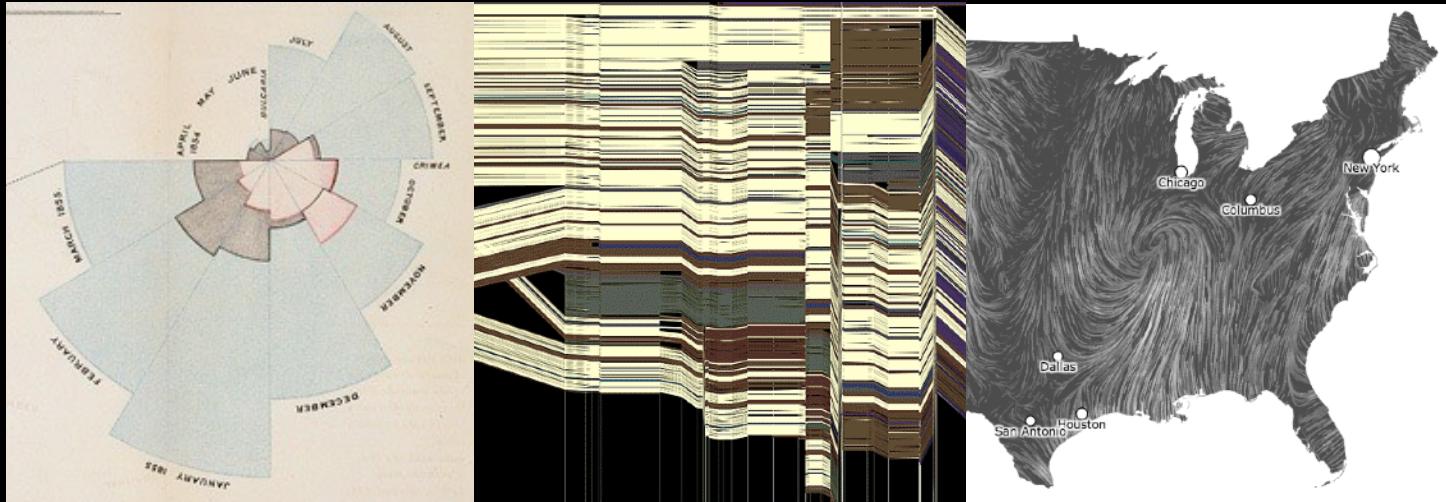


CSE 442 - Data Visualization

Dimensionality Reduction



Jeffrey Heer University of Washington

Dimensionality Reduction

Dimensionality Reduction (DR)

Project nD data to 2D or 3D for viewing. Often used to interpret and sanity check high-dimensional representations fit by machine learning methods.

Different DR methods make different trade-offs: for example to **preserve global structure** (e.g., PCA) or **emphasize local structure** (e.g., nearest-neighbor approaches, including t-SNE and UMAP).

In contrast, multidimensional scaling (MDS) attempts to **preserve pairwise distances**.

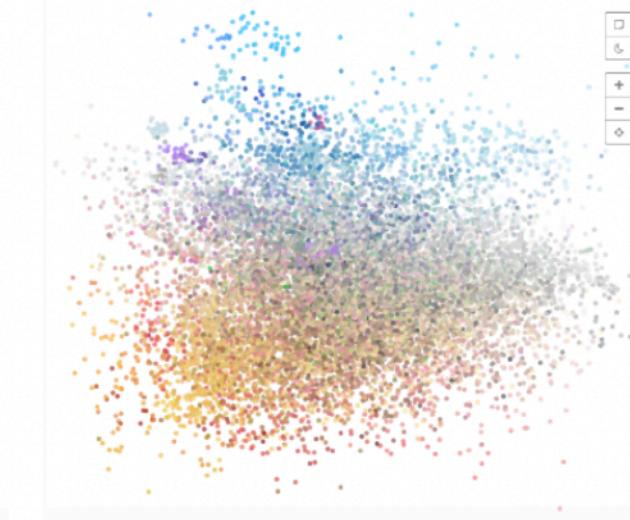
Mapping Emoji Images



t-SNE



UMAP



PCA

Reduction Techniques

LINEAR - PRESERVE GLOBAL STRUCTURE

Principal Components Analysis (PCA)

Linear transformation of basis vectors, ordered by amount of data variance they explain.

NON-LINEAR - PRESERVE LOCAL TOPOLOGY

t-Dist. Stochastic Neighbor Embedding (t-SNE)

Probabilistically model distance, optimize positions.

Uniform Manifold Approx. & Projection (UMAP)

Identify local manifolds, then stitch them together.

Dimensionality Reduction Issues

Reproducible?

