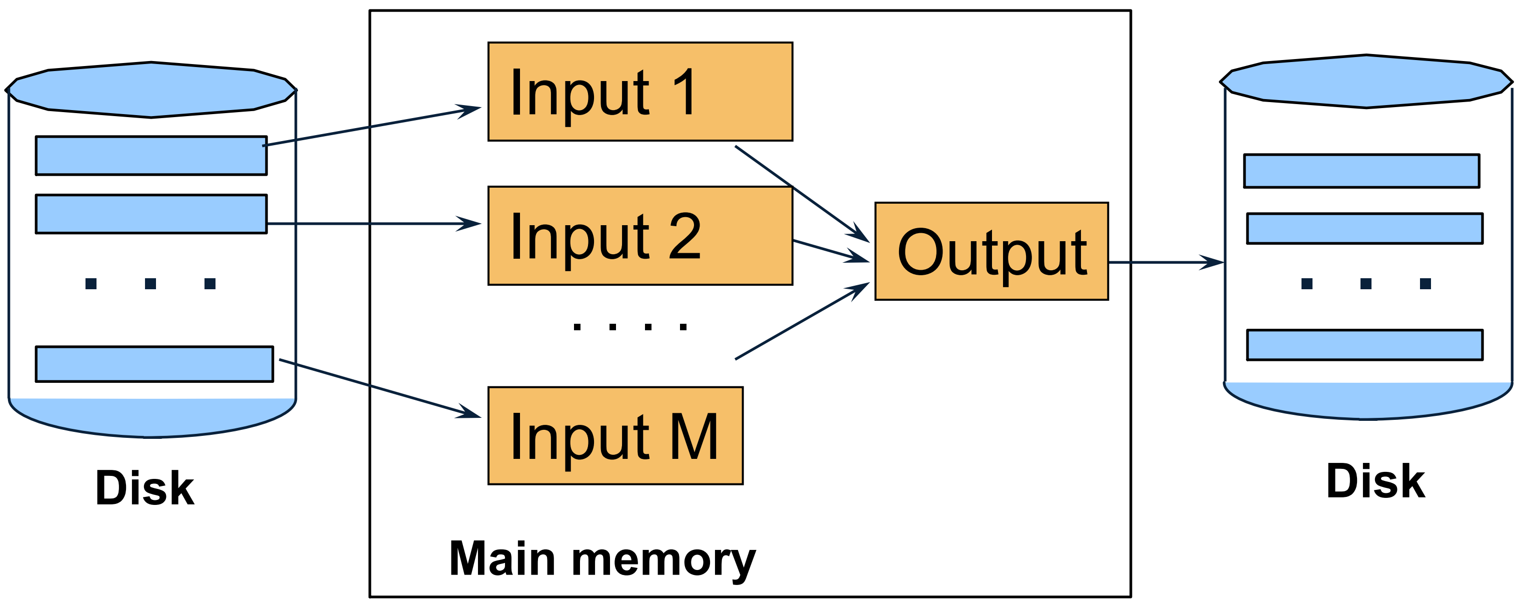
- (2,2)
- (3,3)
- (3,3)
- …

Merge-Join



M_1 = B(R)/M runs for R
M_2 = B(S)/M runs for S
Merge-join M_1 + M_2 runs;
need $M_1 + M_2 <= M$

Partitioned Hash Algorithms

- Partition R it into k buckets:
  $R_1, R_2, R_3, \ldots, R_k$

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  $B(R_i) = B(R)/k$, for all i

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- Goal: each R_i should fit in main memory:
  $B(R_i) \leq M$

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- Goal: each R_i should fit in main memory:
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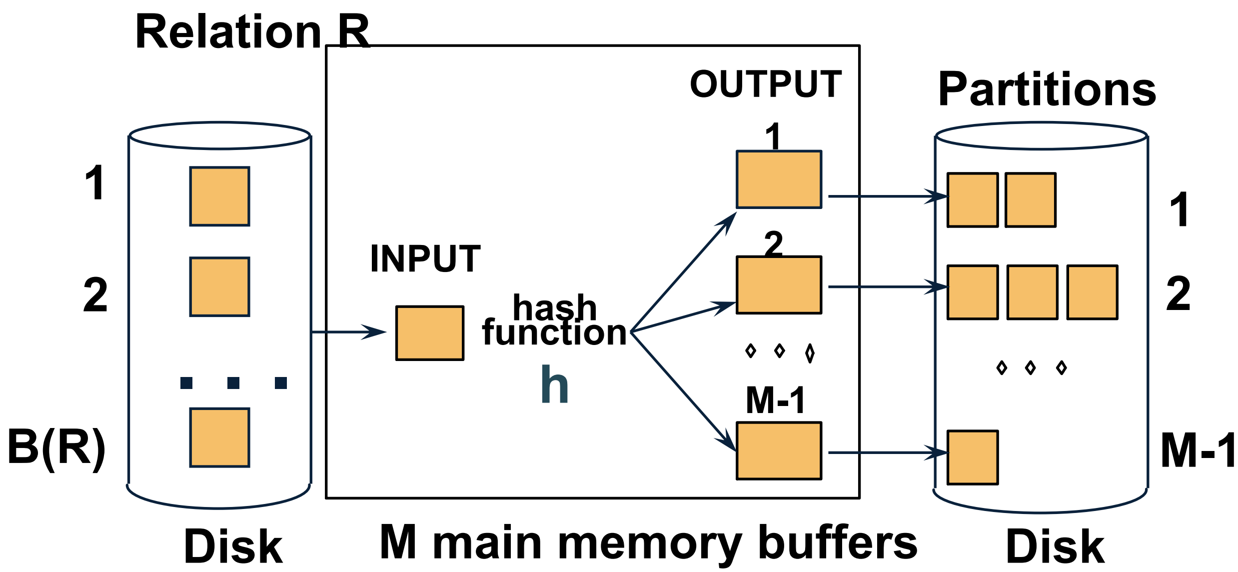
How do we choose k?

