

# Introduction to Database Systems CSE 444

## Lectures 8 & 9 Database Design

April 16 & 18, 2008

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## Outline

- The relational data model: 3.1
- Functional dependencies: 3.4

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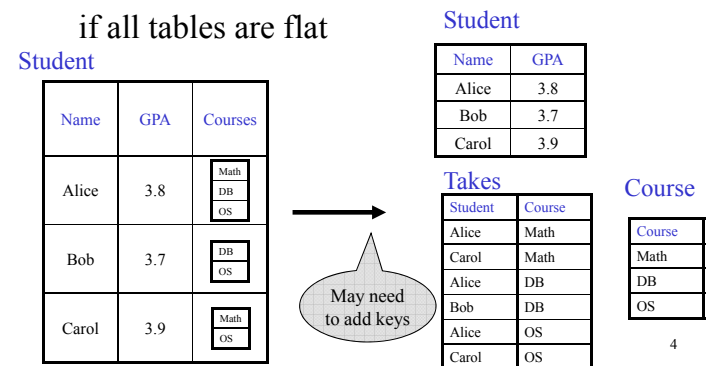
## Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = will study
- 3rd Normal Form = see book

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## First Normal Form (1NF)

- A database schema is in First Normal Form if all tables are flat



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## Relational Schema Design

Conceptual Model:

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Relational Model:  
plus FD's

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Normalization:  
Eliminates **anomalies**

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## Data Anomalies

When a database is poorly designed we get anomalies:

**Redundancy:** data is repeated

**Update anomalies:** need to change in several places

**Delete anomalies:** may lose data when we don't want

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## Relational Schema Design

Recall set attributes (persons with several phones):

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

One person may have multiple phones, but lives in only one city

**Anomalies:**

- Redundancy = repeated data
- Update anomalies = Fred moves to "Bellevue"
- Deletion anomalies = Joe deletes his phone number: what is his city ?

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## Relation Decomposition

**Break the relation into two:**

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121

**Anomalies are gone:**

- No more repeated data
- Easy to move Fred to "Bellevue" (how?)
- Easy to delete all Joe's phone numbers (how?)

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## Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its **functional dependencies**
- Use them to design a better relational schema

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## Functional Dependencies

- A form of constraint
  - hence, part of the schema
- Finding them is part of the database design
- Also used in normalizing the relations

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## Functional Dependencies

Definition:

If two tuples agree on the attributes

$A_1, A_2, \dots, A_n$

then they must also agree on the attributes

$B_1, B_2, \dots, B_m$

Formally:

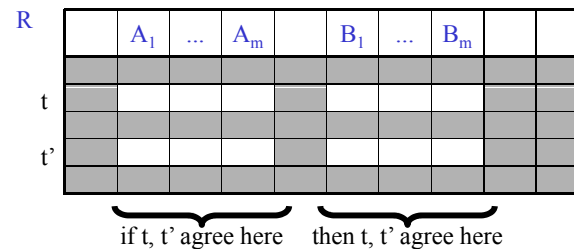
$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

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## When Does an FD Hold

Definition:  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  holds in R if:

$\forall t, t' \in R, (t.A_1=t'.A_1 \wedge \dots \wedge t.A_m=t'.A_m \Rightarrow t.B_1=t'.B_1 \wedge \dots \wedge t.B_n=t'.B_n)$



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## Examples

An FD holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

Position → Phone

but not Phone → Position

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## Example

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

Position → Phone

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## Example

EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
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E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

but not Phone → Position

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## Example

FD's are constraints:

- On some instances they hold
- On others they don't

name → color  
 category → department  
 color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Green	Toys	99

Does this instance satisfy all the FDs ?

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### Example

name → color  
category → department  
color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-suppl.	59

What about this one ?

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### An Interesting Observation

If all these FDs are true:

name → color  
category → department  
color, category → price

Then this FD also holds:

name, category → price

Why ??

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### Goal: Find ALL Functional Dependencies

- Anomalies occur when certain “bad” FDs hold
- We know some of the FDs
- Need to find *all* FDs, then look for the bad ones

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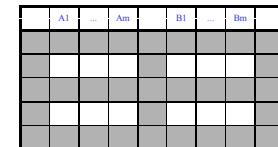
### Armstrong's Rules (1/3)

$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

Is equivalent to

**Splitting rule  
and  
Combing rule**

$A_1, A_2, \dots, A_n \rightarrow B_1$   
 $A_1, A_2, \dots, A_n \rightarrow B_2$   
 . . . . .  
 $A_1, A_2, \dots, A_n \rightarrow B_m$



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## Armstrong's Rules (1/3)

$$A_1, A_2, \dots, A_n \rightarrow A_i$$

Trivial Rule

where  $i = 1, 2, \dots, n$

Why ?