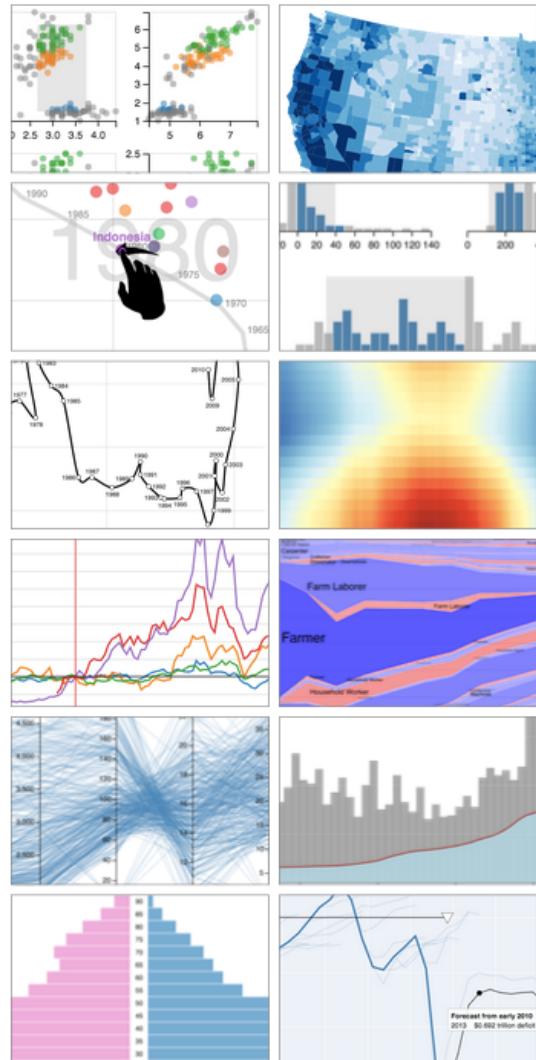
Projections are *data-dependent*. Fitting a new projection with different data can give rise to different results.

Reusable?

PCA and UMAP provide reusable projection functions that can map new points from high-D to low-D. t-SNE (and others, like MDS) do not provide this.

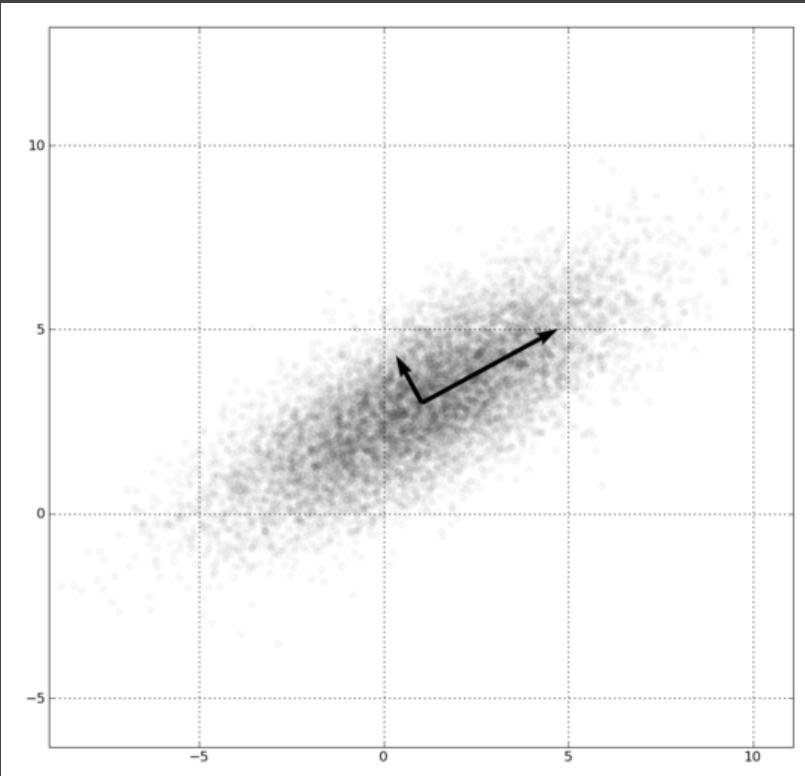
Interpretable?

DR plots are hard to interpret! Try multiple methods and hyperparameter settings. Inspect via interaction!



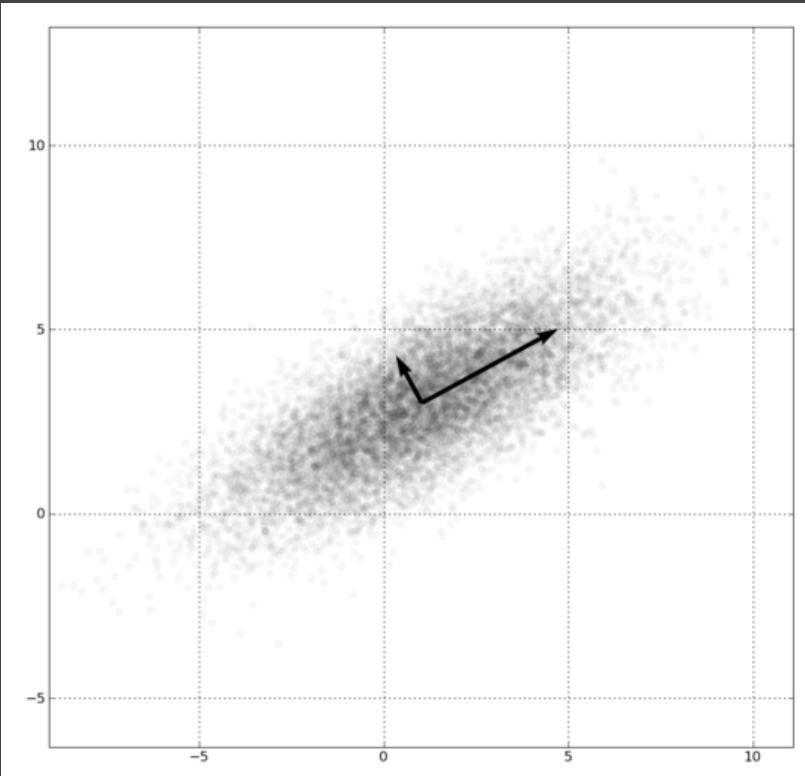


Principal Components Analysis



1. Mean-center the data.
2. Find \perp basis vectors that maximize the data variance.
3. Plot the data using the top vectors.

