# CSE 444: Database Internals 

Lecture 8<br>Operator Algorithms (part 2)

## Announcements

- Lab 2 / part 1 due on Wednesday
- We will not run any tests - So bugs are OK
- Homework 2 due on Friday
- Paper review for master's due on Friday


## Outline

- Join operator algorithms
- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)


## Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)=$ size of $R$ in blocks
- $T(R)=$ number of tuples in $R$
- $\mathrm{V}(\mathrm{R}, \mathrm{a})=$ \# of distinct values of attribute a


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What is the cost in each case?

- Clustered index on a:
- Unclustered index on a:


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What is the cost in each case?

- Clustered index on a: $\quad B(R) / V(R, a)$
- Unclustered index on $a: T(R) / V(R, a)$


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Note: we ignore I/O cost for index pages

## Index Based Selection

- Example: | $B(R)=2000$ |
| :--- |
| $T(R)=100,000$ |
| $V(R, a)=20$ |

$$
\text { cost of } \sigma_{a=v}(R)=?
$$

- Table scan:
- Index based selection:


## Index Based Selection

- Example: $\begin{aligned} & \mathrm{B}(\mathrm{R})=2000 \\ & T(R)=100,000 \\ & \mathrm{~V}(\mathrm{R}, \mathrm{a})=20\end{aligned}$

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- Table scan: $B(R)=2,000$ I/Os
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- Index based selection:
- If index is clustered:
- If index is unclustered:


## Index Based Selection

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| :--- | :--- |
|  |  |
- Table scan: $B(R)=2,000$ I/Os
- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100$ I/Os
- If index is unclustered:


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- Table scan: $B(R)=2,000$ I/Os
- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100$ I/Os
- If index is unclustered: $T(R) / V(R, a)=5,000 \mathrm{I} / \mathrm{Os}$


## Index Based Selection

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- Table scan: $B(R)=2,000$ I/Os
- Index based selection:
- If index is clustered: $B(R) / V(R, a)=100$ I/Os
- If index is unclustered: $T(R) / V(R, a)=5,0001 / O s$

Lesson: Don't build unclustered indexes when $\mathrm{V}(\mathrm{R}, \mathrm{a})$ is small!

## Index Nested Loop Join

$R \bowtie S$

- Assume $S$ has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S
- Cost:
- If index on $S$ is clustered: $B(R)+T(R) B(S) / V(S, a)$
- If index on $S$ is unclustered: $B(R)+T(R) T(S) / V(S, a)$


## Outline

- Join operator algorithms
- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)


## Two-Pass Algorithms

- What if data does not fit in memory?
- Need to process it in multiple passes
- Two key techniques
- Sorting
- Hashing


## Basic Terminology

- A run in a sequence is an increasing subsequence
- What are the runs?

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2,4,99,103,88,77,3,79,100,2,50
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## External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, sent to disk, repeat

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A: Length $=\mathrm{M}$ blocks

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Phase one: load M blocks in memory, sort, sent to disk, repeat
Q: How long are the runs?


A: Length = M blocks
Can increase to length 2M using "replacement selection"

## External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

- Merge $\mathrm{M}-1$ runs into a new run
- Result: runs of length $M(M-1) \approx M^{2}$


If $\mathrm{B}<=\mathrm{M}^{2}$ then we are done

## Example

- Merging three runs to produce a longer run:
$0,14,33,88,92,192,322$
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320
Output:
0


## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
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Output:
0 ,?

## Example

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0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0, 1,?

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
$0,1,2,4,6,7$, ?

## Cost of External Merge Sort

- Read+write+read $=3 B(R)$
- Assumption: $B(R)<=M^{2}$


## Discussion

- What does $B(R)<=M^{2}$ mean?
- How large can R be?


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- Example:
- Page size $=32 \mathrm{~KB}$
- Memory size 32GB: $M=10^{6}$-pages


## Discussion

- What does $B(R)<=M^{2}$ mean?
- How large can R be?
- Example:
- Page size = 32KB
- Memory size 32GB: $M=10^{6}$-pages
- $R$ can be as large as $10^{12}$-pages
$-32 \times 10^{15}$ Bytes $=32$ PB


## Merge-Join

Join $R \bowtie S$

- How?....