Partitioned Hash Algorithms

- We choose k = M-1 Each bucket has size approx. $B(R)/(M-1) \approx B(R)/M$



Assumption: $B(R)/M \leq M$, i.e. $B(R) \leq M^2$

Grace-Join

$R \bowtie S$

Note: grace-join is also called *partitioned hash-join*

Grace-Join

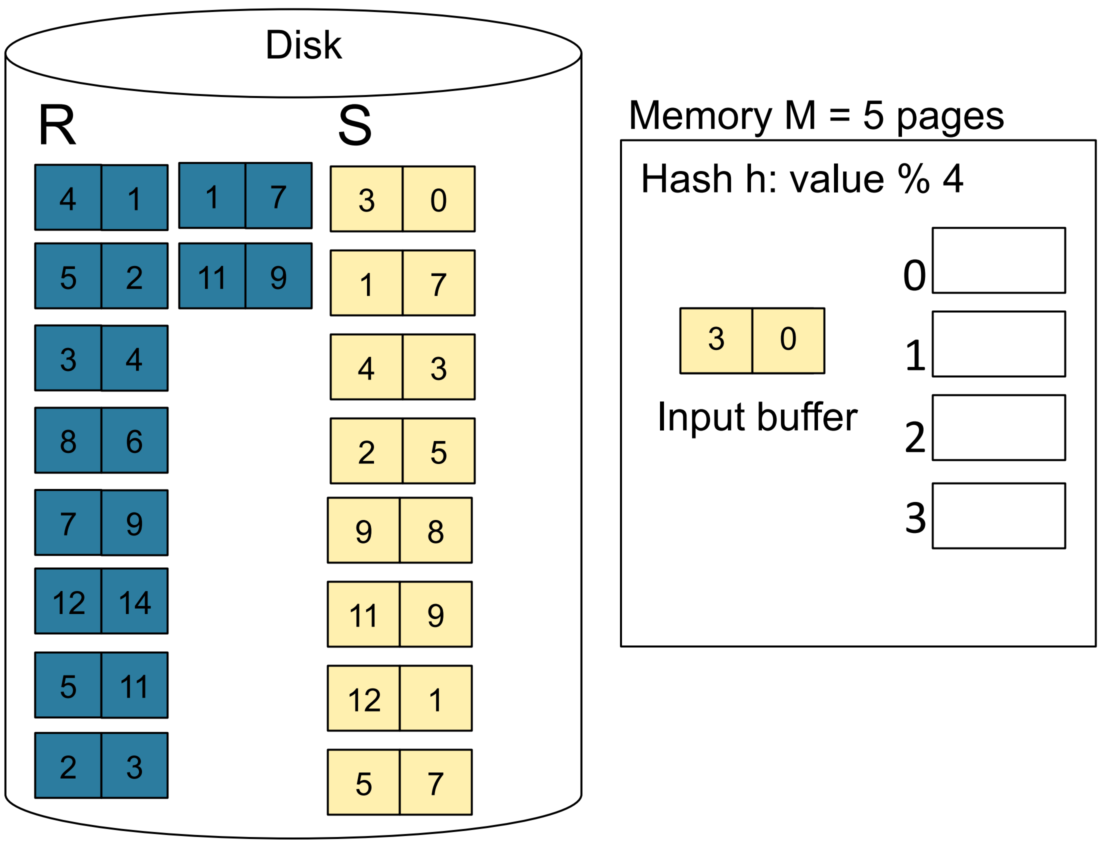
$R \bowtie S$

- Step 1:
  - Hash S into M buckets
  - Send all buckets to disk
- Step 2
  - Hash R into M buckets
  - Send all buckets to disk
- Step 3
  - Join every pair of buckets

Note: grace-join is also called *partitioned hash-join*

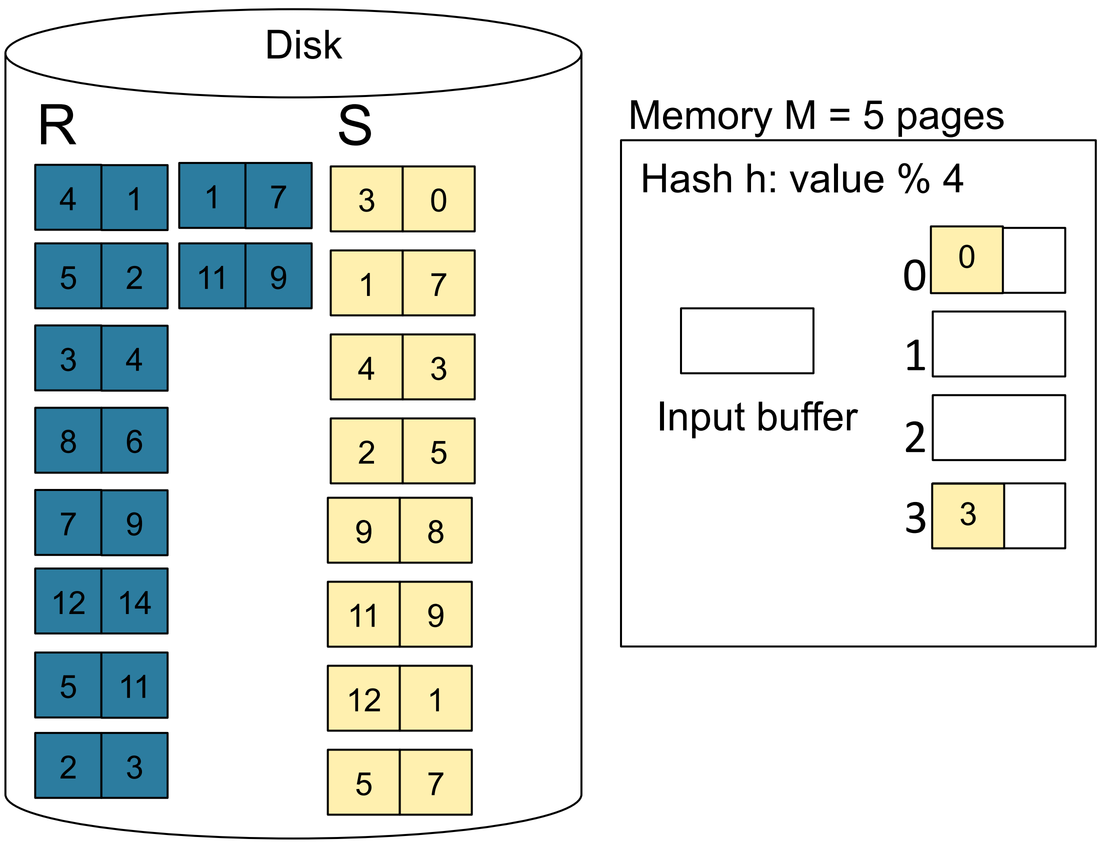
Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into M-1 (=4 buckets)



Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets



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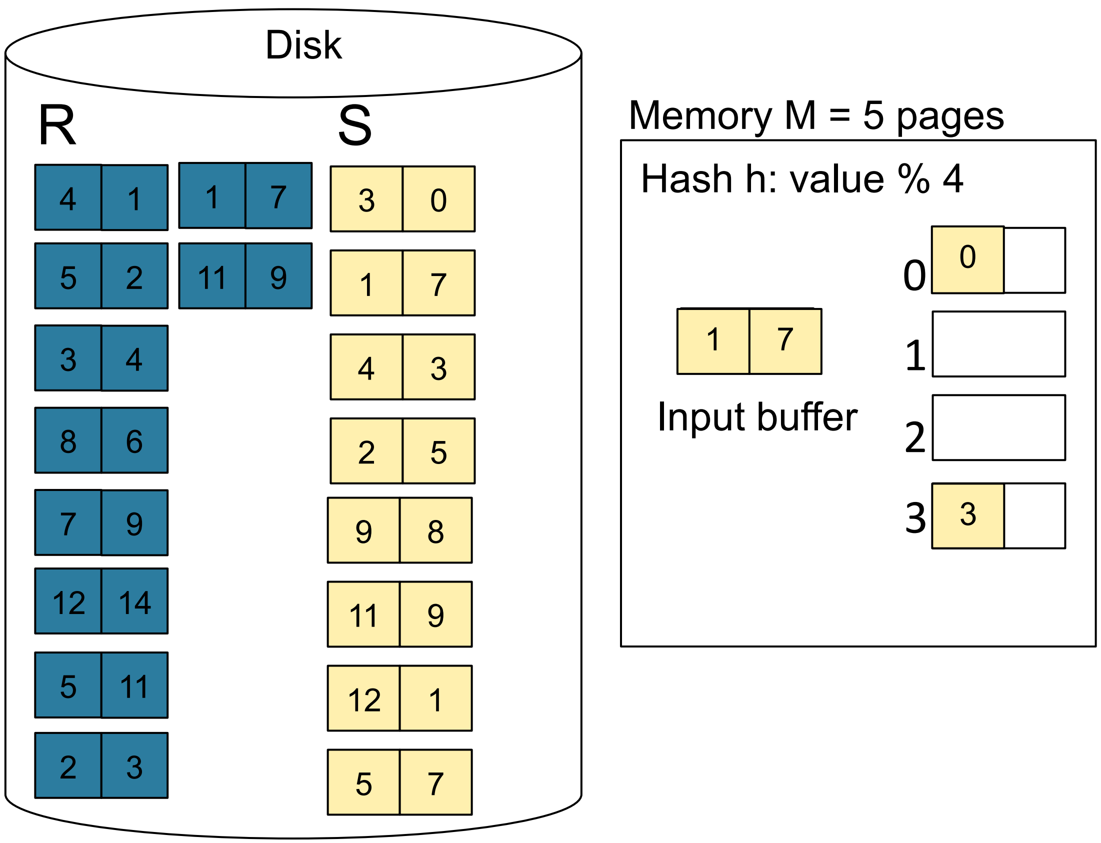

