

Insertion

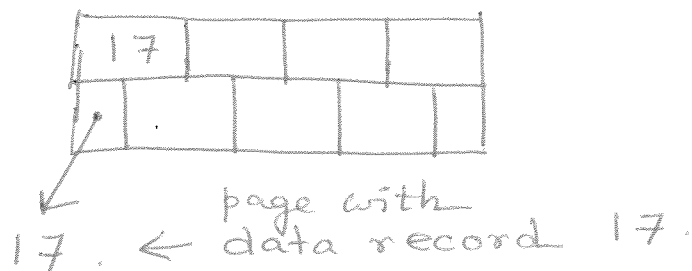
— Start with an empty B^+ tree $d=2$

— Insert

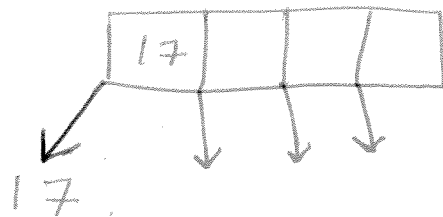
17, 3, 25, 95, 8, 57, 69.

Then 29, 91, 78, 80, 92, 99, 97.

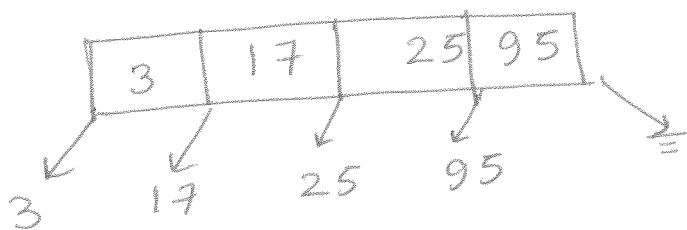
Insert 17



We are going to write it as

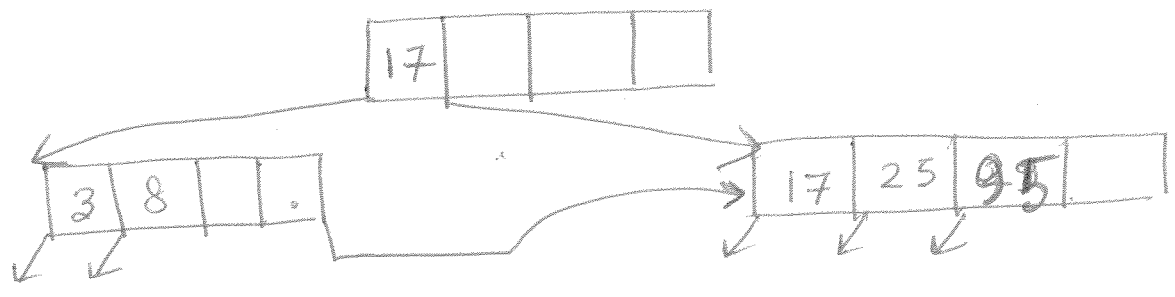


Insert 3, 25, 95



no right next leaf on right.