Principal Components Analysis



Linear transform:
scale and rotate
original space.

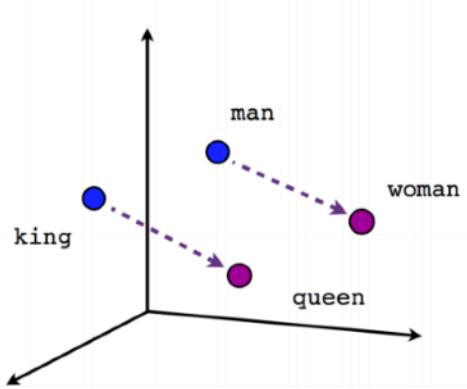
Lines (vectors)
project to lines.

Preserves global
distances.

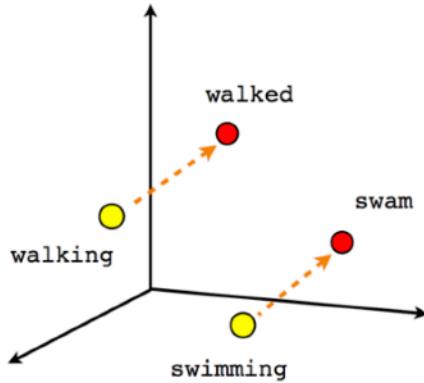
PCA of Genomes [Demiralp et al. '13]



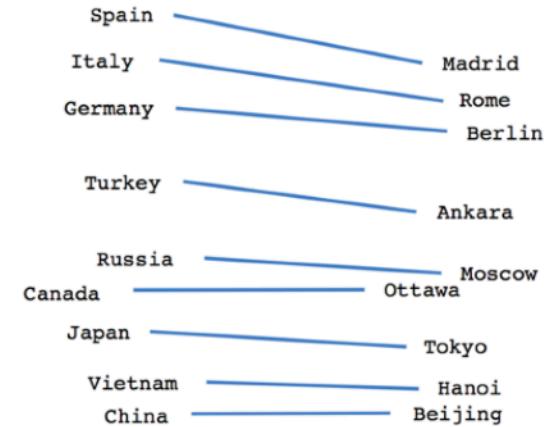
Vector Space Word Embeddings (word2vec, GloVe)



