

# Lecture 9: Database Design

Wednesday, January 25, 2006

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## Closure of a set of Attributes

**Given** a set of attributes  $A_1, \dots, A_n$

The **closure**,  $\{A_1, \dots, A_n\}^+$ , is the set of attributes B  
s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

name  $\rightarrow$  color  
category  $\rightarrow$  department  
color, category  $\rightarrow$  price

Closures:

name<sup>+</sup> = {name, color}

{name, category}<sup>+</sup> = {name, category, color, department, price}

color<sup>+</sup> = {color}

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# Closure Algorithm

Start with  $X = \{A_1, \dots, A_n\}$ .

**Repeat until X doesn't change do:**

**if**  $B_1, \dots, B_n \rightarrow C$  is a FD **and**  
 $B_1, \dots, B_n$  are all in X  
**then** add C to X.

Example:

$\text{name} \rightarrow \text{color}$   
 $\text{category} \rightarrow \text{department}$   
 $\text{color, category} \rightarrow \text{price}$

$\{\text{name, category}\}^+ =$   
 $\{\text{name, category, color, department, price}\}$

Hence:  $\text{name, category} \rightarrow \text{color, department, price}$

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# Example

In class:

$R(A, B, C, D, E, F)$

$A, B \rightarrow C$   
 $A, D \rightarrow E$   
 $B \rightarrow D$   
 $A, F \rightarrow B$

Compute  $\{A, B\}^+$   $X = \{A, B, \quad \quad \quad \}$

Compute  $\{A, F\}^+$   $X = \{A, F, \quad \quad \quad \}$

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## Why Do We Need Closure

- With closure we can find all FD's easily
- To check if  $X \rightarrow A$ 
  - Compute  $X^+$
  - Check if  $A \in X^+$

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## Using Closure to Infer ALL FDs

Example:

A, B	→	C
A, D	→	B
B	→	D

Step 1: Compute  $X^+$ , for every  $X$ :

$A^+ = A, B^+ = BD, C^+ = C, D^+ = D$ $AB^+ = ABCD, AC^+ = AC, AD^+ = ABCD$ $ABC^+ = ABD^+ = ACD^+ = ABCD$ (no need to compute– why ?) $BCD^+ = BCD, ABCD^+ = ABCD$
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Step 2: Enumerate all FD's  $X \rightarrow Y$ , s.t.  $Y \subseteq X^+$  and  $X \cap Y = \emptyset$ :

$AB \rightarrow CD, AD \rightarrow BC, ABC \rightarrow D, ABD \rightarrow C, ACD \rightarrow B$
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## Another Example

- Enrollment(student, major, course, room, time)  
student → major  
major, course → room  
course → time

What else can we infer ? [in class, or at home]

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## Back to Conceptual Design

Now we know how to find more FDs, it's easy

- Search for “bad” FDs
- If there are such, then decompose the table into two tables, repeat for the subtables.
- When done, the database schema is *normalized*

Unfortunately, there are several normal forms...

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# Normal Forms

**First Normal Form** = all attributes are atomic

**Second Normal Form (2NF)** = old and obsolete

**Third Normal Form (3NF)** = will discuss

**Boyce Codd Normal Form (BCNF)** = will discuss

Others...

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# Keys

- A **superkey** is a set of attributes  $A_1, \dots, A_n$  s.t. for any other attribute  $B$ , we have  $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey
  - I.e. set of attributes which is a superkey and for which no subset is a superkey

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## Computing (Super)Keys

- Compute  $X^+$  for all sets  $X$
- If  $X^+ =$  all attributes, then  $X$  is a key
- List only the minimal  $X$ 's

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## Example

Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

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## Example

Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

(name, category) + = name, category, price, color

Hence (name, category) is a key

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## Examples of Keys

Enrollment(student, address, course, room, time)

student  $\rightarrow$  address  
room, time  $\rightarrow$  course  
student, course  $\rightarrow$  room, time

(find keys at home)

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## Eliminating Anomalies

Main idea:

- $X \rightarrow A$  is OK if  $X$  is a (super)key
- $X \rightarrow A$  is not OK otherwise

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## Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$SSN \rightarrow Name, City$

What the key?