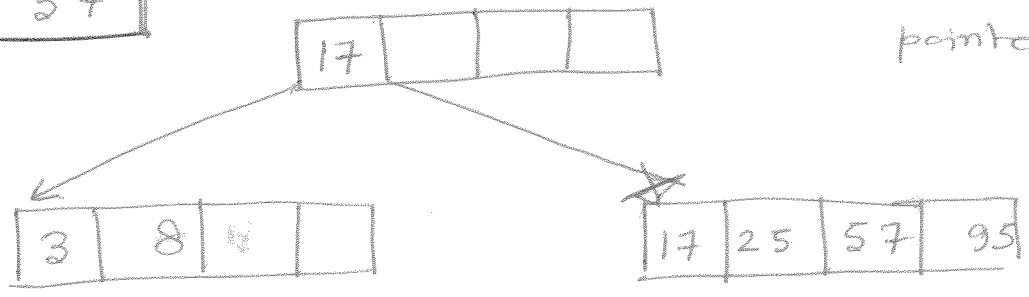
Insert 8



Note: A Leaf is being split
 Copy middle key to right leaf

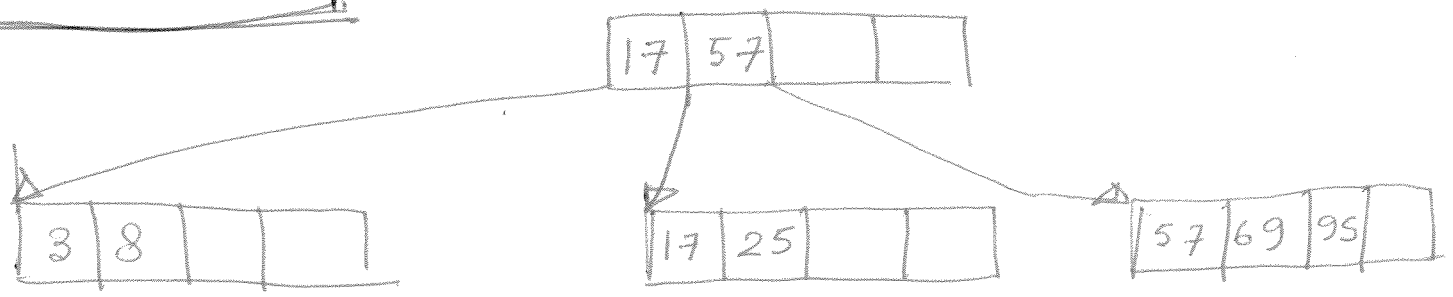
(Later :- copying the key at the middle is not needed when an internal node is split)

Insert 57

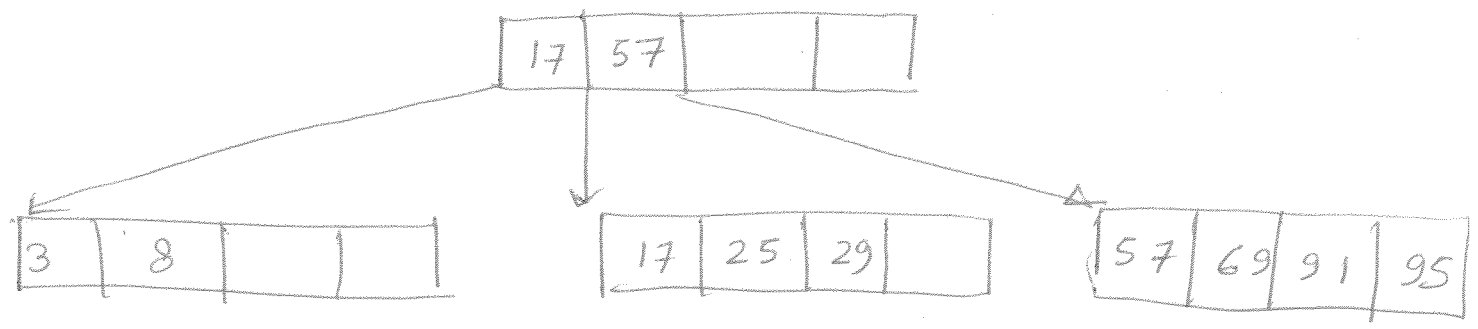


[omitting pointers to data & pointer to right next leaf]