	$A_1$	...	$A_n$	

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## Armstrong's Rules (1/3)

Transitive Closure Rule

If  $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

and  $B_1, B_2, \dots, B_m \rightarrow C_1, C_2, \dots, C_p$

then  $A_1, A_2, \dots, A_n \rightarrow C_1, C_2, \dots, C_p$

Why ?

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	$A_1$	...	$A_n$		$B_1$	...	$B_m$		$C_1$	...	$C_p$	

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## Example (continued)

Start from the following FDs:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

Infer the following FDs:

Inferred FD	Which Rule did we apply ?
4. name, category $\rightarrow$ name	
5. name, category $\rightarrow$ color	
6. name, category $\rightarrow$ category	
7. name, category $\rightarrow$ color, category	
8. name, category $\rightarrow$ price	

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### Example (continued)

Answers:

- 1. name → color
- 2. category → department
- 3. color, category → price

Inferred FD	Which Rule did we apply ?
4. name, category → name	Trivial rule
5. name, category → color	Transitivity on 4, 1
6. name, category → category	Trivial rule
7. name, category → color, category	Split/combine on 5, 6
8. name, category → price	Transitivity on 3, 7

THIS IS TOO HARD ! Let's see an easier way.

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### Closure of a set of Attributes

**Given** a set of attributes  $A_1, \dots, A_n$

The **closure**,  $\{A_1, \dots, A_n\}^+$  = the set of attributes B  
s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

- name → color
- category → department
- color, category → price

Closures:

- name<sup>+</sup> = {name, color}
- {name, category}<sup>+</sup> = {name, category, color, department, price}
- color<sup>+</sup> = {color}

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### Closure Algorithm

$X = \{A_1, \dots, A_n\}$ .

**Repeat until** X doesn't change **do**:

**if**  $B_1, \dots, B_n \rightarrow C$  is a FD **and**  
 $B_1, \dots, B_n$  are all in X  
**then** add C to X.

Example:

- name → color
- category → department
- color, category → price

{name, category}<sup>+</sup> =  
{ name, category, color, department, price }

Hence: name, category → color, department, price

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### Example

In class:

R(A,B,C,D,E,F)

- A, B → C
- A, D → E
- B → D
- A, F → B

Compute {A,B}<sup>+</sup> X = {A, B, }

Compute {A, F}<sup>+</sup> X = {A, F, }

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## Why Do We Need Closure

- With closure we can find all FD's easily
- To check if  $X \rightarrow A$ 
  - Compute  $X^+$
  - Check if  $A \in X^+$

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## Using Closure to Infer ALL FDs

Example:

A, B	→	C
A, D	→	B
B	→	D

Step 1: Compute  $X^+$ , for every X:

$A^+ = A$ , $B^+ = BD$ , $C^+ = C$ , $D^+ = D$
$AB^+ = ABCD$ , $AC^+ = AC$ , $AD^+ = ABCD$ ,
$BC^+ = BCD$ , $BD^+ = BD$ , $CD^+ = CD$
$ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?)
$BCD^+ = BCD$ , $ABCD^+ = ABCD$

Step 2: Enumerate all FD's  $X \rightarrow Y$ , s.t.  $Y \subseteq X^+$  and  $X \cap Y = \emptyset$ :

$AB \rightarrow CD$ , $AD \rightarrow BC$ , $ABC \rightarrow D$ , $ABD \rightarrow C$ , $ACD \rightarrow B$
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## Another Example

- Enrollment(student, major, course, room, time)
  - student  $\rightarrow$  major
  - major, course  $\rightarrow$  room
  - course  $\rightarrow$  time

What else can we infer ? [in class, or at home]

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## Keys

- A **superkey** is a set of attributes  $A_1, \dots, A_n$  s.t. for any other attribute B, we have  $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey
  - i.e. set of attributes which is a superkey and for which no subset is a superkey

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## Computing (Super)Keys

- Compute  $X^+$  for all sets  $X$
- If  $X^+ =$  all attributes, then  $X$  is a key
- List only the minimal  $X$ 's

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## Example

Product(name, price, category, color)

name, category $\rightarrow$ price category $\rightarrow$ color
--

What is the key ?

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## Example

Product(name, price, category, color)

name, category $\rightarrow$ price category $\rightarrow$ color
--

What is the key ?

$(\text{name, category})^+ = \text{name, category, price, color}$

Hence (name, category) is a key

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## Examples of Keys

Enrollment(student, address, course, room, time)

student $\rightarrow$ address room, time $\rightarrow$ course student, course $\rightarrow$ room, time
--

(find keys at home)

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## Eliminating Anomalies

Main idea:

- $X \rightarrow A$  is OK if X is a (super)key
- $X \rightarrow A$  is not OK otherwise

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## Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$SSN \rightarrow Name, City$

What the key?