$\{SSN, PhoneNumber\}$

Hence  $SSN \rightarrow Name, City$   
is a “bad” dependency

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## Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

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## Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

$AB \rightarrow C$   
 $BC \rightarrow A$

or

$A \rightarrow BC$   
 $B \rightarrow AC$

what are the keys here ?

Can you design FDs such that there are *three* keys ?

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# Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if:

If  $A_1, \dots, A_n \rightarrow B$  is a non-trivial dependency  
in R, then  $\{A_1, \dots, A_n\}$  is a superkey for R

In other words: there are no “bad” FDs

Equivalently:

$\forall X$ , either  $(X^+ = X)$  or  $(X^+ = \text{all attributes})$

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# BCNF Decomposition Algorithm

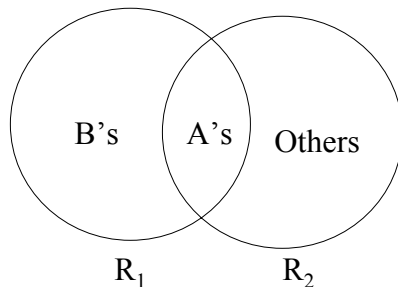
**repeat**

choose  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  that violates BCNF

split R into  $R_1(A_1, \dots, A_m, B_1, \dots, B_n)$  and  $R_2(A_1, \dots, A_m, [\text{others}])$

continue with both  $R_1$  and  $R_2$

**until** no more violations



Is there a  
2-attribute  
relation that is  
not in BCNF ?

In practice, we have  
a better algorithm (coming up<sup>20</sup>)

## Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name, City

What the key?

{SSN, PhoneNumber} use SSN → Name, City  
to split

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## Example

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN → Name, City

SSN	PhoneNumber
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy ?
- Update ?
- Delete ?

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## Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber)  
SSN  $\rightarrow$  name, age  
age  $\rightarrow$  hairColor

Decompose in BCNF (in class):

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## BCNF Decomposition Algorithm

BCNF\_Decompose(R)

find X s.t.:  $X \neq X^+ \neq$  [all attributes]

**if** (not found) **then** “R is in BCNF”

**let**  $Y = X^+ - X$

**let**  $Z =$  [all attributes] -  $X^+$

decompose R into  $R_1(X \cup Y)$  and  $R_2(X \cup Z)$

continue to decompose recursively  $R_1$  and  $R_2$

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Find X s.t.:  $X \neq X^+ \neq$  [all attributes]

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

Iteration 1: Person

SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: P

age<sup>+</sup> = age, hairColor

Decompose: People(SSN, name, age)

Hair(age, hairColor)

Phone(SSN, phoneNumber)

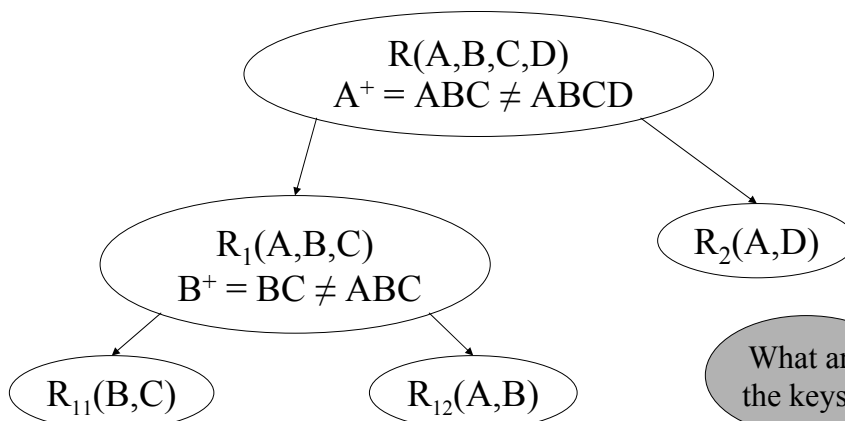
What are the keys ?