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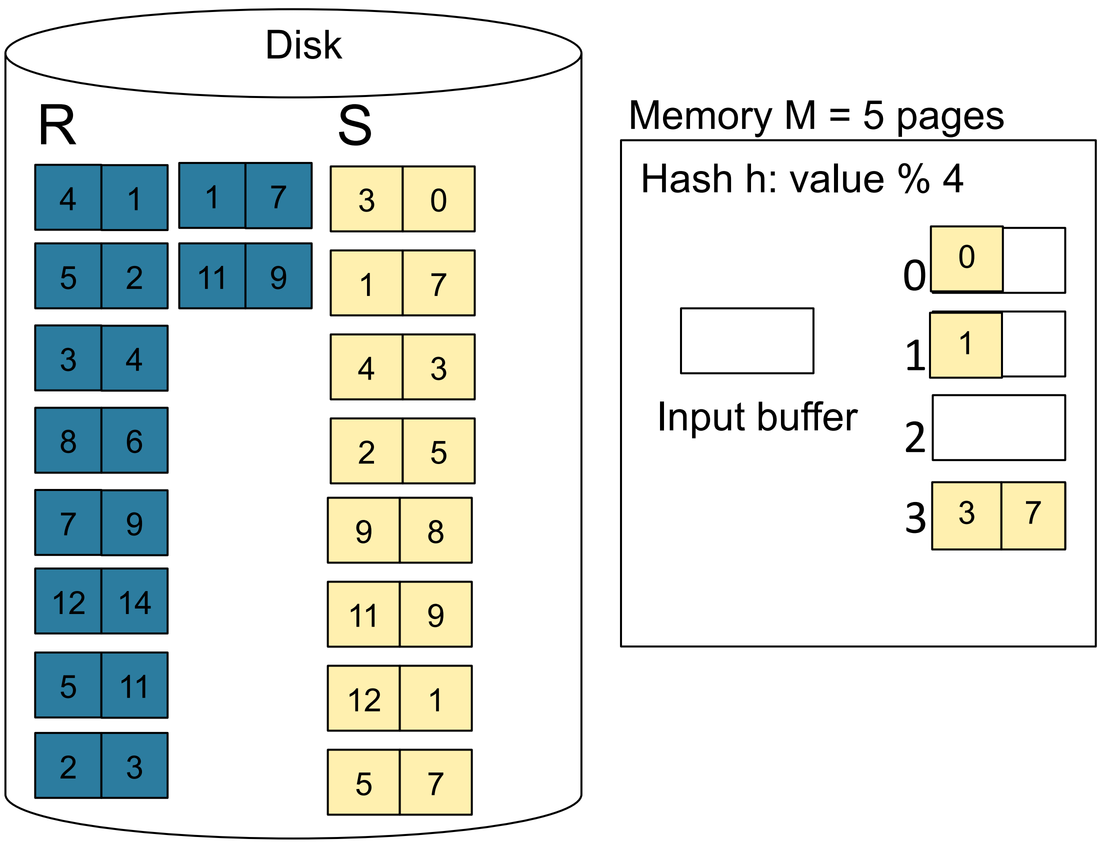

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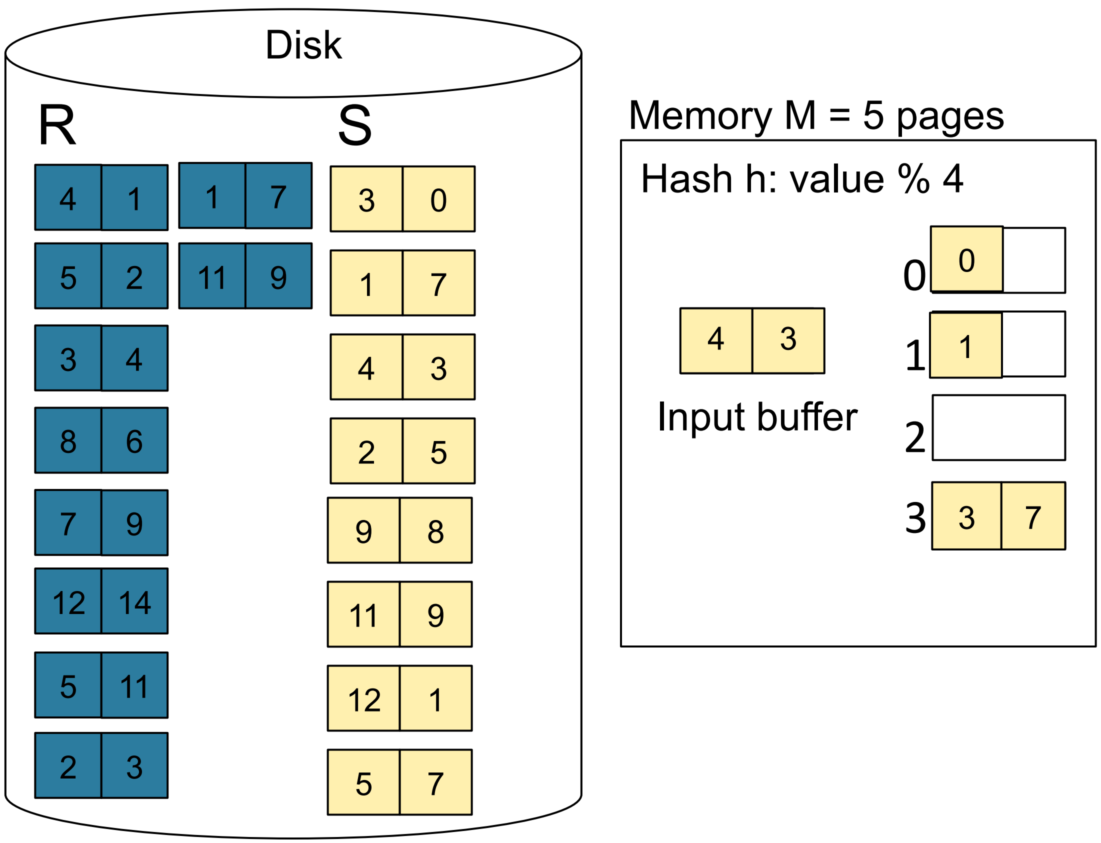
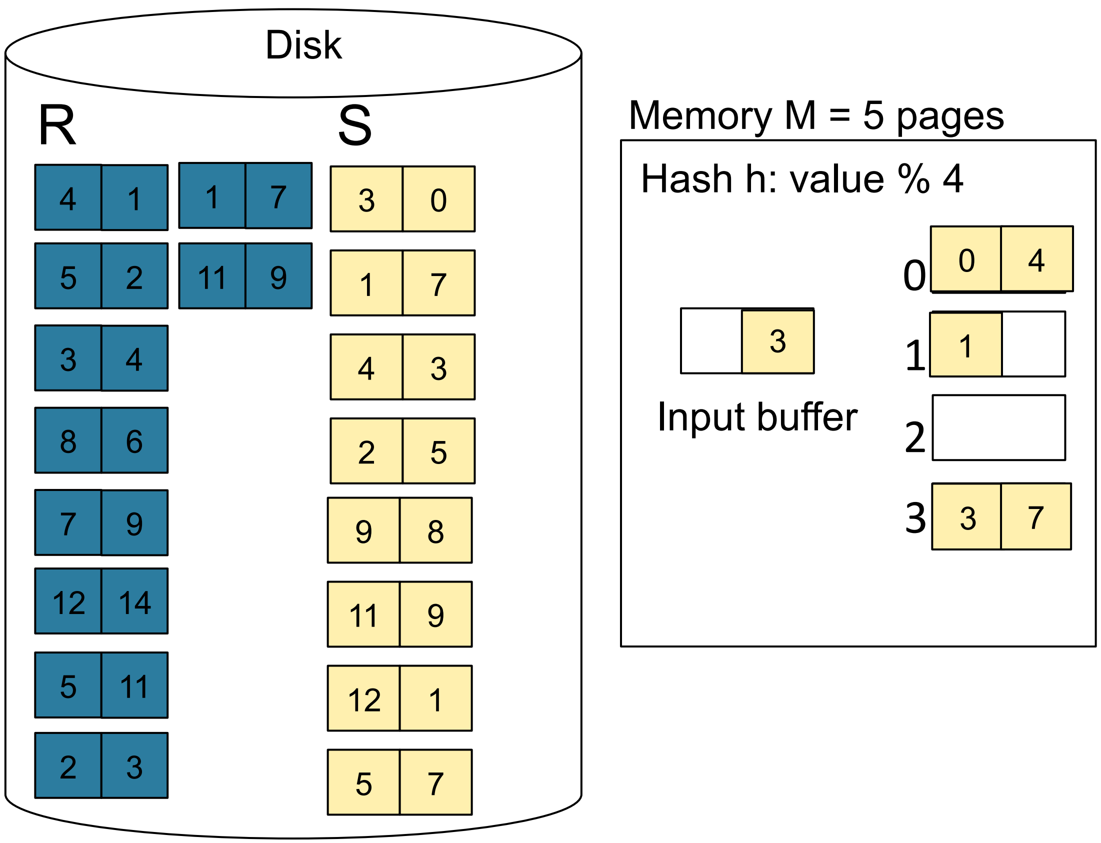


Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets

When a bucket fills up, flush it to disk



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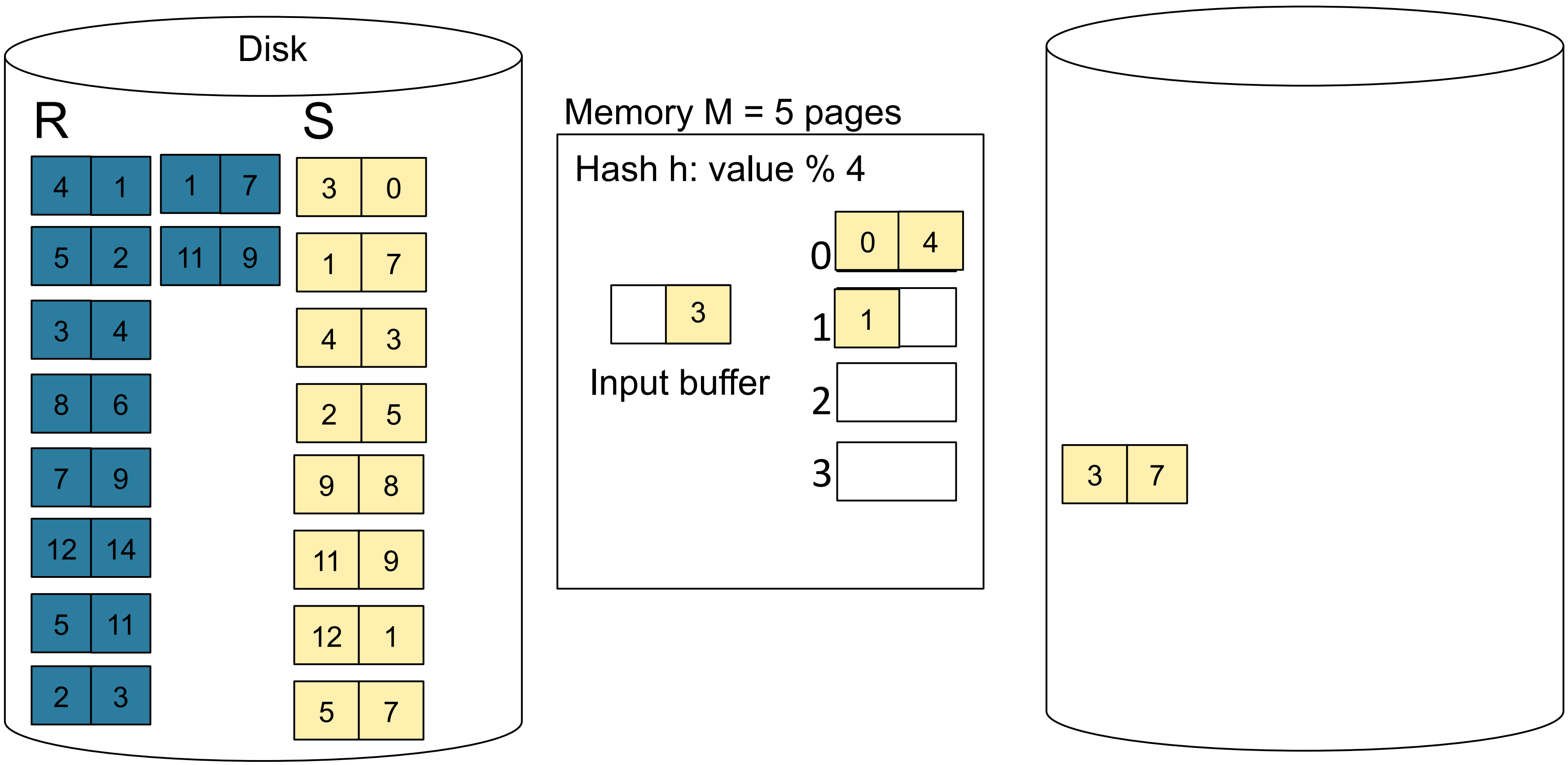