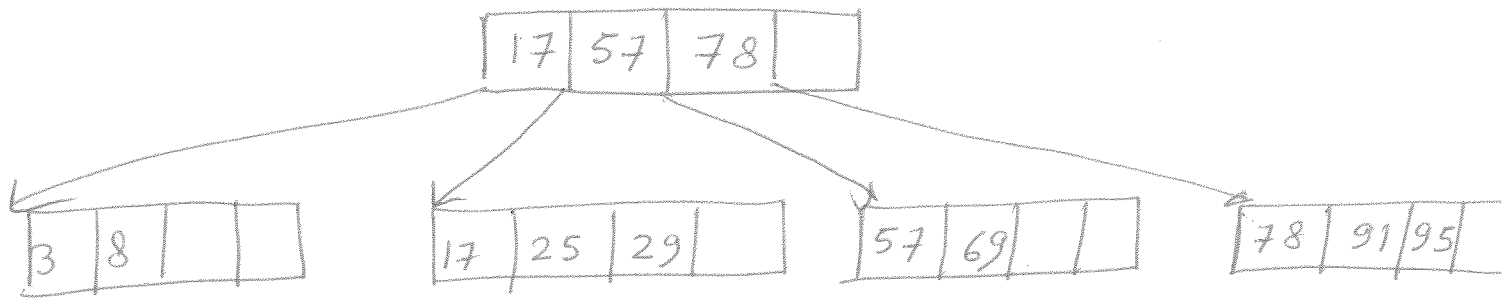
Insert 69



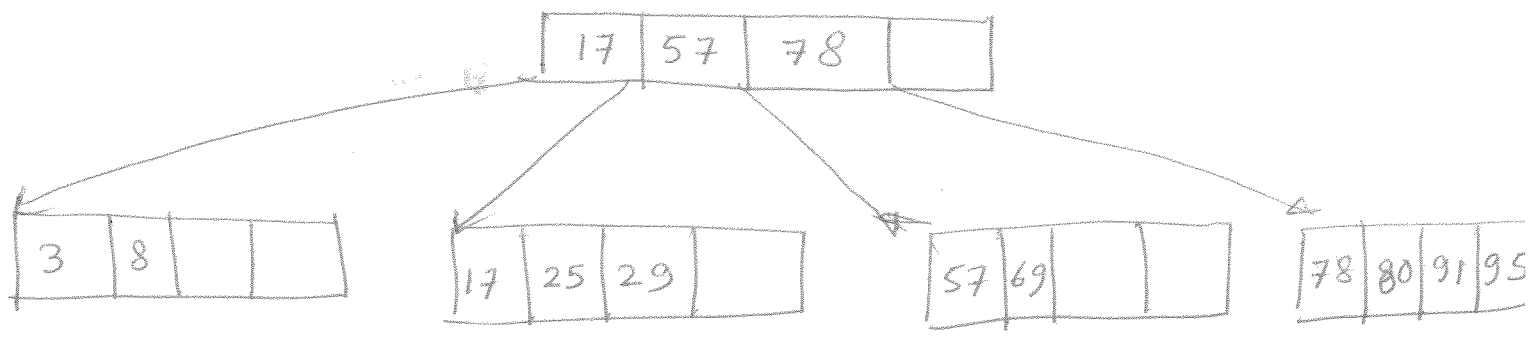
Insert 29, 91



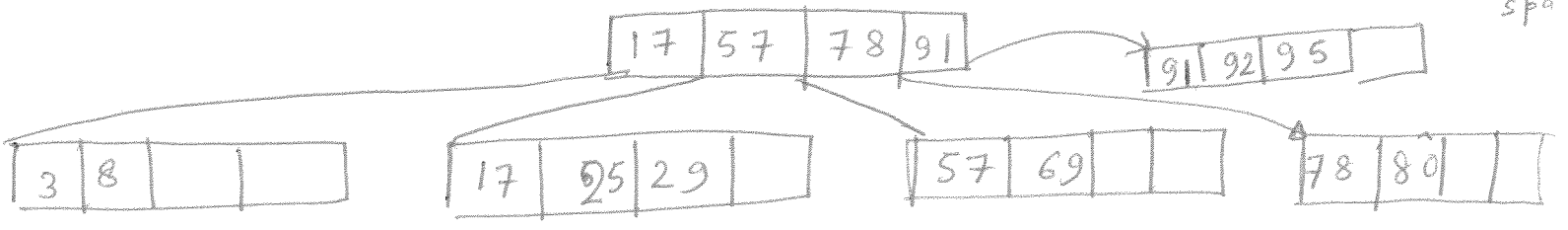
Insert 78



Insert 80