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R(A,B,C,D)

## Example

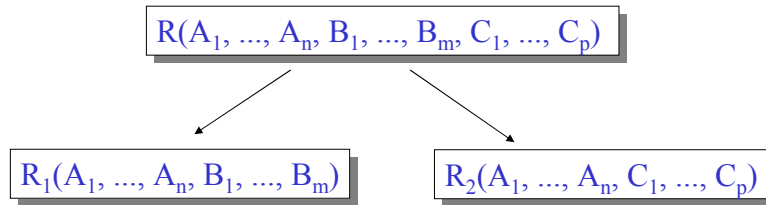
A  $\rightarrow$  B  
B  $\rightarrow$  C



What are the keys ?

What happens if in R we first pick B<sup>+</sup> ? Or AB<sub>26</sub><sup>+</sup> ?

## Decompositions in General

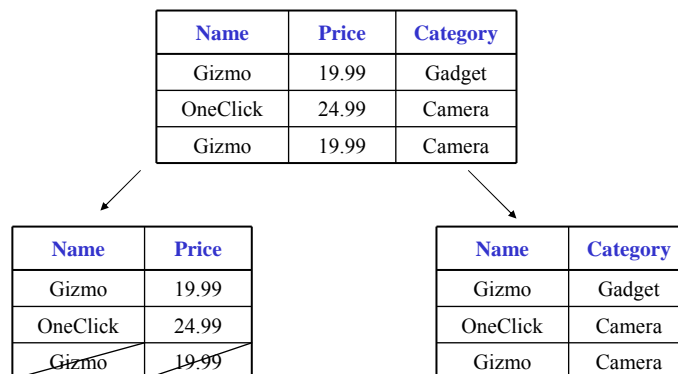


$R_1$  = projection of  $R$  on  $A_1, \dots, A_n, B_1, \dots, B_m$   
 $R_2$  = projection of  $R$  on  $A_1, \dots, A_n, C_1, \dots, C_p$

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## Theory of Decomposition

- Sometimes it is correct:

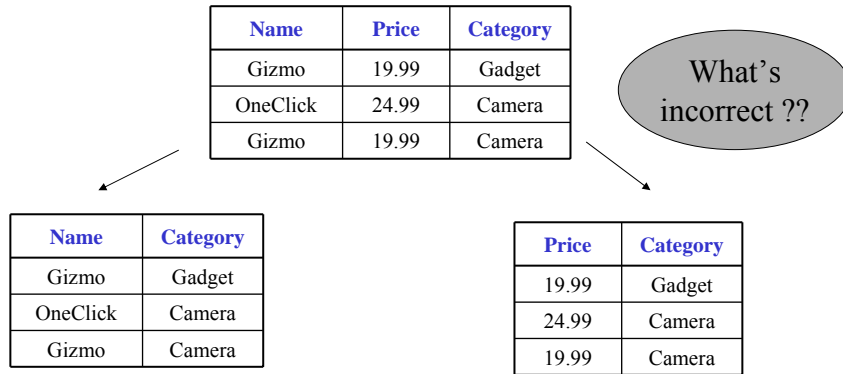


Lossless decomposition

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# Incorrect Decomposition

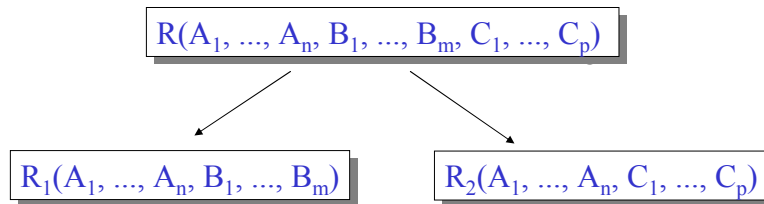
- Sometimes it is not:



Lossy decomposition

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# Decompositions in General



If  $A_1, \dots, A_n \rightarrow B_1, \dots, B_m$   
 Then the decomposition is lossless

Note: don't need  $A_1, \dots, A_n \rightarrow C_1, \dots, C_p$

BCNF decomposition is always lossless. WHY ?

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## 3NF: A Problem with BCNF

Unit	Company	Product

Unit → Company  
Company, Product → Unit

Unit<sup>+</sup> = Unit, Company

<u>Unit</u>	Company

Unit	Product

Unit → Company

We loose the FD: Company, Product → Unit !!

