# CSE 444: Database Internals 

## Lecture 8

Operator Algorithms (part 2)

## Announcements

- Lab 2 / part 1 due on Friday
- Homework 2 due next Wednesday


## Outline

- Join operator algorithms
- One-pass algorithms (Sec. 15.2 and 15.3)
- Index-based algorithms (Sec 15.6)
- Two-pass algorithms (Sec 15.4 and 15.5)


## Index Based Selection

Selection on equality: $\sigma_{a=v}(R)$

- $B(R)=$ size of $R$ in blocks
- $T(R)=$ number of tuples in $R$
- $\mathrm{V}(\mathrm{R}, \mathrm{a})=$ \# of distinct values of attribute a


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What is the cost in each case?

- Clustered index on a:
- Unclustered index on a:


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- Unclustered index on $a: T(R) / V(R, a)$


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What is the cost in each case?

- Clustered index on a: $\quad B(R) / V(R, a)$
- Unclustered index on $\mathrm{a}: \quad \mathrm{T}(\mathrm{R}) / \mathrm{V}(\mathrm{R}, \mathrm{a})$

Note: we ignore I/O cost for index pages

## Index Based Selection

- Example: $\begin{aligned} & B(R)=2000 \\ & T(R)=100,000 \\ & V(R, a)=20\end{aligned}$

$$
\operatorname{cost} \text { of } \sigma_{a=v}(R)=?
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- Table scan:
- Index based selection:


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## Index Nested Loop Join

## $R \bowtie S$

- Assume $S$ has an index on the join attribute
- Iterate over R, for each tuple fetch corresponding tuple(s) from S
- Cost:
- If index on $S$ is clustered: $B(R)+T(R) B(S) / V(S, a)$
- If index on $S$ is unclustered: $B(R)+T(R) T(S) / V(S, a)$


## Outline

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## Two-Pass Algorithms

- What if data does not fit in memory?
- Need to process it in multiple passes
- Two key techniques
- Sorting
- Hashing


## Basic Terminology

- A run in a sequence is an increasing subsequence
- What are the runs?
$2,4,99,103,88,77,3,79,100,2,50$


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Phase one: load M blocks in memory, sort, sent to disk, repeat

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A: Length $=\mathrm{M}$ blocks

## External Merge-Sort: Step 1

Phase one: load M blocks in memory, sort, sent to disk, repeat


Can increase to length 2M using "replacement selection"

## External Merge-Sort: Step 2

Phase two: merge M runs into a bigger run

## Example

- Merging three runs to produce a longer run:
$0,14,33,88,92,192,322$
2, 4, 7, 43, 78, 103, 523
1, 6, 9, 12, 33, 52, 88, 320

Output:
0

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
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## Example

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1, 6, 9, 12, 33, 52, 88, 320

Output:
0, 1,?

## Example

- Merging three runs to produce a longer run:

0, 14, 33, 88, 92, 192, 322
2, 4, 7, 43, 78, 103, 523
$1,6,9,12,33,52,88,320$

Output:
$0,1,2,4,6,7$, ?

## External Merge-Sort: Step 2

- Merge $\mathrm{M}-1$ runs into a new run
- Result: runs of length $M(M-1) \approx M^{2}$


If $B<=M^{2}$ then we are done

## Cost of External Merge Sort

- Read+write+read $=3 B(R)$
- Assumption: $\mathrm{B}(\mathrm{R})<=\mathrm{M}^{2}$


## Discussion

- What does $B(R)<=M^{2}$ mean? How large can $R$ be?


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- Memory size 32GB: M = 106-pages


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- What does $B(R)<=M^{2}$ mean? How large can R be?
- Example:
- Page size = 32KB
- Memory size 32GB: M = 106-pages
- $R$ can be as large as $10^{12}$-pages
$-32 \times 10^{15}$ Bytes $=32 \mathrm{~PB}$


## Merge-Join

## Join $R \bowtie S$ <br> - How? ....

## Merge-Join

Join $R \bowtie S$

- Step 1a: initial runs for R
- Step 1b: initial runs for $S$
- Step 2: merge and join


## Merge-Join



## Partitioned Hash Algorithms

- Partition R it into k buckets:

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R_{1}, R_{2}, R_{3}, \ldots, R_{k}
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B\left(R_{i}\right)=B(R) / k, \text { for all } i
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- Goal: each $R_{i}$ should fit in main memory: $B\left(R_{i}\right) \leq M$


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- Goal: each $R_{i}$ should fit in main memory: $B\left(R_{i}\right) \leq M$

How do we choose k?