Male-Female



Verb tense

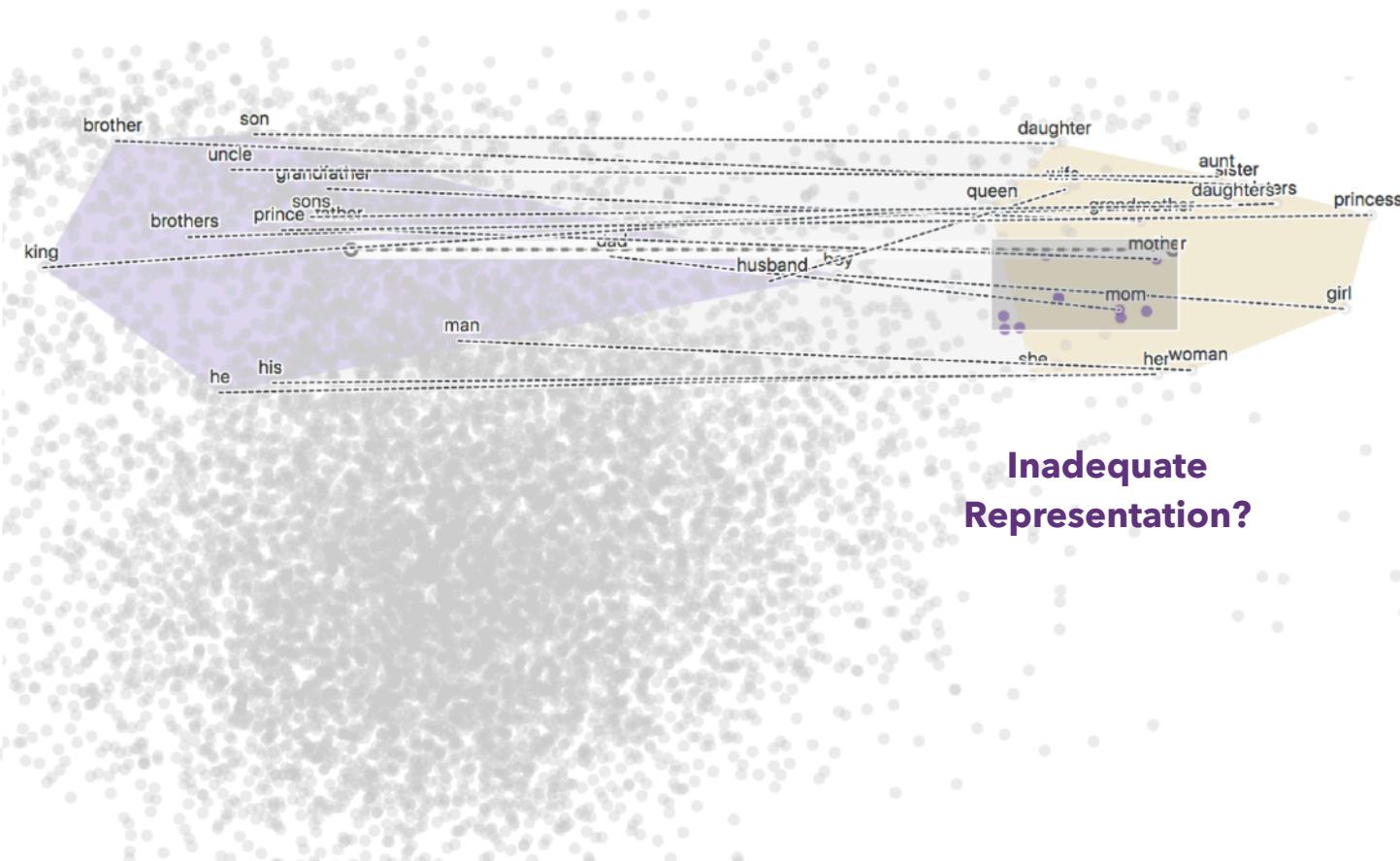


Country-Capital

Mapping Machine-Learned Latent Spaces

[Liu et al. 2019]

| Brushed | |
|-------------|--------------|
| mother | + |
| ms. | + |
| wedding | + |
| pink | Bias? |
| mom | + |
| nurse | + |
| bedroom | + |
| ladies | + |
| householder | + |
| butterfly | + |



Non-Linear Techniques

Distort the space, trade-off preservation of global structure to emphasize local neighborhoods. Use topological (nearest neighbor) analysis.

Two popular contemporary methods:

t-SNE - probabilistic interpretation of distance

UMAP - tries to balance local/global trade-off

t-SNE

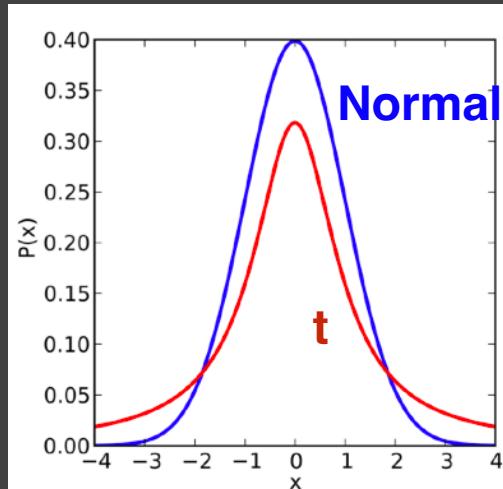
[Maaten & Hinton 2008]

1. Model probability \mathbf{P} of one point “choosing” another as its neighbor in the original space, using a Gaussian distribution defined using the distance between points. Nearer points have higher probability than distant ones.

t-SNE

[Maaten & Hinton 2008]

2. Define a similar probability \mathbf{Q} in the low-dimensional (2D or 3D) embedding space, using a Student's t distribution (hence the "t-" in "t-SNE"!). The t -distribution is heavy-tailed, allowing distant points to be even further apart.



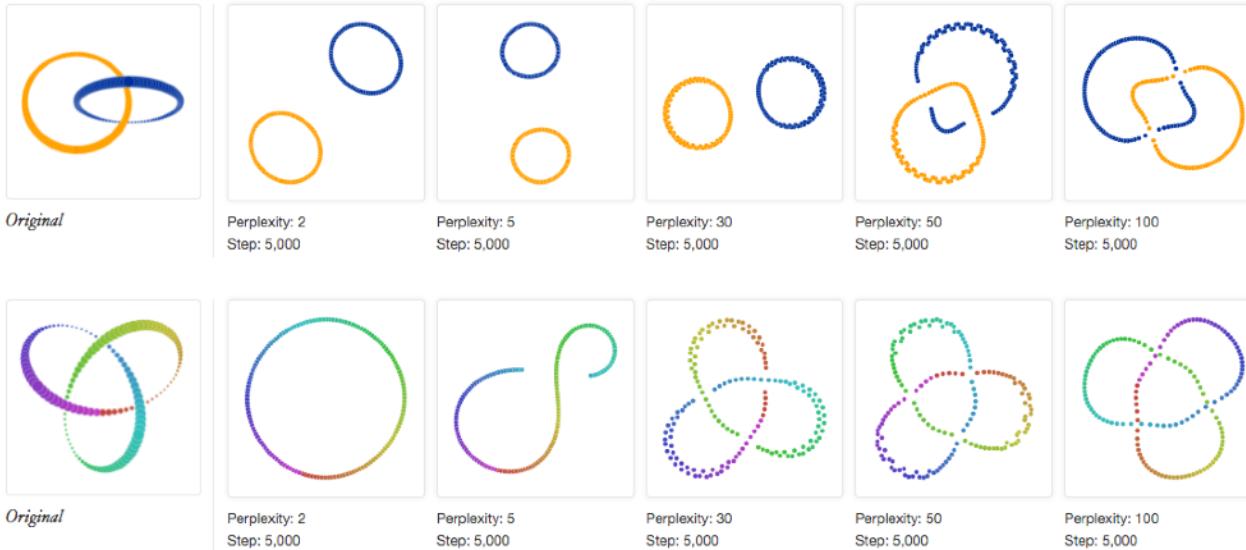
t-SNE

[Maaten & Hinton 2008]

1. Model probability \mathbf{P} of one point “choosing” another as its neighbor in the original space, using a Gaussian distribution defined using the distance between points. Nearer points have higher probability than distant ones.
2. Define a similar probability \mathbf{Q} in the low-dimensional (2D or 3D) embedding space, using a Student’s t distribution (*hence the “ t -” in “t-SNE”!*). The t -distribution is heavy-tailed, allowing distant points to be even further apart.
3. Optimize to find the positions in the embedding space that minimize the Kullback-Leibler divergence between the \mathbf{P} and \mathbf{Q} distributions: $KL(P \parallel Q)$