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## So What's the Problem?

<u>Unit</u>	Company
Galaga99	UW
Bingo	UW

Unit	Product
Galaga99	Databases
Bingo	Databases

Unit → Company

No problem so far. All *local* FD's are satisfied.

Let's put all the data back into a single table again:

Unit	Company	Product
Galaga99	UW	Databases
Bingo	UW	Databases

Violates the FD:

Company, Product → Unit

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## The Problem

- We started with a table R and FD
- We decomposed R into BCNF tables  $R_1, R_2, \dots$  with their own  $FD_1, FD_2, \dots$
- We can reconstruct R from  $R_1, R_2, \dots$
- But we cannot reconstruct FD from  $FD_1, FD_2, \dots$

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## Solution: 3rd Normal Form (3NF)

A simple condition for removing anomalies from relations:

A relation R is in 3rd normal form if :

Whenever there is a nontrivial dependency  $A_1, A_2, \dots, A_n \rightarrow B$  for R, then  $\{A_1, A_2, \dots, A_n\}$  a super-key for R,  
or B is part of a key.

Tradeoff:

BCNF = no anomalies, but may lose some FDs

3NF = keeps all FDs, but may have some anomalies

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## 3NF Decomposition Algorithm

3NF\_Decompose(R)

**let** K = [all attributes that are part of some key]

find X s.t.:  $X^+ - X - K \neq \emptyset$  and  $X^+ \neq$  [all attributes]

**if** (not found) **then** “R is already in 3NF”

**let** Y =  $X^+ - X - K$

**let** Z = [all attributes] -  $(X \cup Y)$

decompose into R1( $X \cup Y$ ) and R2( $X \cup Z$ )

decompose, recursively, R1 and R2

## Example of 3NF decomposition

R(A,B,C,D,E):

AB  $\rightarrow$  C

C  $\rightarrow$  D

D  $\rightarrow$  B

D  $\rightarrow$  E

Keys: (need to compute  $X^+$ , for several Xs)

AB, AC, AD

K = {A, B, C, D}

Pick X = C

$C^+ = BCDE$

C  $\rightarrow$  BDE is a BCNF violation

For 3NF: remove B, D (part of K):

C  $\rightarrow$  E is a 3NF violation

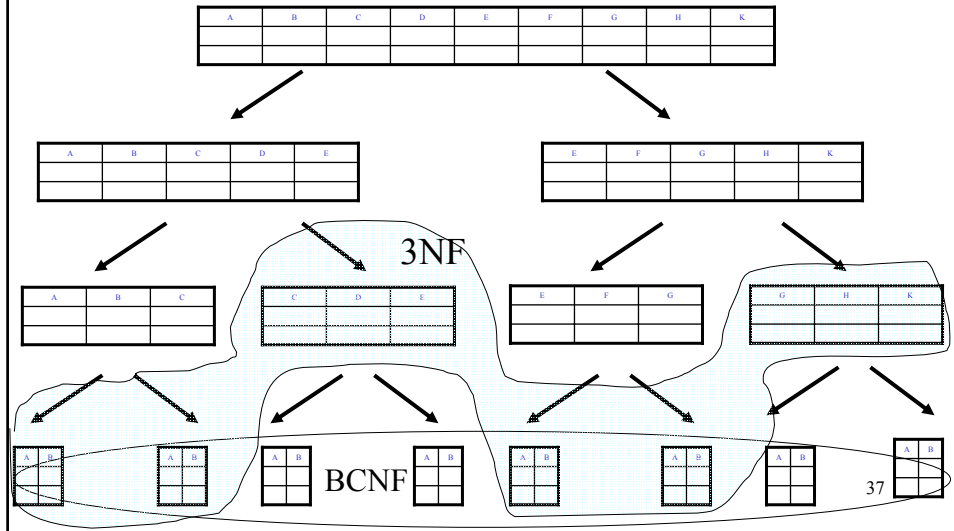
Decompose: R1(C, E), R2(A,B,C,D)

R1 is in 3NF

R2 is in 3NF (because its keys: AB, AC, AD)

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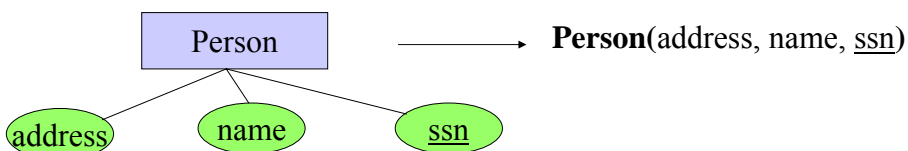
## 3NF v.s. BCNF Decomposition



## FD's for E/R Diagrams

Given a relation constructed from an E/R diagram, what is its key?