Partitioned Hash-Join Example

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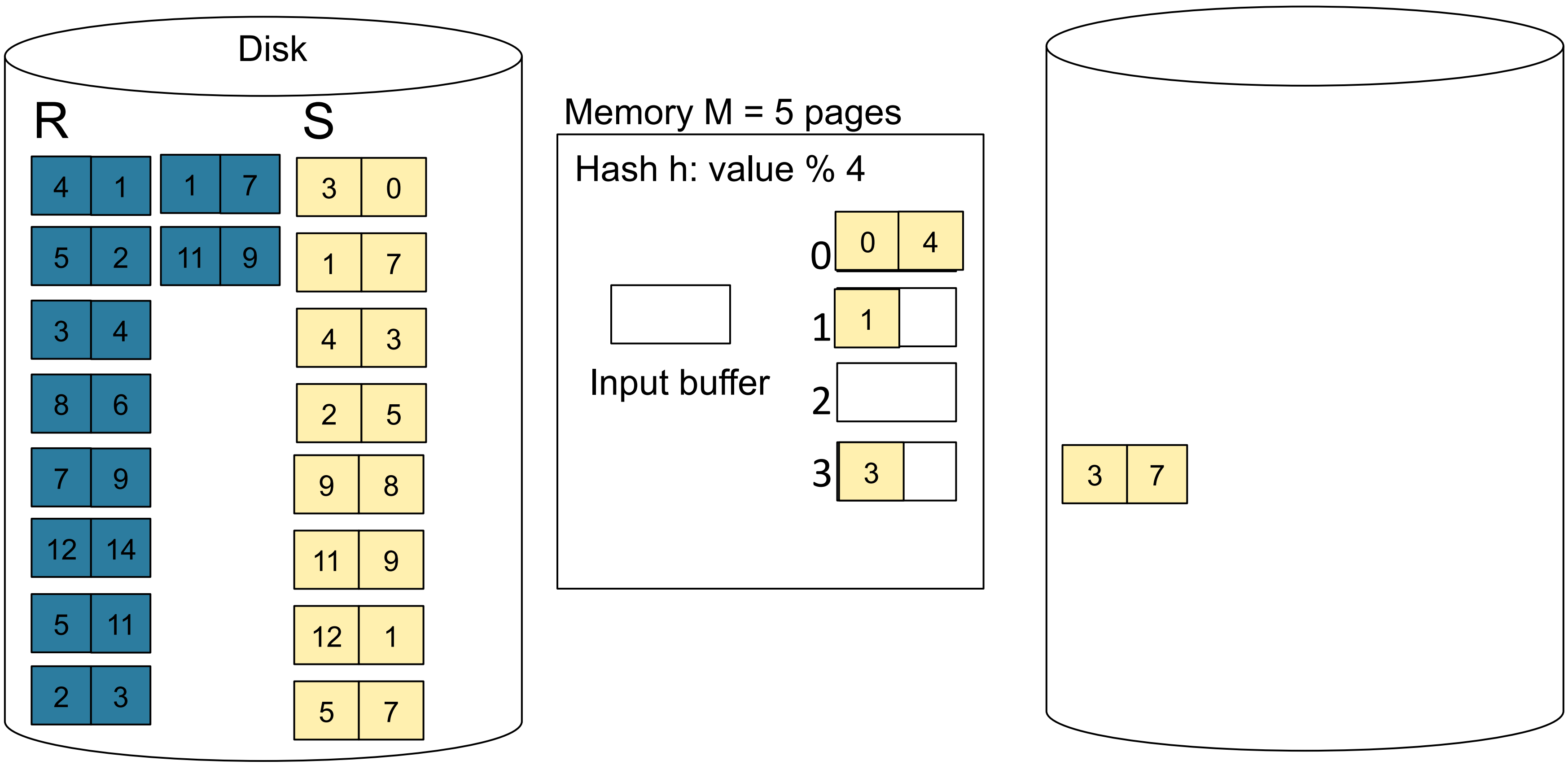
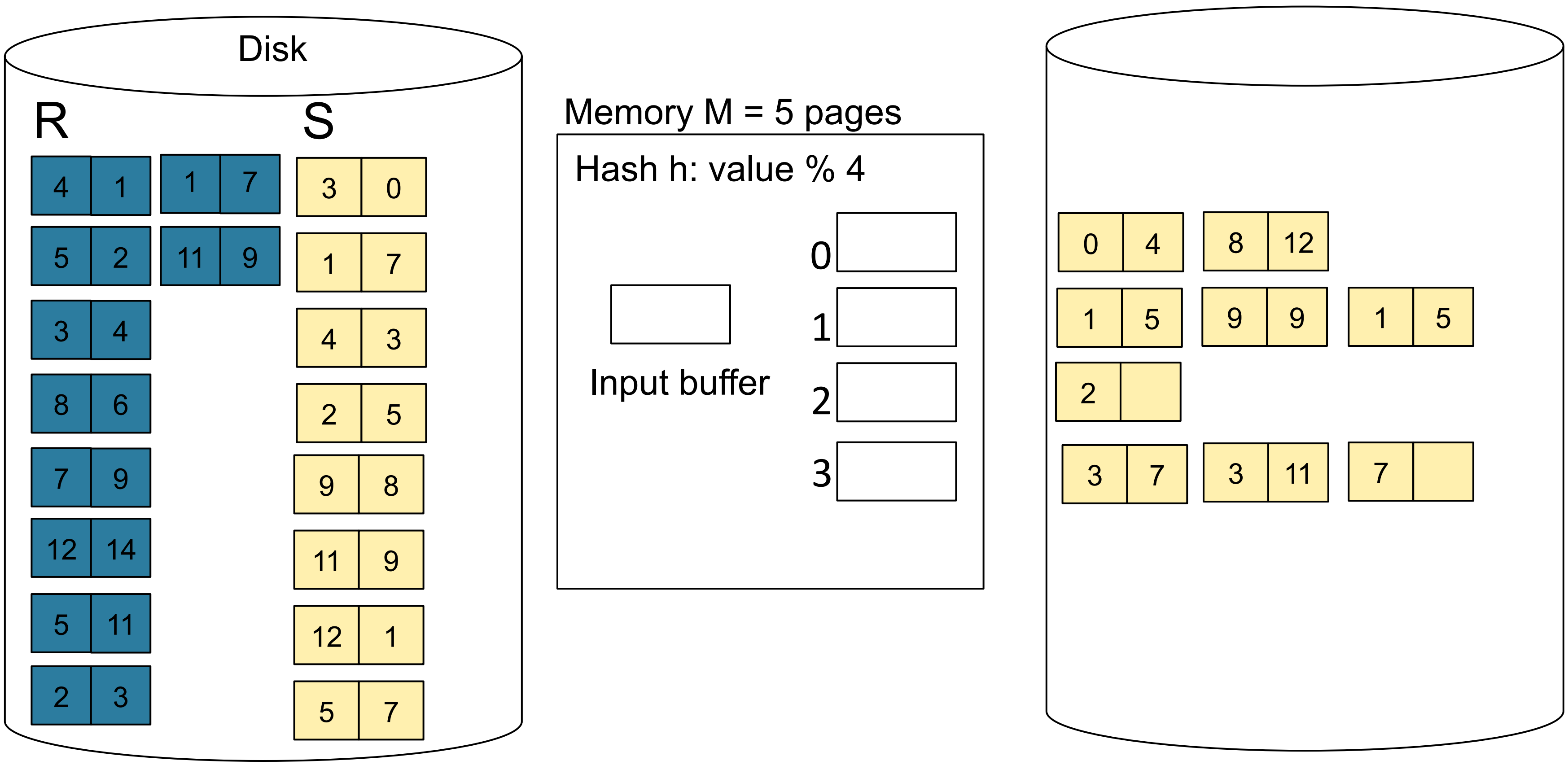


Partitioned Hash-Join Example

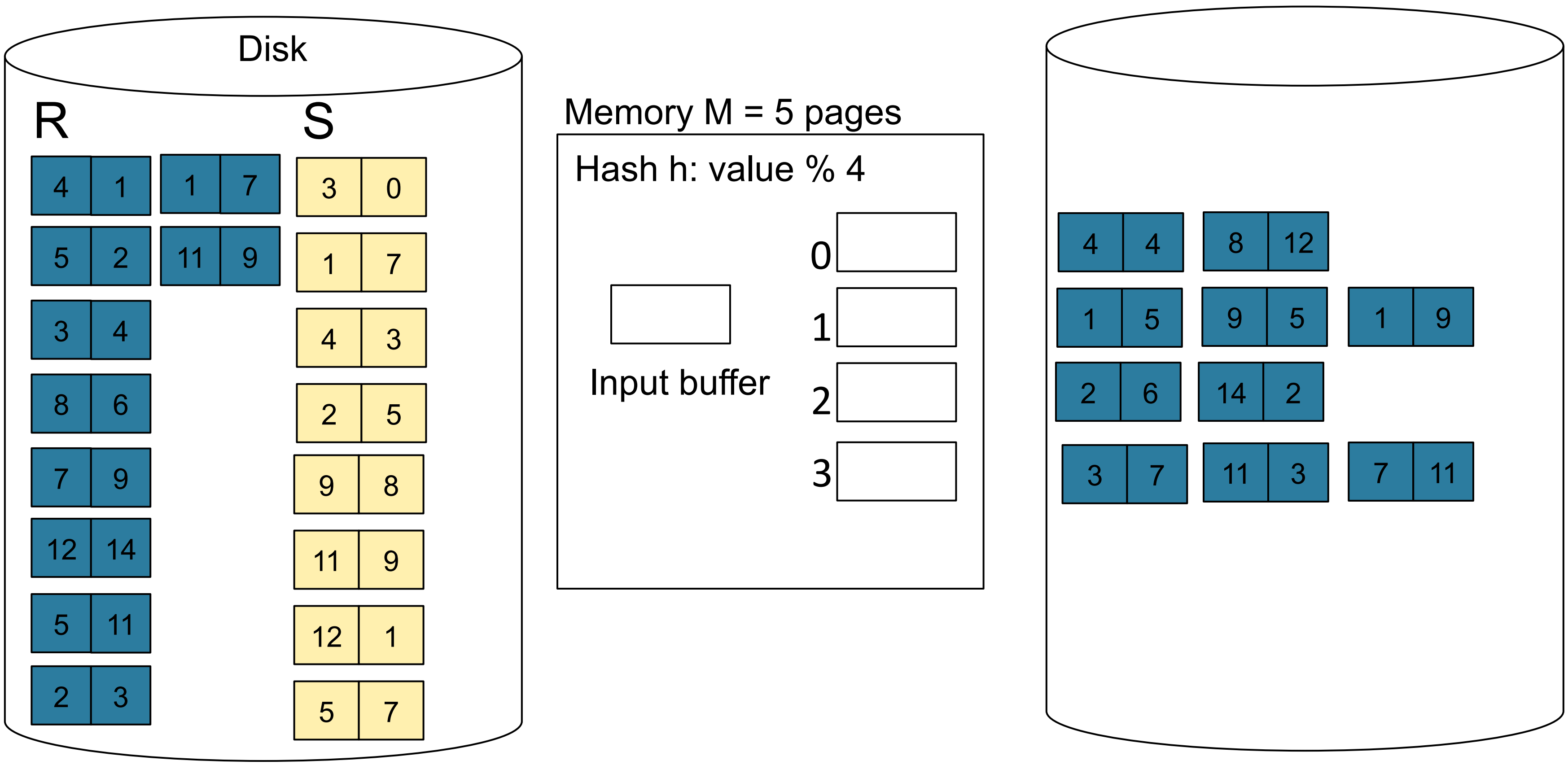
Step 1: Read relation S one page at a time and hash into the 4 buckets