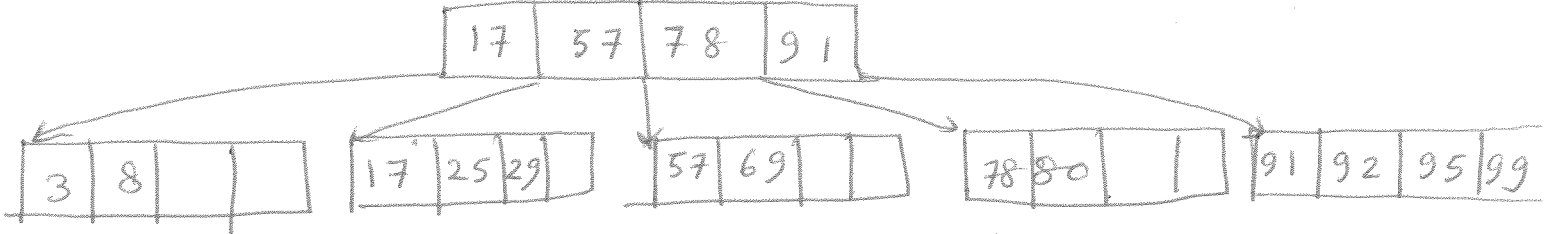


Insert 92



(sorry no spa)

Insert 99

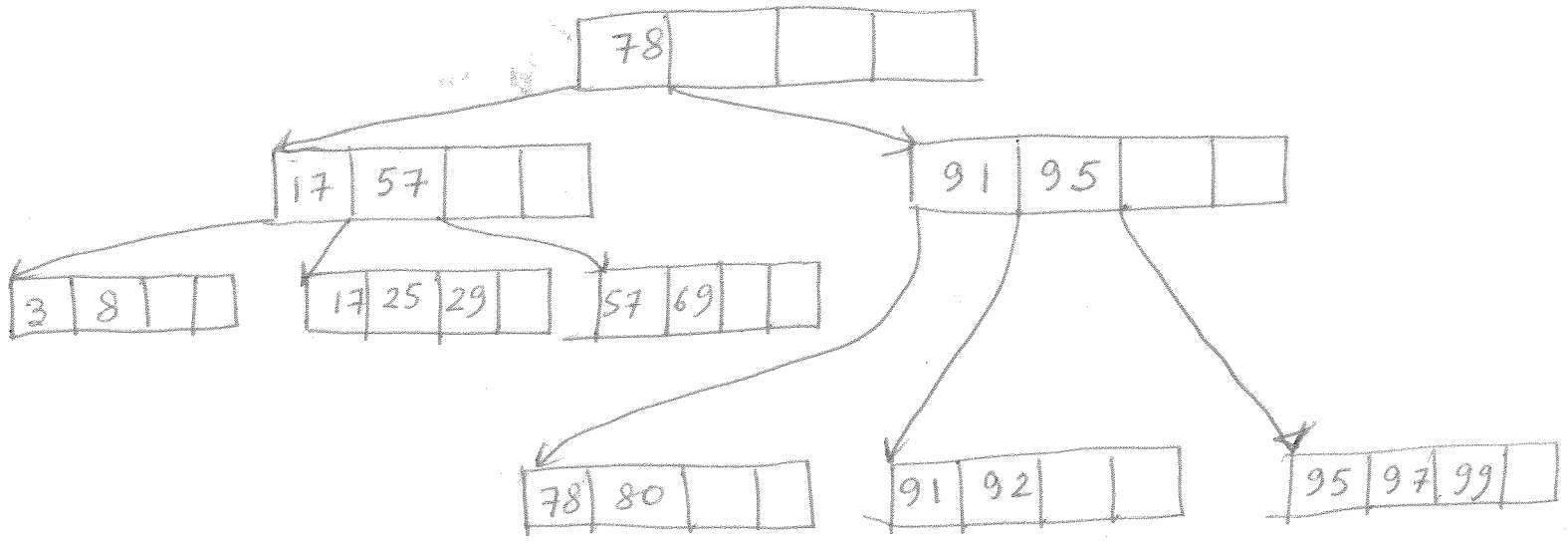


Insert 97

Need to split root, height increases!