$\{SSN, PhoneNumber\}$

Hence  $SSN \rightarrow Name, City$   
is a "bad" dependency 38

## Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

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## Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

$AB \rightarrow C$   
 $BC \rightarrow A$       or       $A \rightarrow BC$   
 $B \rightarrow AC$

what are the keys here ?

Can you design FDs such that there are *three* keys ? 40

## Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if:

If  $A_1, \dots, A_n \rightarrow B$  is a non-trivial dependency in R, then  $\{A_1, \dots, A_n\}$  is a superkey for R

In other words: there are no “bad” FDs

Equivalently:

$\forall X$ , either  $(X^+ = X)$  or  $(X^+ = \text{all attributes})$

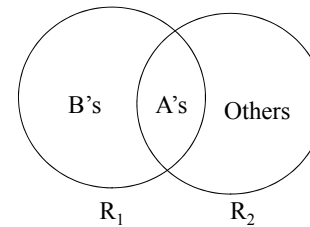
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## BCNF Decomposition Algorithm

**repeat**

choose  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  that violates BCNF  
 split R into  $R_1(A_1, \dots, A_m, B_1, \dots, B_n)$  and  $R_2(A_1, \dots, A_m, [\text{others}])$   
 continue with both  $R_1$  and  $R_2$

**until** no more violations



Is there a 2-attribute relation that is not in BCNF ?

In practice, we have a better algorithm (coming<sup>42</sup> up)

## Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$SSN \rightarrow \text{Name, City}$

What the key?

$\{SSN, \text{PhoneNumber}\}$  use  $SSN \rightarrow \text{Name, City}$  to split

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## Example

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

$SSN \rightarrow \text{Name, City}$

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy ?
- Update ?
- Delete ?

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## Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber)  
 SSN → name, age  
 age → hairColor

Decompose in BCNF (in class):

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## BCNF Decomposition Algorithm

BCNF\_Decompose(R)

find X s.t.:  $X \neq X^+ \neq$  [all attributes]

**if** (not found) **then** “R is in BCNF”

**let**  $Y = X^+ - X$

**let**  $Z =$  [all attributes] -  $X^+$

decompose R into  $R_1(X \cup Y)$  and  $R_2(X \cup Z)$

continue to decompose recursively  $R_1$  and  $R_2$

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Find X s.t.:  $X \neq X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)  
 SSN → name, age  
 age → hairColor

Iteration 1: Person

$SSN^+ = SSN, name, age, hairColor$

Decompose into: P(SSN, name, age, hairColor)  
 Phone(SSN, phoneNumber)

Iteration 2: P

$age^+ = age, hairColor$

Decompose: People(SSN, name, age)  
 Hair(age, hairColor)  
 Phone(SSN, phoneNumber)

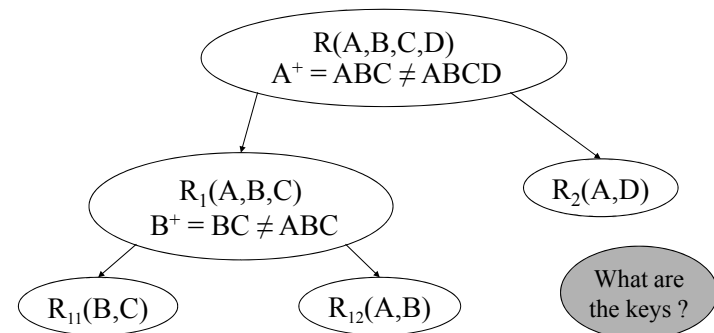
What are the keys ?

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R(A,B,C,D)

A → B  
 B → C

## Example



What happens if in R we first pick  $B^+$  ? Or  $AB^+$  ?

### Decompositions in General

$$R(A_1, \dots, A_n, B_1, \dots, B_m, C_1, \dots, C_p)$$

$R_1(A_1, \dots, A_n, B_1, \dots, B_m)$

$R_2(A_1, \dots, A_n, C_1, \dots, C_p)$

$R_1 =$  projection of  $R$  on  $A_1, \dots, A_n, B_1, \dots, B_m$   
 $R_2 =$  projection of  $R$  on  $A_1, \dots, A_n, C_1, \dots, C_p$

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### Theory of Decomposition

- Sometimes it is correct:

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Price
Gizmo	19.99
OneClick	24.99
<del>Gizmo</del>	<del>19.99</del>

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Lossless decomposition

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### Incorrect Decomposition

- Sometimes it is not:

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

Lossy decomposition

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### Decompositions in General

$$R(A_1, \dots, A_n, B_1, \dots, B_m, C_1, \dots, C_p)$$

If  $A_1, \dots, A_n \rightarrow B_1, \dots, B_m$   
Then the decomposition is lossless

Note: don't need  $A_1, \dots, A_n \rightarrow C_1, \dots, C_p$

BCNF decomposition is always lossless. WHY ?

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