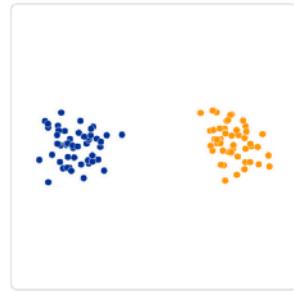
Visualizing t-SNE

[Wattenberg et al. '16]

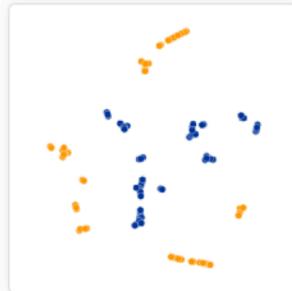


Results can be highly sensitive to the algorithm parameters!
Are you seeing real structures, or algorithmic hallucinations?

Hyperparameters matter!



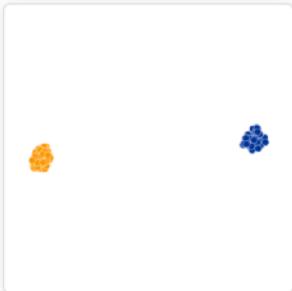
Original



Perplexity: 2
Step: 5,000



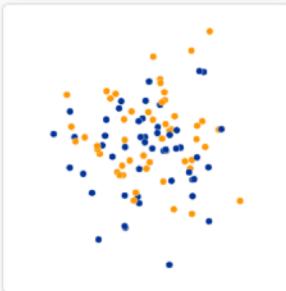
Perplexity: 5
Step: 5,000



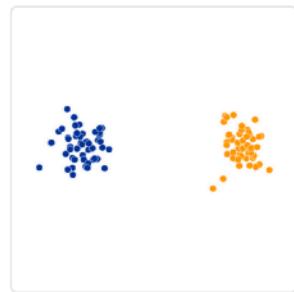
Perplexity: 30
Step: 5,000



Perplexity: 50
Step: 5,000



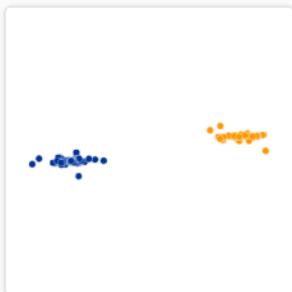
Perplexity: 100
Step: 5,000



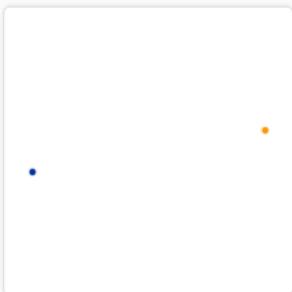
Original



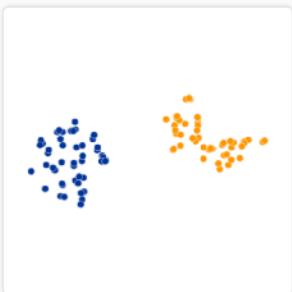
Perplexity: 30
Step: 10



Perplexity: 30
Step: 20



Perplexity: 30
Step: 60

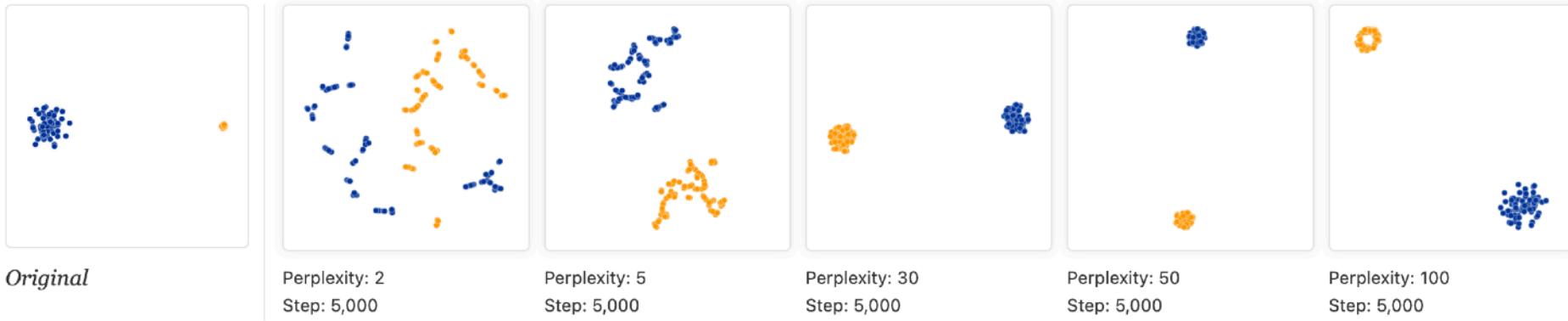


Perplexity: 30
Step: 120



Perplexity: 30
Step: 1,000

Cluster sizes mean nothing...