Note :- 78 is not being copied to right new node, unlike leaves.

why? we have left pointer from 91 which can point to [78, 80,]

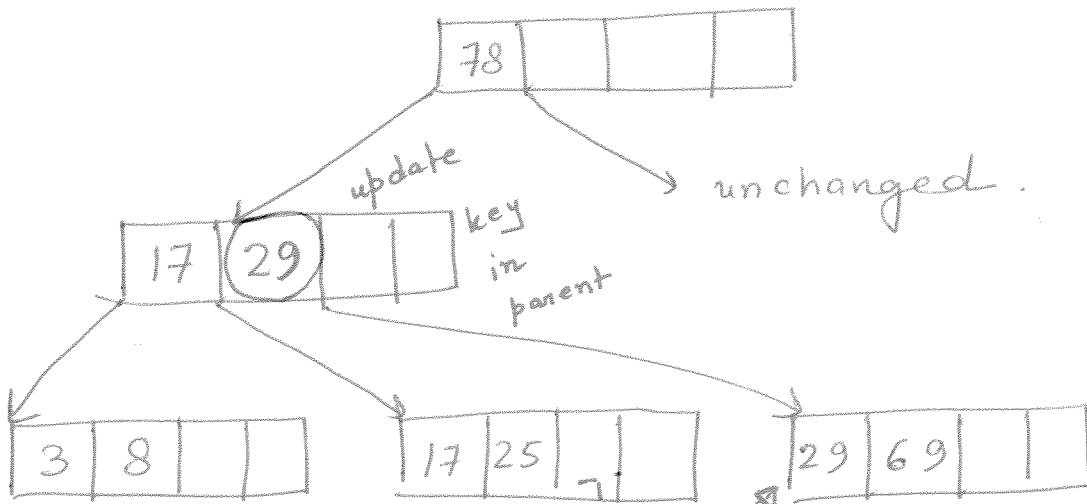


Deletion

Now we will delete all nodes in the following order:

- 57, 3, 99, 29, 17, 25, 95,
- 8, 78, 92, 69, 97, 91

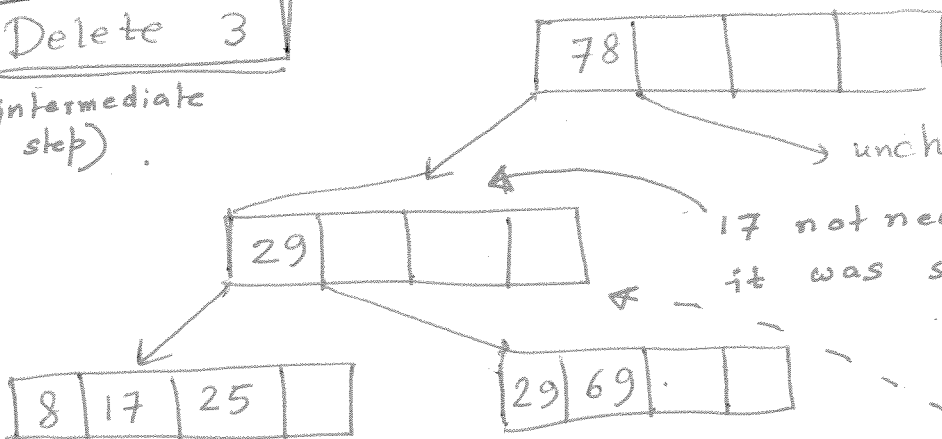
Delete 57



Note :- Next leaf on right may not be a sibling (= same parent)

