Lecture 6: Functional Dependencies

Friday, October 6 2000

Outline

- Definition and keys (3.5)
- Rules about FD's (3.6)
- Design of relational database schemas (3.7.1-3.7.6)

Functional Dependencies

Definition:

If two tuples agree on the attributes

$A_1, A_2, \ldots A_n$

then they must also agree on the attributes

$B_1, B_2, \ldots B_m$

Formally: $A_1, A_2, \ldots A_n \longrightarrow B_1, B_2, \ldots B_m$

Motivating example for the study of functional dependencies:

Name	Social Security Number	Phone Number

Examples

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E1847	John	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	lawyer

- EmpID $\longrightarrow$ Name, Phone, Position
- Position $\longrightarrow$ Phone
- but Phone $\not\longrightarrow$ Position

In General

- To check A $\longrightarrow$ B, erase all other columns

…	A	…	B	
	X1		Y1	
	X2		Y2	
	…		…	

- check if the remaining relation is many-one (called ***functional*** in mathematics)

Example

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E1847	John	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	lawyer

More Examples

Product: name → price, manufacturer
Person: ssn → name, age
Company: name → stock price, president

Key of a relation is a set of attributes that:

- functionally determines all the attributes of the relation
- none of its subsets determines all the attributes.

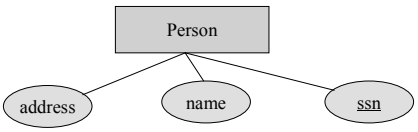
Superkey: a set of attributes that contains a key.

Finding the Keys of a Relation

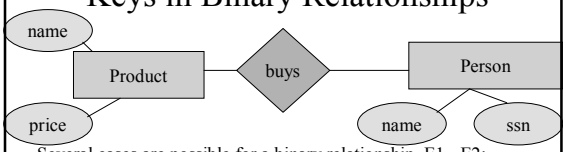
Given a relation constructed from an E/R diagram, what is its key?

Rules:

1. If the relation comes from an entity set, the key of the relation is the set of attributes which is the key of the entity set.



Keys in Binary Relationships



Several cases are possible for a binary relationship E1 - E2:

1. Many-many: the key includes the key of E1 together with the key of E2.

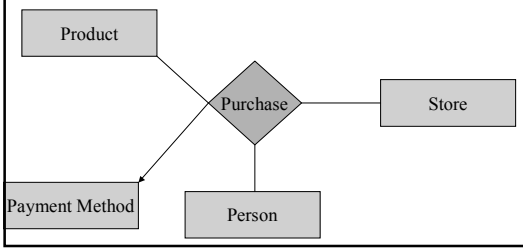
What happens for:

2. Many-one:

3. One-one:

Keys in Multiway Relationships

If there is an arrow from the relationship to E, then we don't need the key of E as part of the relation key.



Rules in FD's

Splitting/Combining Rule:

$A_1, A_2, \ldots A_n \longrightarrow B_1, B_2, \ldots B_m$ Is equivalent to

$A_1, A_2, \ldots A_n \longrightarrow B_1$
$A_1, A_2, \ldots A_n \longrightarrow B_2$
…
$A_1, A_2, \ldots A_n \longrightarrow B_m$

Splitting rule and Combing rule

Rules in FD's (continued)

Trivial Dependency

$A_1, A_2, \ldots A_n \longrightarrow A_i$ Always holds.

Why ?

Rules in FD's (continued)

Transitive Closure Rule:

If $A_1, A_2, \ldots A_n \longrightarrow B_1, B_2 \ldots, B_m$

and $B_1, B_2, \ldots B_m \longrightarrow C_1, C_2 \ldots, C_p$

then $A_1, A_2, \ldots A_n \longrightarrow C_1, C_2 \ldots, C_p$

Why ?

Closure of a set of Attributes

Given a set of attributes {***A1, …, An}*** and a set of dependencies S.
Problem: find all attributes *B* such that:
any relation which satisfies S also satisfies:
A1, …, An → B

The **closure** of *{A1, …, An}*, denoted *{A1, …, An}*$^+$, is the set of all such attributes *B*

Closure Algorithm

Start with X={A1, …, An}.

Repeat until X doesn't change **do**:

if $B_1, B_2, \ldots B_n \longrightarrow C$ is in S, **and**
$B_1, B_2 \ldots B_n$ are all in X, **and**
C is not in X

then

add C to X.

