Introduction to Database Systems
CSE 444

Lecture 20:
Query Execution: Relational Algebra

November 16, 2007

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DBMS Architecture

How does a SQL engine work ?

- SQL query → relational algebra plan
- Relational algebra plan → Optimized plan
- Execute each operator of the plan

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Relational Algebra

- Formalism for creating new relations from existing ones
- Its place in the big picture:

Declartive query language → Algebra → Implementation

SQL, relational calculus

Relational algebra
Relational bag algebra

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Relational Algebra

- Five operators:
  - Union: $\cup$
  - Difference: -
  - Selection: σ
  - Projection: Π
  - Cartesian Product: $\times$
- Derived or auxiliary operators:
  - Intersection, complement
  - Joins (natural,equi-join, theta join, semi-join)
  - Renaming: ρ

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1. Union and 2. Difference

- R1 $\cup$ R2
- Example:
  - ActiveEmployees $\cup$ RetiredEmployees
- R1 – R2
- Example:
  - AllEmployees -- RetiredEmployees

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What about Intersection ?

- It is a derived operator
- R1 $\cap$ R2 = R1 – (R1 – R2)
- Also expressed as a join (will see later)
- Example
  - UnionizedEmployees $\cap$ RetiredEmployees

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3. Selection

- Returns all tuples which satisfy a condition
- Notation: $\sigma_c(R)$
- Examples
  - $\sigma_{\text{Salary} > 40000}$ (Employee)
  - $\sigma_{\text{name} = \text{"Smith"}}$ (Employee)
- The condition c can be =, <, ≤, >, ≥, <>

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SSN	Name	Salary
1234545	John	200000
5423341	Smith	600000
4352342	Fred	500000

$\sigma_{\text{Salary} > 40000}$ (Employee)

SSN	Name	Salary
5423341	Smith	600000
4352342	Fred	500000

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4. Projection

- Eliminates columns, then removes duplicates
- Notation: $\Pi_{A1,\ldots,An}(R)$
- Example: project social-security number and names:
  - $\Pi_{\text{SSN, Name}}$ (Employee)
  - Output schema: Answer(SSN, Name)

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SSN	Name	Salary
1234545	John	200000
5423341	John	600000
4352342	John	200000

$\Pi_{\text{Name,Salary}}$ (Employee)

Name	Salary
John	20000
John	60000

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5. Cartesian Product

- Each tuple in R1 with each tuple in R2
- Notation: R1 × R2
- Example:
  - Employee × Dependents
- Very rare in practice; mainly used to express joins

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Cartesian Product Example

Employee

Name	SSN
John	999999999
Tony	777777777

Dependents

EmployeeSSN	Dname
999999999	Emily
777777777	Joe

Employee x Dependents

Name	SSN	EmployeeSSN	Dname
John	999999999	999999999	Emily
John	999999999	777777777	Joe
Tony	777777777	999999999	Emily
Tony	777777777	777777777	Joe

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Relational Algebra

- Five operators:
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  - Difference: -
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Renaming

- Changes the schema, not the instance
- Notation: $\rho_{B1,\ldots,Bn}$ (R)
- Example:
  - $\rho_{LastName,\ SocSocNo}$ (Employee)
  - Output schema: Answer(LastName, SocSocNo)

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Renaming Example

Employee

Name	SSN
John	999999999
Tony	777777777

$\rho_{LastName,\ SocSocNo}$ **(Employee)**

LastName	SocSocNo
John	999999999
Tony	777777777

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Natural Join

- Notation: R1 $\bowtie$ R2
- Meaning: R1 $\bowtie$ R2 = $\Pi_A(\sigma_C(R1 \times R2))$
- Where:
  - The selection σ_C checks equality of all common attributes
  - The projection eliminates the duplicate common attributes

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Natural Join Example

Employee

Name	SSN
John	999999999
Tony	777777777

Dependents

SSN	Dname
999999999	Emily
777777777	Joe

Employee $\bowtie$ Dependents =

$\Pi_{Name,\ SSN,\ Dname}(\sigma_{SSN=SSN2}$(Employee x $\rho_{SSN2,\ Dname}$(Dependents))

Name	SSN	Dname
John	999999999	Emily
Tony	777777777	Joe

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Natural Join

- R=

A	B
X	Y
X	Z
Y	Z
Z	V

S=

B	C
Z	U
V	W
Z	V

- R $\bowtie$ S=

A	B	C
X	Z	U
X	Z	V
Y	Z	U
Y	Z	V
Z	V	W

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Natural Join

- Given the schemas R(A, B, C, D), S(A, C, E), what is the schema of $R \bowtie S$?
- Given R(A, B, C), S(D, E), what is $R \bowtie S$?
- Given R(A, B), S(A, B), what is $R \bowtie S$?