Delete 3

(intermediate step)



But unbalanced has < 2 keys

~~Delete~~ Now: -

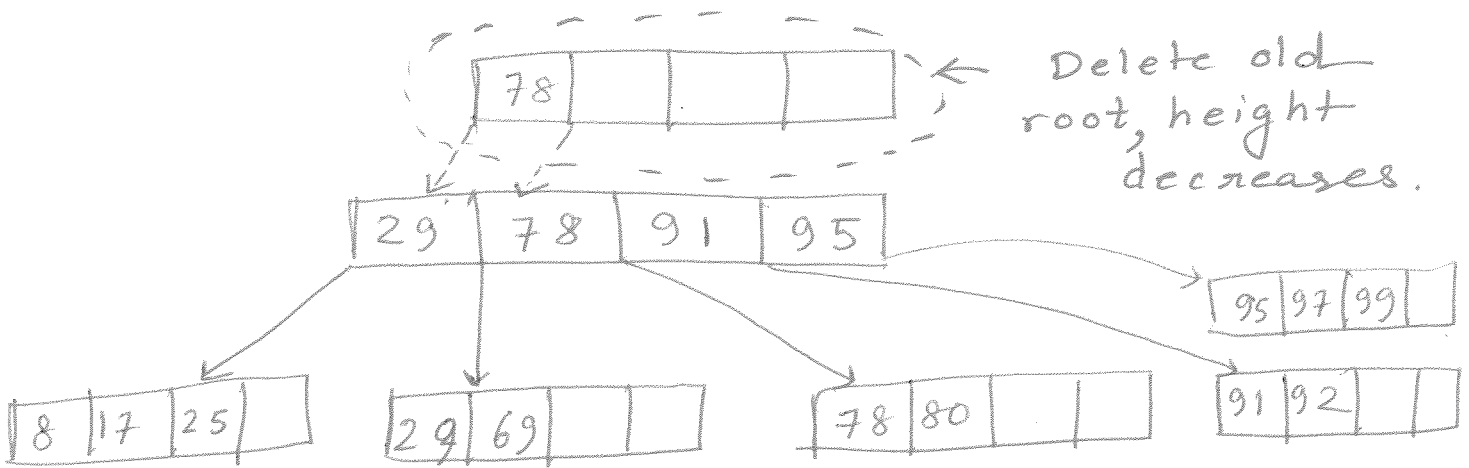
Merge

29			
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 with its sibling.

Note 1: - cannot borrow from sibling

Note 2: - Need to include key separating these siblings from parent (here 78)



Note 3: We always have space to include the key separating the siblings.

left sibling: $\leq d-1$ keys (that's why needed more keys)

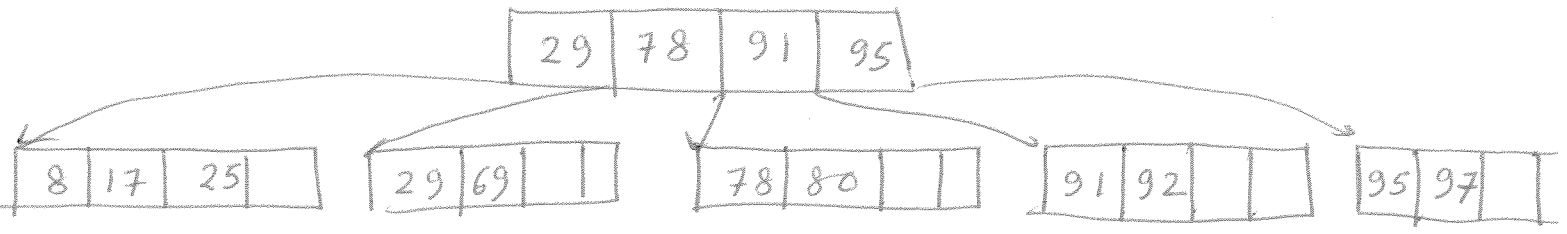
right sibling: $\leq d$ keys (that's why we could not borrow from it)

Total $\leq 2d-1$ keys.