Example

A B → C
A D → E
B → D
A F → B

Closure of {A,B}: X = {A, B, }

Closure of {A, F}: X = {A, F, }

Why Is the Algorithm Correct ?

- Show the following by induction:
  - For every *B* in *X*:
    - *A1, …, An → B*
- Initially *X = {A1, …, An}* -- holds
- Induction step: *B1, …, Bm* in *X*
  - Implies *A1, …, An → B1, …, Bm*
  - We also have *B1, …, Bm → C*
  - By transitivity we have *A1, …, An → C*
- This shows that the algorithm is *sound*; need to show it is *complete*

Relational Schema Design

Main idea:

- Start with some relational schema
- Find out its FD's
- Use them to design a better relational schema

Relational Schema Design

Recall set attributes (persons with several phones):

Name	SSN	Phone Number
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(206) 572-4312
Joe	909-438-44	(908) 464-0028
Joe	909-438-44	(212) 555-4000

Problems:

- redundancy
- update anomalies
- deletion anomalies

Note: SSN is NOT a key here

Relation Decomposition

Break the relation into two:

SSN	Name
123-321-99	Fred
909-438-44	Joe

SSN	Phone Number
123-321-99	(201) 555-1234
123-321-99	(206) 572-4312
909-438-44	(908) 464-0028
909-438-44	(212) 555-4000

Decompositions in General

Let *R* be a relation with attributes $A_1, A_2, \ldots A_n$

Create two relations *R1* and *R2* with attributes

$B_1, B_2, \ldots B_m$ $C_1, C_2, \ldots C_1$

Such that:

$B_1, B_2, \ldots B_m \cup C_1, C_2, \ldots C_1 = A_1, A_2, \ldots A_n$

And

-- *R1* is the projection of *R* on $B_1, B_2, \ldots B_m$

-- *R2* is the projection of *R* on $C_1, C_2, \ldots C_1$

Incorrect Decomposition

- Sometimes it is incorrect:

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
DoubleClick	29.99	Camera

Decompose on : **Name, Category** and **Price, Category**

Incorrect Decomposition

Name	Category
Gizmo	Gadget
OneClick	Camera
DoubleClick	Camera

Price	Category
19.99	Gadget
24.99	Camera
29.99	Camera

When we put it back:

Cannot recover information

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
OneClick	29.99	Camera
DoubleClick	24.99	Camera
DoubleClick	29.99	Camera

Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

A relation R is in BCNF if and only if:

Whenever there is a nontrivial dependency $A_1, A_2, \ldots A_n \rightarrow B$ for R , it is the case that $\{ A_1, A_2, \ldots A_n \}$ a super-key for R.

In English (though a bit vague):

Whenever a set of attributes of *R* is determining another attribute, should determine ***all*** the attributes of *R*.

Example

Name	SSN	Phone Number
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(206) 572-4312
Joe	909-438-44	(908) 464-0028
Joe	909-438-44	(212) 555-4000

What are the dependencies?

What are the keys?

Is it in BCNF?

And Now?

SSN	Name
123-321-99	Fred
909-438-44	Joe

SSN	Phone Number
123-321-99	(201) 555-1234
123-321-99	(206) 572-4312
909-438-44	(908) 464-0028
909-438-44	(212) 555-4000

What About This?

Name	Price	Category
Gizmo	$19.99	gadgets

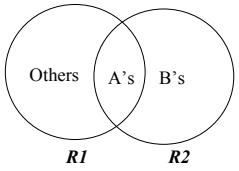
Question:

Find an example of a 2-attribute relation that is not in BCNF.

More Careful Strategy

Find a dependency that violates the BCNF condition:

$A_1, A_2, \ldots A_n \longrightarrow B_1, B_2, \ldots B_m$



Example Decomposition

Name	Social-security-number	Age	Eye Color	Phone Number

Functional dependencies:

Name + Social-security-number $\longrightarrow$ Age, Eye Color

What if we also had an attribute Draft-worthy, and the FD:

Age $\longrightarrow$ Draft-worthy

Decomposition Based on BCNF is Necessarily Correct

Attributes A, B, C. FD: A $\longrightarrow$ C

Relations R1[A,B] R2[A,C]

Tuples in R1: (a,b), (a,b')

Tuples in R2: (a,c), (a,c')

Tuples in the join of R1 and R2: (a,b,c), (a,b,c'), (a,b',c), (a,b',c')

Can (a,b,c') be a bogus tuple? What about (a,b',c') ?