Cluster distances are illusive



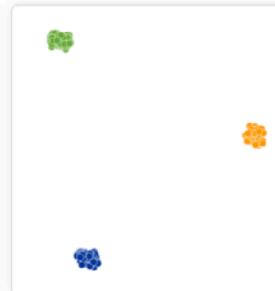
Original



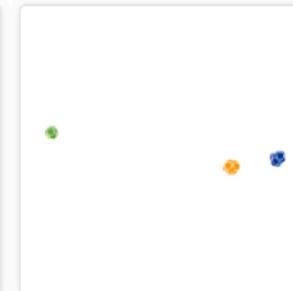
Perplexity: 2
Step: 5,000



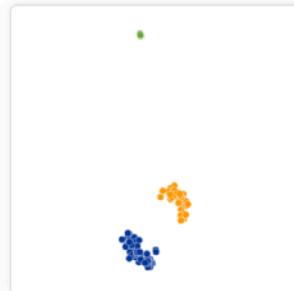
Perplexity: 5
Step: 5,000



Perplexity: 30
Step: 5,000



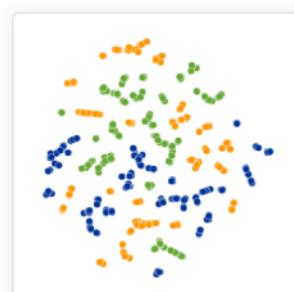
Perplexity: 50
Step: 5,000



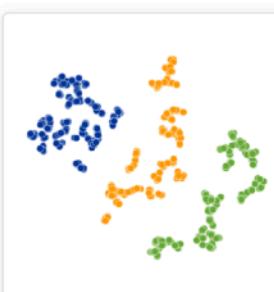
Perplexity: 100
Step: 5,000



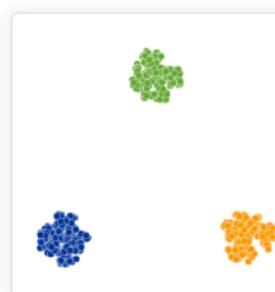
Original



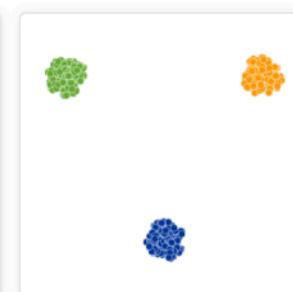
Perplexity: 2
Step: 5,000



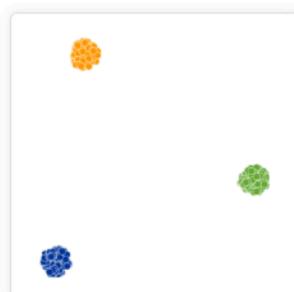
Perplexity: 5
Step: 5,000



Perplexity: 30
Step: 5,000

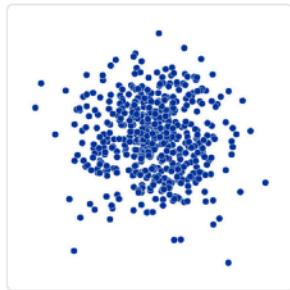


Perplexity: 50
Step: 5,000

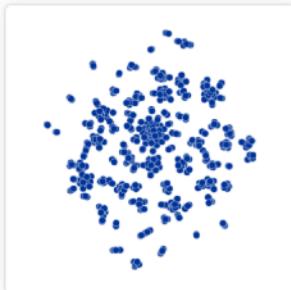


Perplexity: 100
Step: 5,000

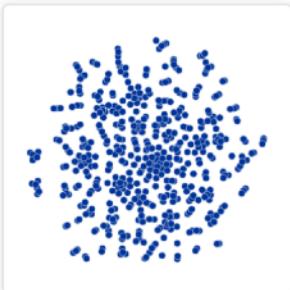
Random noise may not look like it



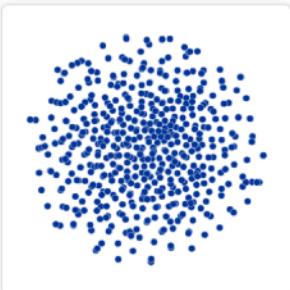
Original



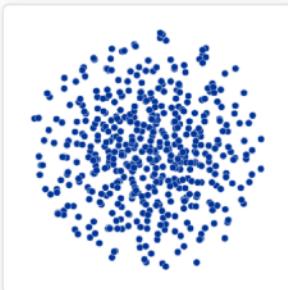
Perplexity: 2
Step: 5,000



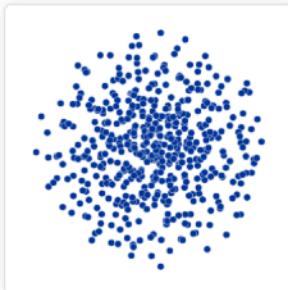
Perplexity: 5
Step: 5,000



Perplexity: 30