At the end, we get relation S back on disk split into 4 buckets



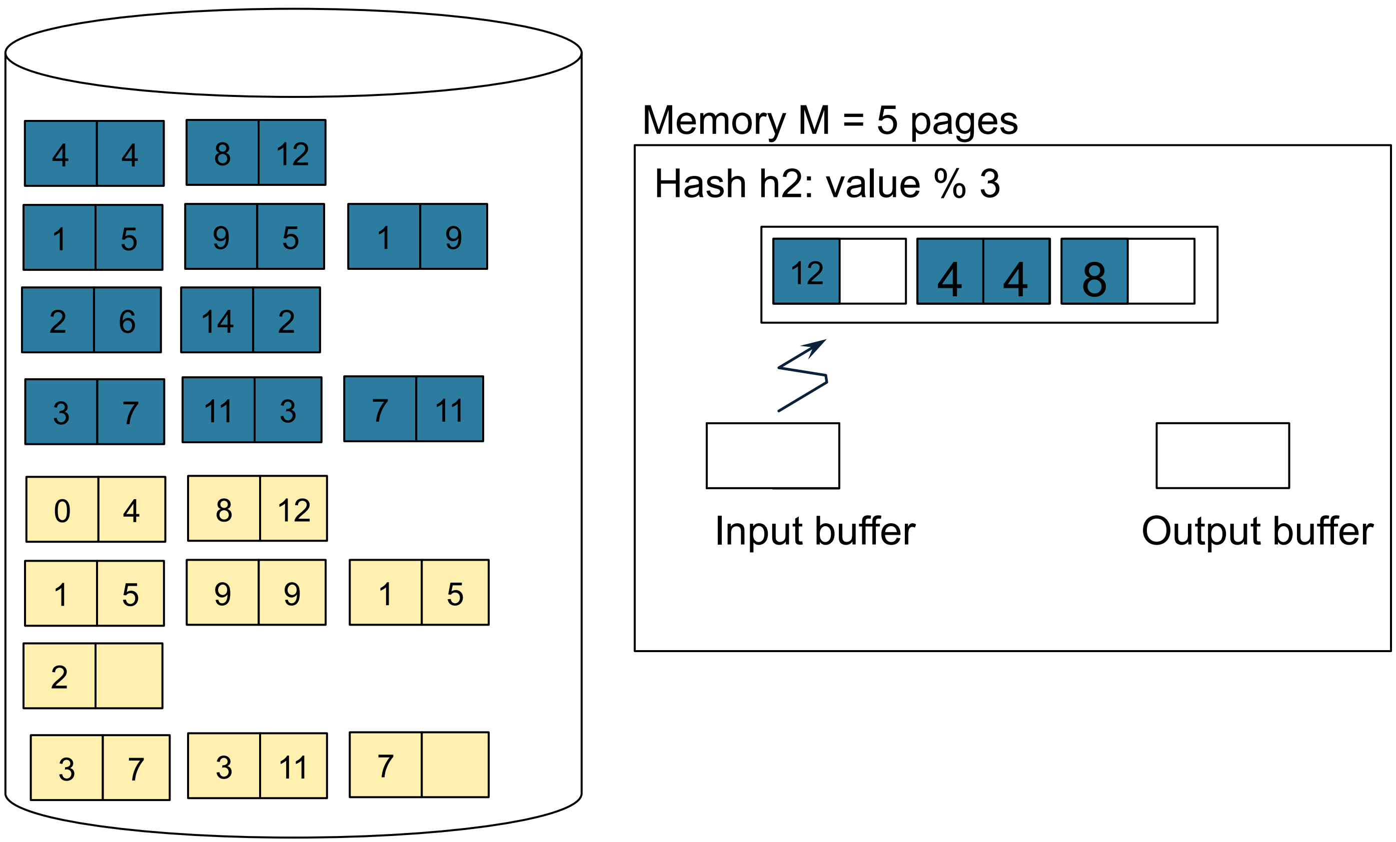
Partitioned Hash-Join Example

Step 2: Read relation R one page at a time and hash into same 4 buckets



Partitioned Hash-Join Example

Step 3: Read one partition of R and create hash table in memory using a different hash functior