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Theta Join

- A join that involves a predicate
- $R1 \bowtie_{\theta} R2 \;=\; \sigma_{\theta}(R1 \times R2)$
- Here θ can be any condition

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Eq-join

- A theta join where θ is an equality
- $R \bowtie_{A=B} R2 \;=\; \sigma_{A=B}(R1 \times R2)$
- Example:
  - Employee $\bowtie_{SSN=SSN}$ Dependents
- Most useful join in practice

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Semijoin

- $R \ltimes S \;=\; \Pi_{A1,\ldots,An}(R \bowtie S)$
- Where $A_1, \ldots, A_n$ are the attributes in R
- Example:
  - Employee $\ltimes$ Dependents

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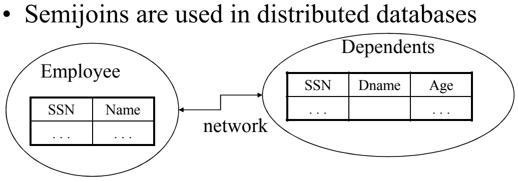
Semijoins in Distributed Databases

- Semijoins are used in distributed databases



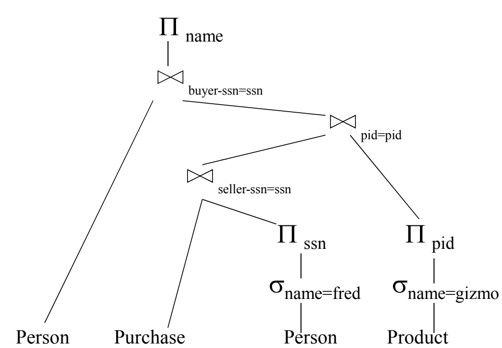
$\boxed{\text{Employee} \bowtie_{ssn=ssn} (\sigma_{age>71}(\text{Dependents}))}$

$T = \Pi_{SSN}\, \sigma_{age>71}(\text{Dependents})$

$R = \text{Employee} \ltimes T$

Answer = $R \bowtie$ Dependents

Complex RA Expressions



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Operations on Bags

A **bag** = a set with repeated elements

All operations need to be defined carefully on bags

- {a,b,b,c}∪{a,b,b,b,e,f,f}={a,a,b,b,b,b,b,c,e,f,f}
- {a,b,b,b,c,c} – {b,c,c,c,d} = {a,b,b,d}
- $\sigma_C(R)$: preserve the number of occurrences
- $\Pi_A(R)$: no duplicate elimination
- Cartesian product, join: no duplicate elimination

Important ! Relational Engines work on bags, not sets !

Reading assignment: 5.3 – 5.4

Note: RA has Limitations !

- Cannot compute "transitive closure"

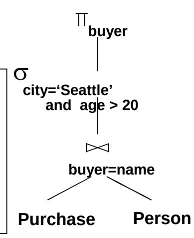
Name1	Name2	Relationship
Fred	Mary	Father
Mary	Joe	Cousin
Mary	Bill	Spouse
Nancy	Lou	Sister

- Find all direct and indirect relatives of Fred
- Cannot express in RA !!! Need to write C program

From SQL to RA

Purchase(buyer, product, city)
Person(name, age)

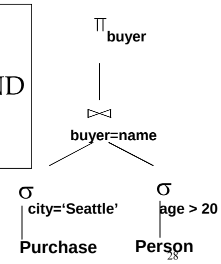
SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
P.city='Seattle' AND
Q.age > 20



Also…

Purchase(buyer, product, city)
Person(name, age)

SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
P.city='Seattle' AND
Q.age > 20



Non-monontone Queries (in class)

Purchase(buyer, product, city)
Person(name, age)

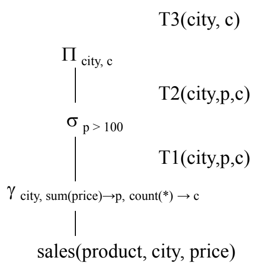
SELECT DISTINCT P.product
FROM Purchase P
WHERE P.city='Seattle' AND
not exists (select *
from Purchase P2, Person Q
where P2.product = P.product
and P2.buyer = Q.name
and Q.age > 20)

Extended Logical Algebra Operators (operate on Bags, not Sets)

- Union, intersection, difference
- Selection σ
- Projection Π
- Join $\bowtie$
- Duplicate elimination δ
- Grouping γ
- Sorting τ

Logical Query Plan

SELECT city, count(*)
FROM sales
GROUP BY city
HAVING sum(price) > 100



T1, T2, T3 = temporary tables

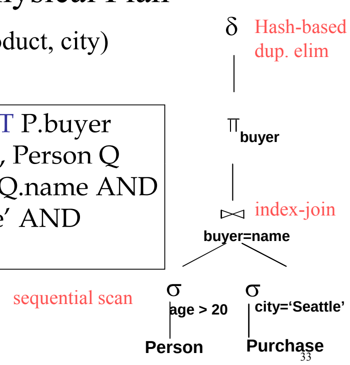
Logical v.s. Physical Algebra

- We have seen the logical algebra so far:
  - Five basic operators, plus group-by, plus sort
- The Physical algebra refines each operator into a concrete algorithm

Physical Plan

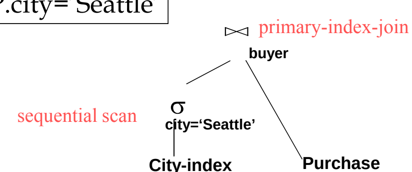
Purchase(buyer, product, city)
Person(name, age)

SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
P.city='Seattle' AND
Q.age > 20



Physical Plans Can Be Subtle

SELECT *
FROM Purchase P
WHERE P.city='Seattle'



Where did the join come from ?