## Partitioned Hash Algorithms

- We choose $k=M-1$ Each bucket has size approx. $B(R) /(M-1) \approx B(R) / M$


$$
\text { Assumption: } B(R) / M \leq M \text {, i.e. } B(R) \leq M^{2}
$$

## Grace-Join

## $R \bowtie S$

Note: grace-join is also called

## Grace-Join

## $R \bowtie S$

- Step 1:
- Hash S into M buckets
- send all buckets to disk
- Step 2
- Hash R into M buckets
- Send all buckets to disk
- Step 3
- Join every pair of buckets

Note: grace-join is also called partitioned hash-join

## Grace-Join

- Partition both relations using hash fn h : R tuples in partition i will only match $S$ tuples in partition i.



## Grace-Join

- Partition both relations using hash fn h : R tuples in partition i will only match S tuples in partition i.



## Partitions <br> of R \& S

Read in a partition of R, hash it using h2 (<> h!). Scan matching partition of S, search for matches.


Disk

Join Result
Hash table for partition


Disk

## Grace Join

- Cost: 3B(R) + 3B(S)
- Assumption: $\min (B(R), B(S))<=M^{2}$


## Hybrid Hash Join Algorithm

- Partition $S$ into $k$ buckets $t$ buckets $S_{1}, \ldots, S_{t}$ stay in memory k-t buckets $\mathrm{S}_{\mathrm{t}+1}, \ldots, \mathrm{~S}_{\mathrm{k}}$ to disk
- Partition R into k buckets
- First t buckets join immediately with $S$
- Rest k-t buckets go to disk
- Finally, join k-t pairs of buckets:
$\left(R_{t+1}, S_{t+1}\right),\left(R_{t+2}, S_{t+2}\right), \ldots,\left(R_{k}, S_{k}\right)$


## Hybrid Hash Join Algorithm



## Hybrid Join Algorithm

- How to choose k and t?


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- How to choose k and t?
- Choose k large but s.t.
k <= M


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$$

- Choose t/k large but s.t. $t / k$ * $B(S)<=M$


## Hybrid Join Algorithm

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## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.
- Choose t/k large but s.t. $t / k$ * $B(S)<=M$
- Together:
$t / k$ * $B(S)+k-t<=M$


## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.
- Choose t/k large but s.t.

- Together:
$t / k$ * $B(S)+k-t<=M$
- Assuming t/k * $B(S) \gg k-t: \quad t / k=M / B(S)$


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$$
\text { Total size of first } t \text { buckets }
$$

## Hybrid Join Algorithm

- How to choose k and t?
- Choose k large but s.t.
- Choose t/k large but s.t.

- Together:
$t / k$ * $B(S)+k-t<=M$
- Assuming t/k * $B(S) \gg k-t: \quad t / k=M / B(S)$



## Hybrid Join Algorithm

Even better: adjust t dynamically

- Start with $\mathrm{t}=\mathrm{k}$ : all buckets are in main memory
- Read blocks from S, insert tuples into buckets
- When out of memory:
- Send one bucket to disk
- t:= t-1
- Worst case:
- All buckets are sent to disk ( $\mathrm{t}=0$ )
- Hybrid join becomes grace join


## Hybrid Join Algorithm

Cost of Hybrid Join:

- Grace join: 3B(R) + 3B(S)
- Hybrid join:
- Saves 2 I/Os for $t / k$ fraction of buckets
- Saves 2t/k(B(R) + B(S)) I/Os
- Cost:

$$
(3-2 t / k)(B(R)+B(S))=(3-2 M / B(S))(B(R)+B(S))
$$

## Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm ?


## Hybrid Join Algorithm

- What is the advantage of the hybrid algorithm ?

It degrades gracefully when $S$ larger than $M$ :

- When $\mathrm{B}(\mathrm{S})<=\mathrm{M}$
- Main memory hash-join has cost $B(R)+B(S)$
- When $B(S)>M$
- Grace-join has cost $3 B(R)+3 B(S)$
- Hybrid join has cost (3-2t/k)(B(R) + B(S))


## Summary of External Join Algorithms

- Block Nested Loop: B(S) + B(R)*B(S)/M
- Index Join: $B(R)+T(R) B(S) / V(S, a)$
- Partitioned Hash: 3B(R)+3B(S);
$-\min (B(R), B(S))<=M^{2}$
- Merge Join: 3B(R)+3B(S)
$-B(R)+B(S)<=M^{2}$


## Summary of Query Execution

- For each logical query plan
- There exist many physical query plans
- Each plan has a different cost
- Cost depends on the data
- Additionally, for each query
- There exist several logical plans
- Next lecture: query optimization
- How to compute the cost of a complete plan?
- How to pick a good query plan for a query?