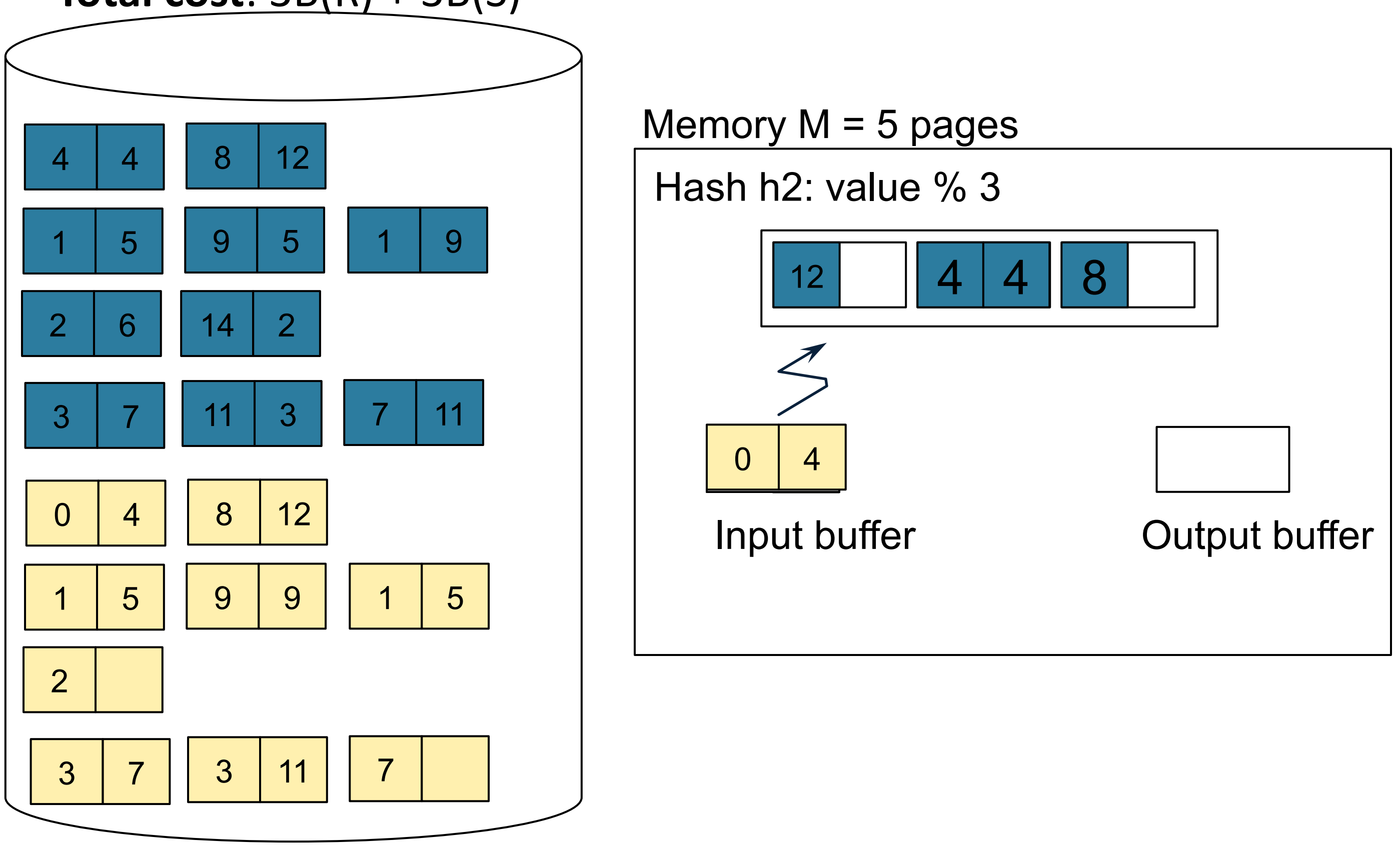


Partitioned Hash-Join Example

Step 4: Scan matching partition of S and probe the hash table

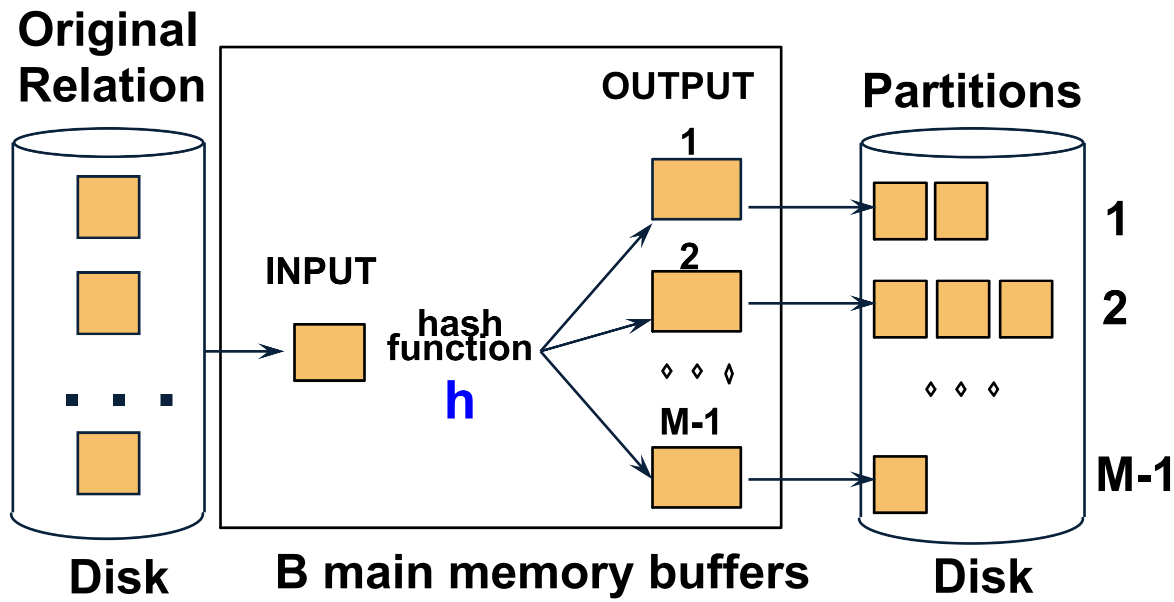
Step 5: Repeat for all the buckets

Total cost: 3B(R) + 3B(S)



Grace-Join

- Partition both relations using hash fn **h**: R tuples in partition i will only match S tuples in partition i.



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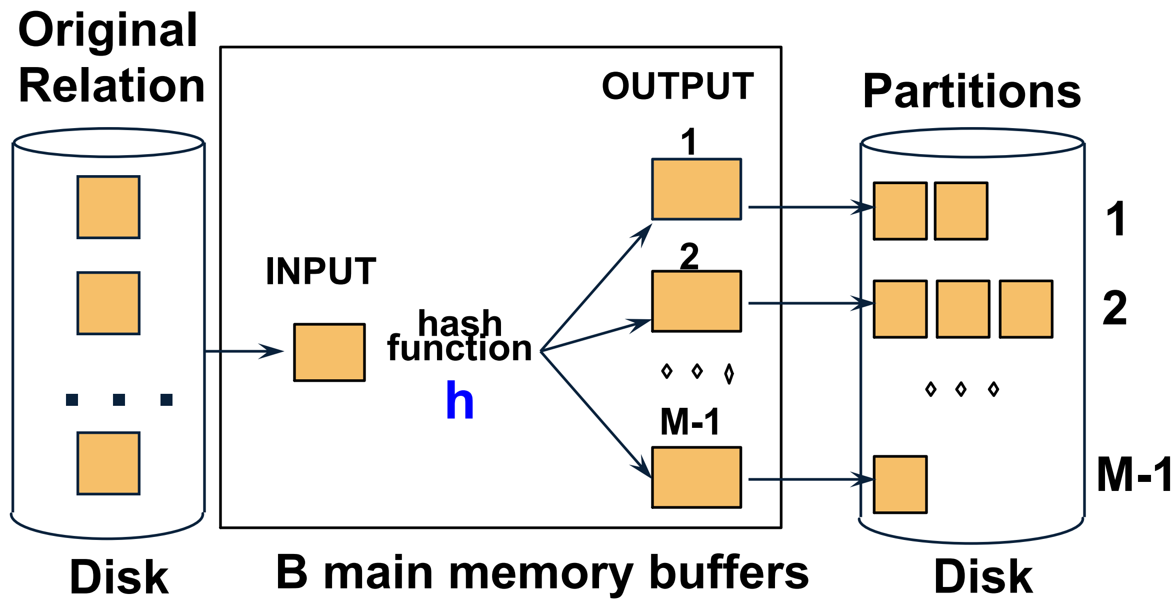
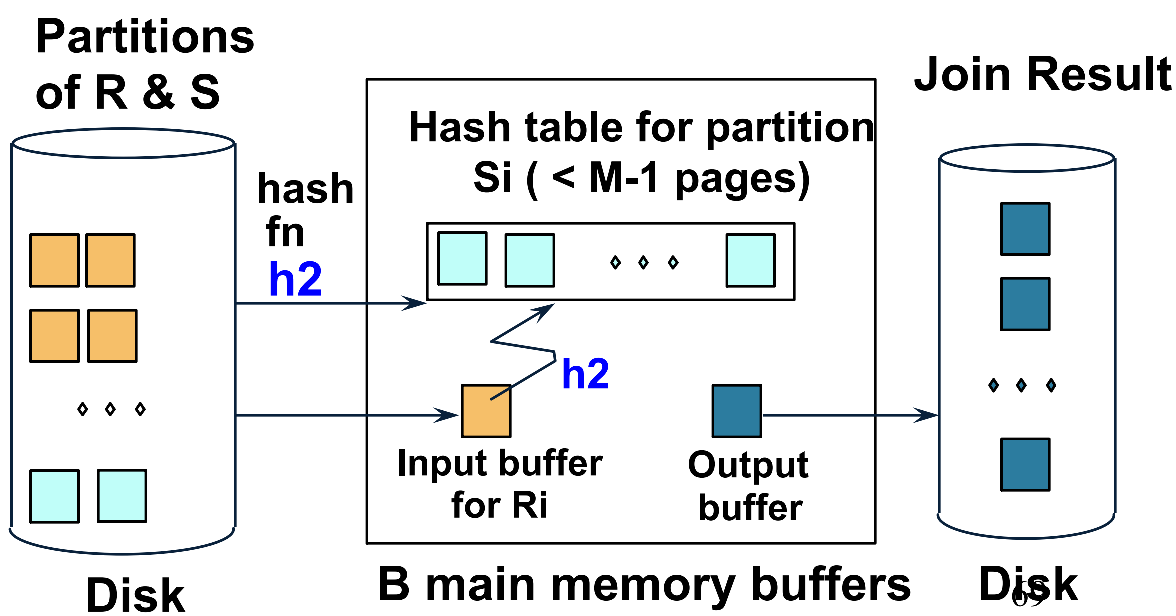