## Merge-Join

Join $R \bowtie S$

- Step 1a: generate initial runs for $R$
- Step 1b: generate initial runs for $S$
- Step 2: merge and join
- Either merge first and then join
- Or merge \& join at the same time


## Merge-Join Example

## Setup: Want to join R and S

- Relation R has 10 pages with 2 tuples per page
- Relation $S$ has 8 pages with 2 tuples per page

Values shown are values of join attribute for each given tuple


## Merge-Join Example

Step 1: Read $M$ pages of $R$ and sort in memory


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## Merge-Join Example

Step 1: Read $M$ pages of $R$ and sort in memory, then write to disk


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## Merge-Join Example

Step 1: Repeat for next $M$ pages until all $R$ is processed


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## Merge-Join Example

Step 1: Do the same with S


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## Merge-Join Example

Step 4: Join while merging sorted runs
Run 1 of $R$ Run 2 of $R$

| 1 | 2 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | 4 | 3 | 5 |
| 4 | 5 | 7 | 9 |
| 6 | 7 | 11 | 11 |
| 8 | 9 | 12 | 14 |
| Run 1 of $S$ |  | Run 2 of S |  |
| 0 | 1 | 1 | 5 |
| 2 | 3 | 7 | 9 |
| 3 | 4 | 11 | 12 |
| 5 | 7 |  |  |
| 8 | 9 |  |  |

Total cost: $3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})$

Step 4: Join while merging
Output tuples

- $(1,1)$
- $(1,1)$
- $(1,1)$
- $(1,1)$


## Merge-Join Example

Step 4: Join while merging sorted runs
Run 1 of $R$ Run 2 of $R$

| 1 | 2 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | 4 | 3 | 5 |
| 4 | 5 | 7 | 9 |
| 6 | 7 | 11 | 11 |
| 8 | 9 | 12 | 14 |
| Run 1 of $S$ |  | Run 2 of $S$ |  |
| 0 | 1 | 1 | 5 |
| 2 | 3 | 7 | 9 |
| 3 | 4 | 11 | 12 |
| 5 | 7 |  |  |
| 8 | 9 |  |  |

Total cost: $3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})$

Step 4: Join while merging
Output tuples

- $(1,1)$
- $(1,1)$
- $(1,1)$
- $(1,1)$
- $(2,2)$
- $(2,2)$


## Merge-Join Example

Step 4: Join while merging sorted runs


## Merge-Join


$M_{1}=B(R) / M$ runs for $R$
$M_{2}=B(S) / M$ runs for $S$
Merge-join $M_{1}+M_{2}$ runs;
need $\mathrm{M}_{1}+\mathrm{M}_{2}<=\mathrm{M}$

## Partitioned Hash Algorithms

- Partition R it into k buckets:
$R_{1}, R_{2}, R_{3}, \ldots, R_{k}$


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- Goal: each $\mathrm{R}_{\mathrm{i}}$ should fit in main memory: $B\left(R_{i}\right) \leq M$


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- Goal: each $\mathrm{R}_{\mathrm{i}}$ should fit in main memory: $B\left(R_{i}\right) \leq M$

How do we choose k?

## Partitioned Hash Algorithms

- We choose $k=$ M-1 Each bucket has size approx. $B(R) /(M-1) \approx B(R) / M$


$$
\text { Assumption: } B(R) / M \leq M \text {, i.e. } B(R) \leq M^{2}
$$

## Grace-Join

$R \bowtie S$


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$R \bowtie S$

- Step 1:

Note: grace-join is also called partitioned hash-join

- Hash S into M buckets
- Send all buckets to disk
- Step 2
- Hash R into M buckets
- Send all buckets to disk
- Step 3
- Join every pair of buckets


## Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into $\mathrm{M}-1$ (=4 buckets)


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## Partitioned Hash-Join Example

Step 1: Read relation S one page at a time and hash into the 4 buckets At the end, we get relation $S$ back on disk split into 4 buckets


## Partitioned Hash-Join Example

Step 2: Read relation $R$ one page at a time and hash into same 4 buckets


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## Partitioned Hash-Join Example

Step 3: Read one partition of $R$ and create hash table in memory using a different hash functior


## Partitioned Hash-Join Example

Step 4: Scan matching partition of $S$ and probe the hash table
Step 5: Repeat for all the buckets
Total cost: $3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})$


Memory M = 5 pages

$$
\text { Hash h2: value \% } 3
$$



## Grace-Join

- Partition both relations using hash fn h : R tuples in partition i will only match S tuples in partition i.