Step: 5,000

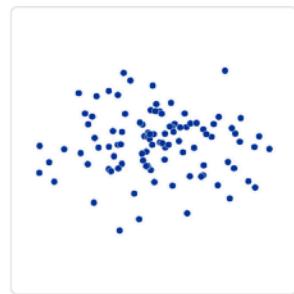


Perplexity: 50
Step: 5,000

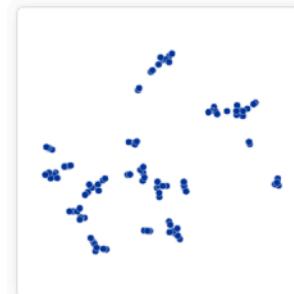


Perplexity: 100
Step: 5,000

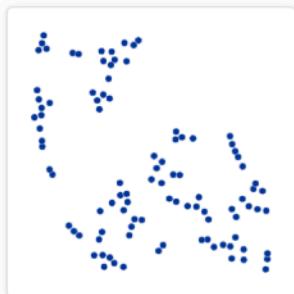
You can see shapes, sometimes



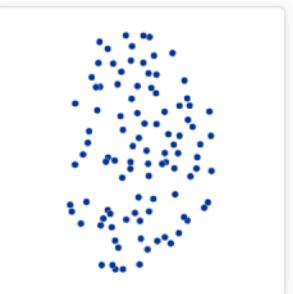
Original



Perplexity: 2
Step: 5,000



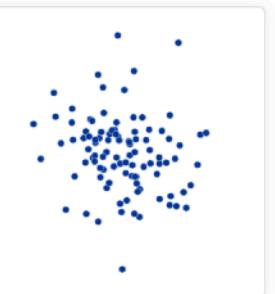
Perplexity: 5
Step: 5,000



Perplexity: 30
Step: 5,000



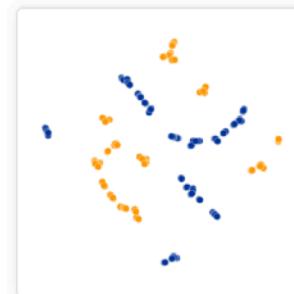
Perplexity: 50
Step: 5,000



Perplexity: 100
Step: 5,000



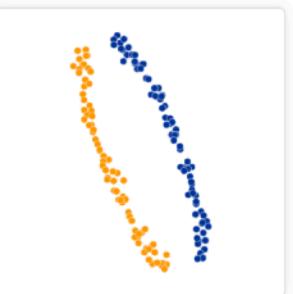
Original



Perplexity: 2
Step: 5,000



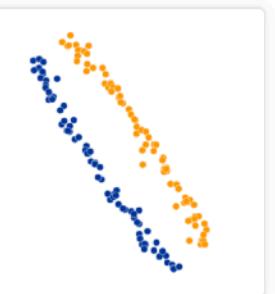
Perplexity: 5
Step: 5,000



Perplexity: 30
Step: 5,000

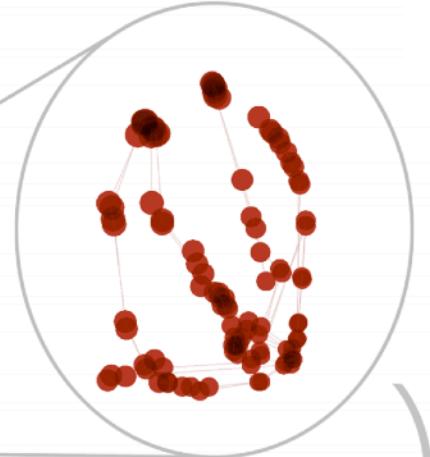
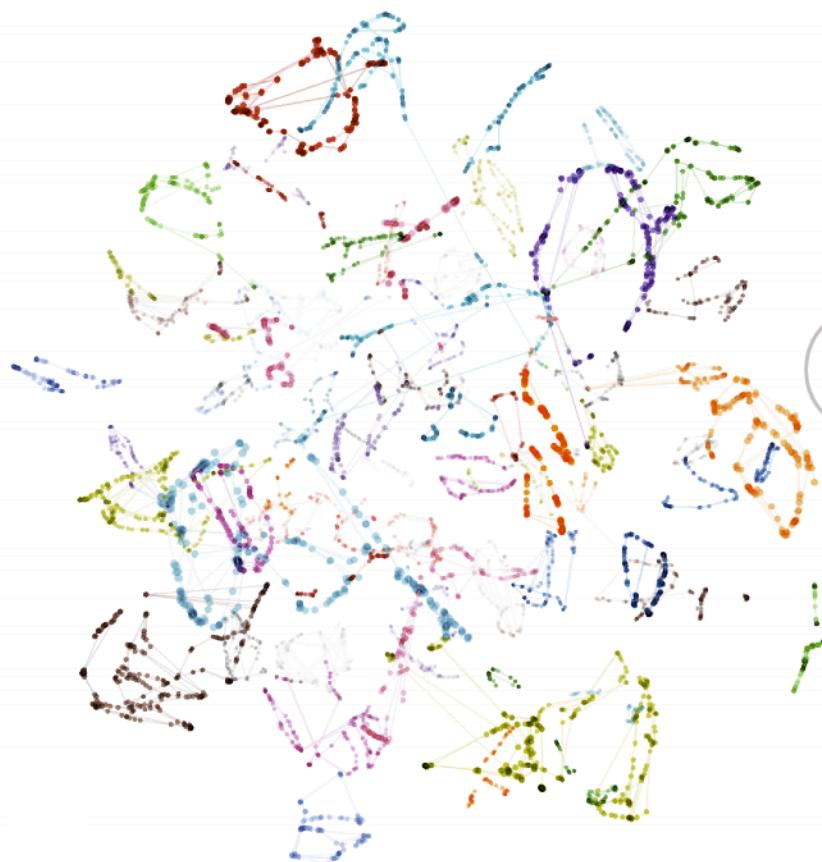


Perplexity: 50
Step: 5,000



Perplexity: 100
Step: 5,000

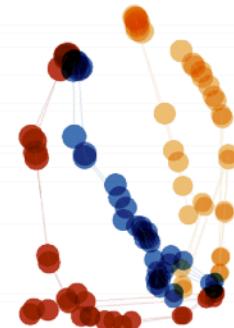
Multi-Lingual Word Embedding [Google 2016]



ENGLISH
The stratosphere extends from about 10km to about 50km in altitude.

KOREAN
성층권은 고도 약 10km부터 약 50km까지 확장됩니다.