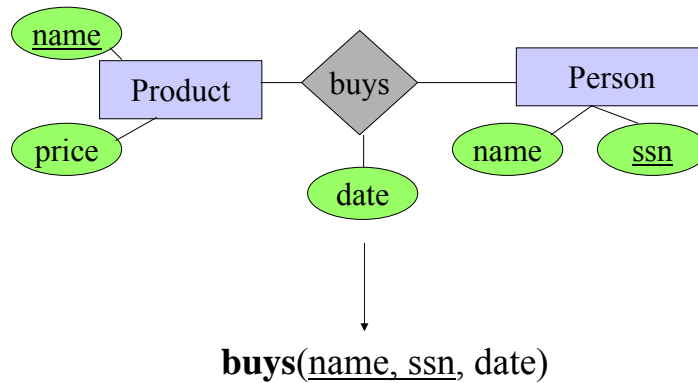
**Rule 1:** If the relation comes from an entity set, the key of the relation is the set of attributes which is the key of the entity set.



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## FD's for E/R Diagrams

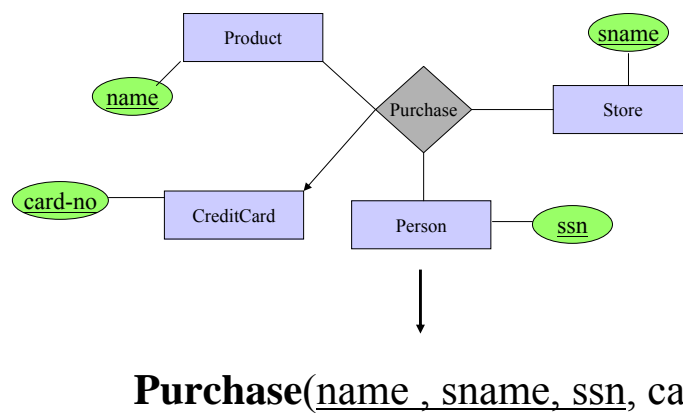
**Rule 2:** If the relation comes from a many-many relationship, the key of the relation is the set of all attribute keys in the relations corresponding to the entity sets



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## FD's for E/R Diagrams

**Except:** if there is an arrow from the relationship to E, then we don't need the key of E as part of the relation key.



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## FD's for E/R Diagrams

More rules:

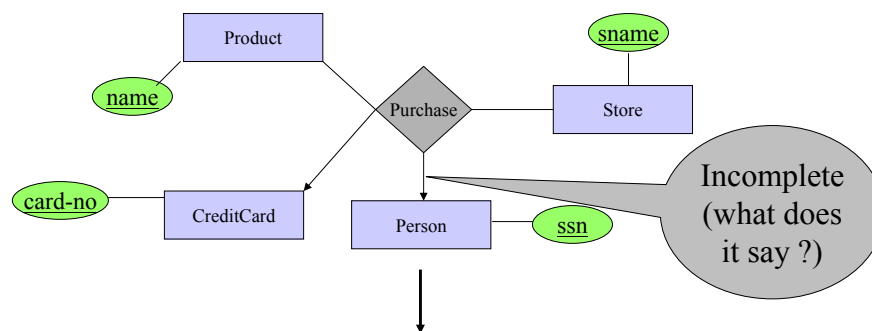
- Many-one, one-many, one-one relationships
- Multi-way relationships
- Weak entity sets

(Try to find them yourself, or check book)

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## FD's for E/R Diagrams

Say: "the CreditCard determines the Person"



**Purchase**(name, sname, ssn, card-no)

card-no → ssn

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