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- Partition both relations using hash fn h : R tuples in partition i will only match S tuples in partition i.



## Grace Join

- Cost: 3B(R) + 3B(S)
- Assumption: $\min (B(R), B(S))<=M^{2}$


## Hybrid Hash Join Algorithm

- Partition S into k buckets
$t$ buckets $S_{1}, \ldots, S_{t}$ stay in memory $k$-t buckets $S_{t+1}, \ldots, S_{k}$ to disk
- Partition R into k buckets
- First t buckets join immediately with $S$
- Rest k-t buckets go to disk
- Finally, join k-t pairs of buckets:
$\left(R_{t+1}, S_{t+1}\right),\left(R_{t+2}, S_{t+2}\right), \ldots,\left(R_{k}, S_{k}\right)$


## Hybrid Hash Join Algorithm



## Hybrid Join Algorithm

- How to choose $k$ and $t$ ?


## Hybrid Join Algorithm

- How to choose k and t ?
- Choose k large but s.t. $\mathrm{k}<=\mathrm{M}$


## Hybrid Join Algorithm

- How to choose $k$ and $t$ ?
- Choose k large but s.t.

One block/bucket in memory
$\mathrm{k}<=\mathrm{M}$

## Hybrid Join Algorithm

- How to choose $k$ and $t$ ?
- Choose k large but s.t.

One block/bucket in memory k <= M

- Choose t/k large but s.t.
$t / k$ * $B(S)<=M$


## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.
- Choose t/k large but s.t.



## Hybrid Join Algorithm

- How to choose $k$ and $t$ ?
- Choose k large but s.t.
- Choose t/k large but s.t.
$t / k$ * $B(S)<=M$
- Together:
$t / k * B(S)+k-t<=M$


## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.

One block/bucket in memory

- Choose t/k large but s.t.
$t / k$ * $B(S)<=M$
- Together:
$t / k * B(S)+k-t<=M$
- Assuming $t / k$ * $B(S) \gg k-t: \quad t / k=M / B(S)$


## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.

One block/bucket in memory

- Choose t/k large but s.t.
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## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.
- Choose t/k large but s.t.
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- Together:
$t / k$ * $B(S)+k-t<=M$
- Assuming $t / k$ * $B(S) \gg k-t: \quad t / k=M / B(S)$


## Hybrid Join Algorithm

Even better: adjust t dynamically

- Start with $\mathrm{t}=\mathrm{k}$ : all buckets are in main memory
- Read blocks from S, insert tuples into buckets
- When out of memory:
- Send one bucket to disk
- t:= t-1
- Worst case:
- All buckets are sent to disk ( $\mathrm{t}=0$ )
- Hybrid join becomes grace join


## Hybrid Join Algorithm

Cost of Hybrid Join:

- Grace join: 3B(R) + 3B(S)
- Hybrid join:
- Saves $2 \mathrm{I} / \mathrm{Os}$ for $\mathrm{t} / \mathrm{k}$ fraction of buckets
- Saves 2t/k(B(R) + B(S)) I/Os
- Cost:
$(3-2 t / k)(B(R)+B(S))=(3-2 M / B(S))(B(R)+B(S))$


## Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm?


## Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm?

It degrades gracefully when S larger than M :

- When B(S) <= M
- Main memory hash-join has cost $B(R)+B(S)$
- When $B(S)>M$
- Grace-join has cost 3B(R) + 3B(S)
- Hybrid join has cost ( $3-2 t / k)(B(R)+B(S))$


## Summary of External Join Algorithms

- Block Nested Loop: $\mathrm{B}(\mathrm{S})+\mathrm{B}(\mathrm{R}) * \mathrm{~B}(\mathrm{~S}) / \mathrm{M}$
- Index Join: B(R) + T(R)B(S)/V(S,a)
- Partitioned Hash: 3B(R)+3B(S);
$-\min (B(R), B(S))<=M^{2}$
- Merge Join: 3B(R)+3B(S)
$-B(R)+B(S)<=M^{2}$


## Summary of Query Execution

- For each logical query plan
- There exist many physical query plans
- Each plan has a different cost
- Cost depends on the data
- Additionally, for each query
- There exist several logical plans
- Next lecture: query optimization
- How to compute the cost of a complete plan?
- How to pick a good query plan for a query?