- Read in a partition of R, hash it using **h2 (<> h!)**. Scan matching partition of S, search for matches.



Grace Join

- Cost: 3B(R) + 3B(S)
- Assumption: min(B(R), B(S)) <= M^2

Hybrid Hash Join Algorithm

- Partition S into k buckets

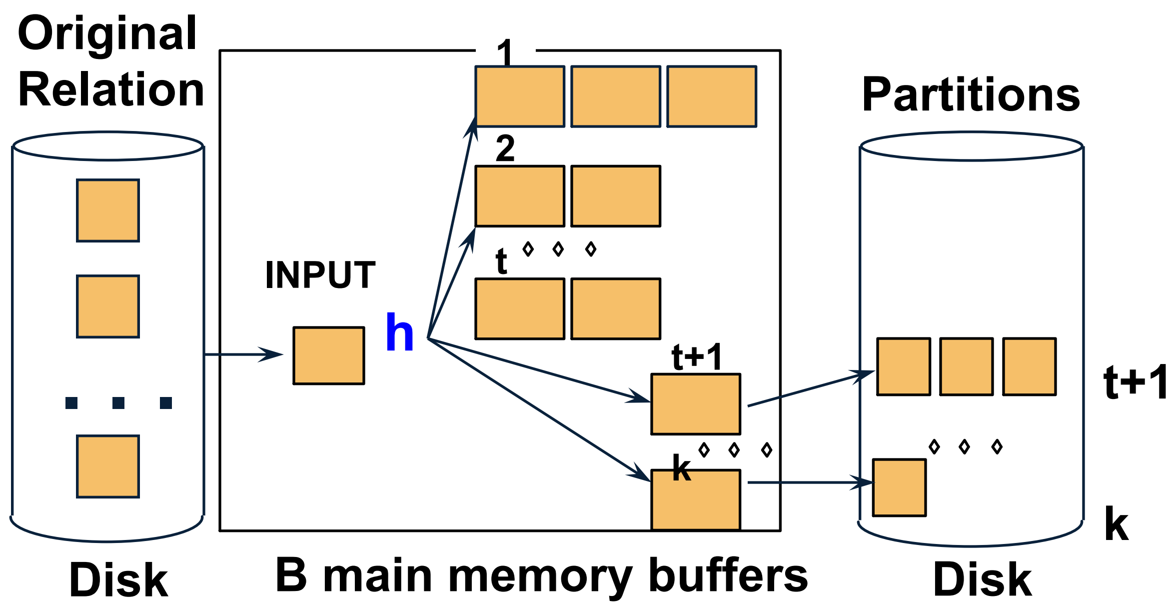
  t buckets S_1 , …, S_t stay in memory

  k-t buckets S_{t+1}, …, S_k to disk

- Partition R into k buckets
  - First t buckets join immediately with S
  - Rest k-t buckets go to disk
- Finally, join k-t pairs of buckets:

  (R_{t+1},S_{t+1}), (R_{t+2},S_{t+2}), …, (R_k,S_k)

Hybrid Hash Join Algorithm



Hybrid Join Algorithm

- How to choose k and t?

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  - Choose k large but s.t. k <= M

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One block/bucket in memory

Hybrid Join Algorithm

- How to choose k and t?

  One block/bucket in memory

  – Choose k large but s.t. k <= M

  – Choose t/k large but s.t. t/k * B(S) <= M

Hybrid Join Algorithm

- How to choose k and t?
  - Choose k large but s.t. $k <= M$ — One block/bucket in memory
  - Choose t/k large but s.t. $t/k * B(S) <= M$ — First t buckets in memory

Hybrid Join Algorithm

- How to choose k and t?
  - Choose k large but s.t. One block/bucket in memory $k <= M$
  - Choose t/k large but s.t. First t buckets in memory $t/k * B(S) <= M$
  - Together: $t/k * B(S) + k-t <= M$

Hybrid Join Algorithm

- How to choose k and t?
  - Choose k large but s.t. $k <= M$ (One block/bucket in memory)
  - Choose t/k large but s.t. $t/k * B(S) <= M$ (First t buckets in memory)
  - Together: $t/k * B(S) + k-t <= M$
- Assuming $t/k * B(S) >> k-t$: $t/k = M/B(S)$

Hybrid Join Algorithm

- How to choose k and t?

  One block/bucket in memory

  – Choose k large but s.t. $k <= M$

  First t buckets in memory

  – Choose t/k large but s.t. $t/k * B(S) <= M$

  – Together: $t/k * B(S) + k-t <= M$

- Assuming $t/k * B(S) >> k-t$: $t/k = M/B(S)$

Total size of first t buckets

Hybrid Join Algorithm

- How to choose k and t?

One block/bucket in memory

  – Choose k large but s.t. $k <= M$

First t buckets in memory

  – Choose t/k large but s.t. $t/k * B(S) <= M$

  – Together: $t/k * B(S) + k\text{-}t <= M$

- Assuming $t/k * B(S) >> k\text{-}t$: $t/k = M/B(S)$

Total size of first t buckets

Number of remaining buckets

Hybrid Join Algorithm

Even better: adjust t dynamically

- Start with t = k: all buckets are in main memory
- Read blocks from S, insert tuples into buckets
- When out of memory:
  - Send one bucket to disk
  - t := t-1
- Worst case:
  - All buckets are sent to disk (t=0)
  - Hybrid join becomes grace join

Hybrid Join Algorithm

Cost of Hybrid Join:

- Grace join: $3B(R) + 3B(S)$
- Hybrid join:
  - Saves 2 I/Os for t/k fraction of buckets
  - Saves $2t/k(B(R) + B(S))$ I/Os
  - Cost:
    $(3-2t/k)(B(R) + B(S)) = (3-2M/B(S))(B(R) + B(S))$

Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm?

Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm?

It degrades gracefully when S larger than M:

- When B(S) <= M
  - Main memory hash-join has cost B(R) + B(S)
- When B(S) > M
  - Grace-join has cost 3B(R) + 3B(S)
  - Hybrid join has cost (3-2t/k)(B(R) + B(S))

Summary of External Join Algorithms

- Block Nested Loop: B(S) + B(R)*B(S)/(M-1)

- Index Join: B(R) + T(R)B(S)/V(S,a) (unclustered)

- Partitioned Hash: 3B(R)+3B(S);
  - min(B(R),B(S)) <= M^2

- Merge Join: 3B(R)+3B(S)
  - B(R)+B(S) <= M^2

Summary of Query Execution

- For each logical query plan
  - There exist many physical query plans
  - Each plan has a different cost
  - Cost depends on the data
- Additionally, for each query
  - There exist several logical plans
- Next lecture: query optimization
  - How to compute the cost of a complete plan?
  - How to pick a good query plan for a query?