So how we have room to include the key from parent (here 78)

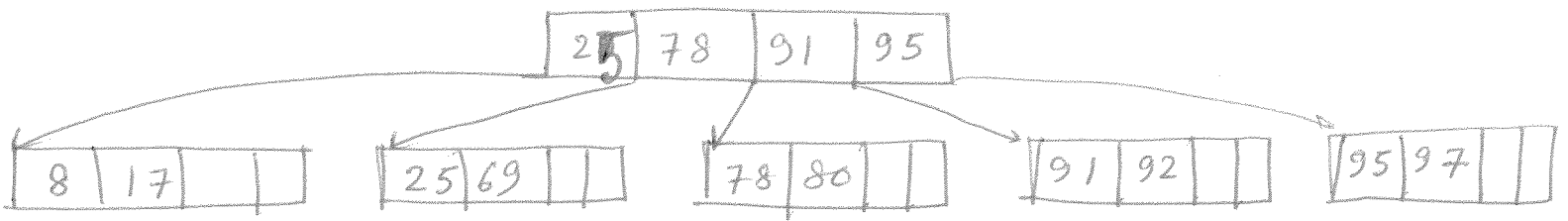
Delete 99

Easy!



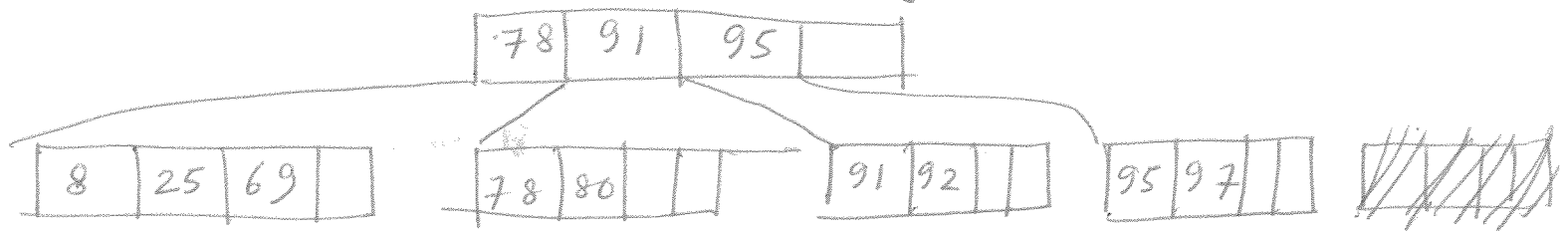
Delete 29

can borrow from left sibling,
update key in parent



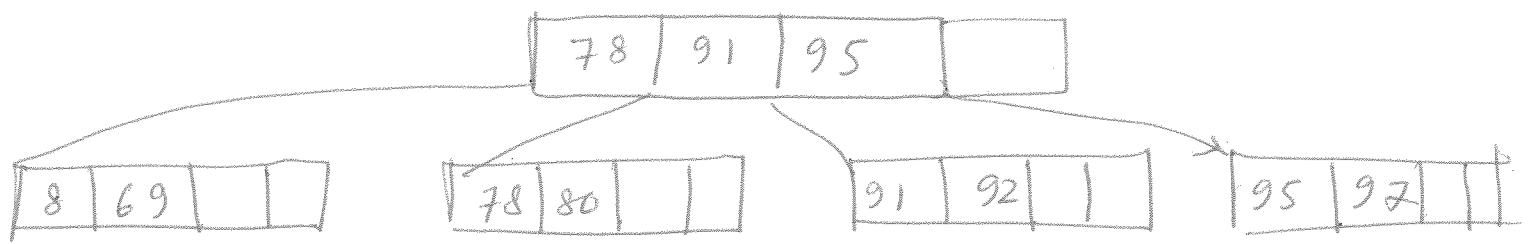
Delete 17

can't borrow, merge with right sibling
Delete separating key (25) from parent



Delete 25

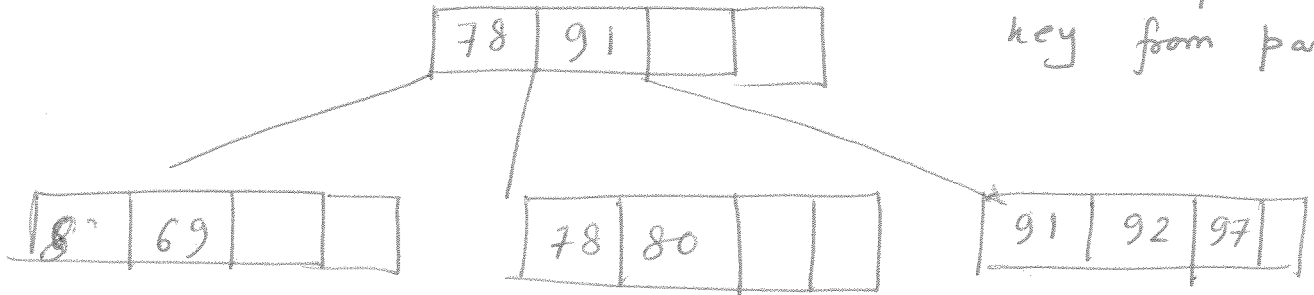
Easy!



Delete 95

Merge with ^{left} ~~right~~ sibling

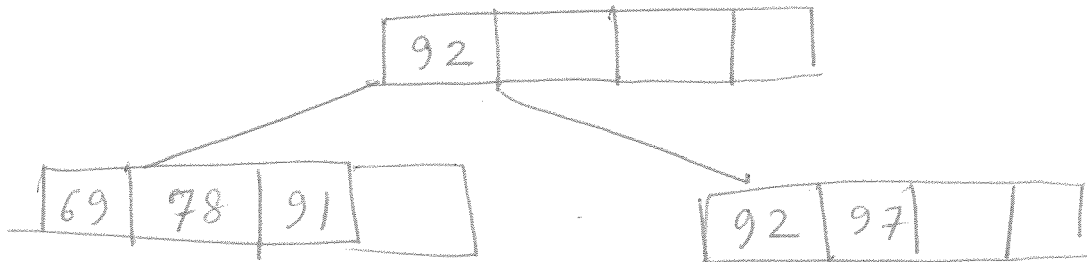
delete separating key from parent



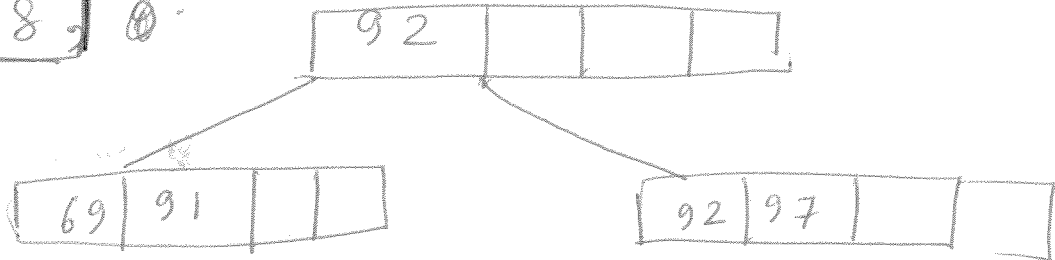
Delete 80, 8

Now you can do the rest!

~~skipping~~ some steps are skipped here.



Delete 78, 91



Delete 92

Merge leaves, root is empty, height decreases again.



Delete 69, 97, 91

Too easy!