JAPANESE
成層圏は、高度 10km から 50km の範囲にあります。



UMAP

[McInnes et al. 2018]

Form weighted nearest neighbor graph, then layout the graph in a manner that balances embedding of local and global structure.

“Our algorithm is competitive with t-SNE for visualization quality and arguably preserves more of the global structure with superior run time performance.” - McInnes et al. 2018

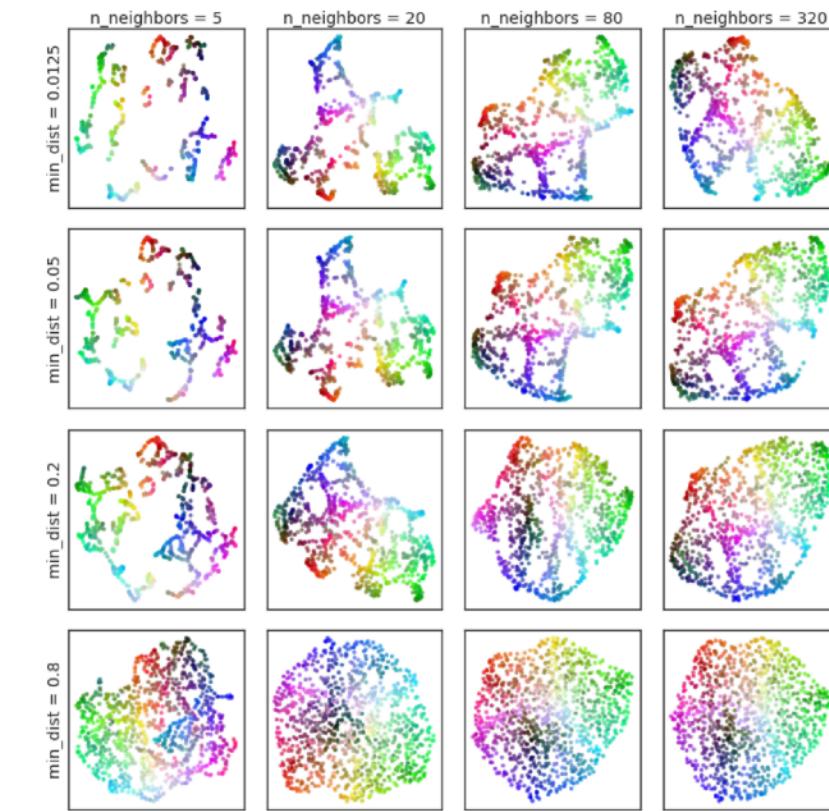


Figure 1: Variation of UMAP hyperparameters n and min_dist result in different embeddings. The data is uniform random samples from a 3-dimensional color-cube, allowing for easy visualization of the original 3-dimensional coordinates in the embedding space by using the corresponding RGB colour. Low values of n spuriously interpret structure from the random sampling noise – see Section 6 for further discussion of this phenomena.

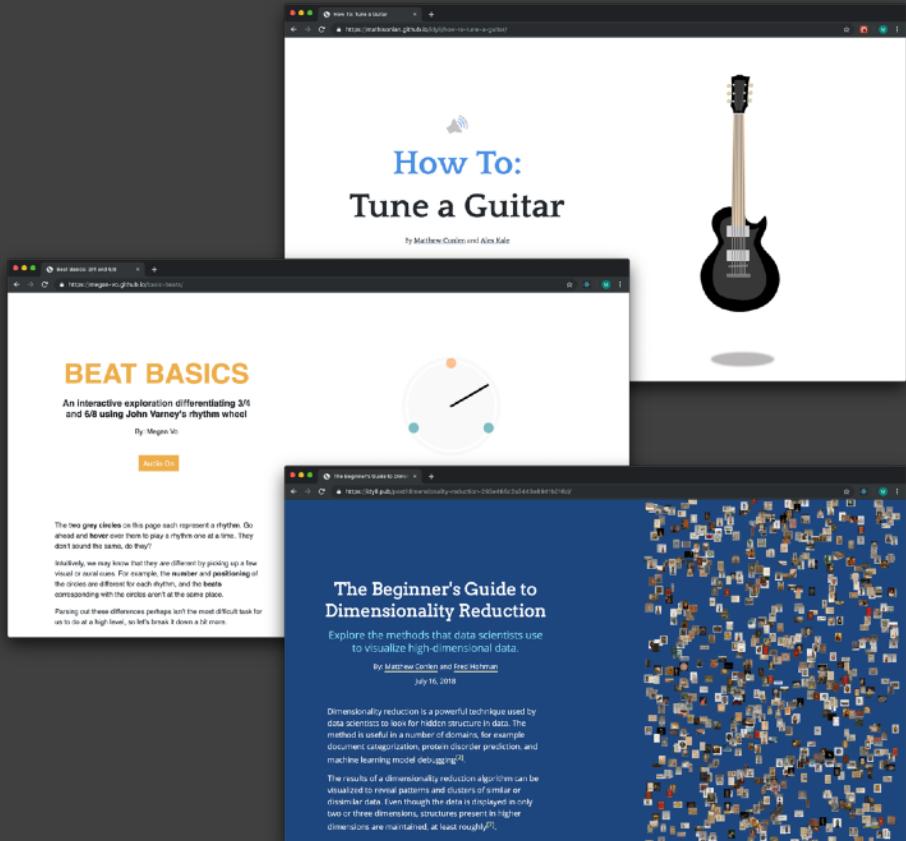
User Engagement with Interactive Articles

Provide an overview of usage patterns of interactive features.

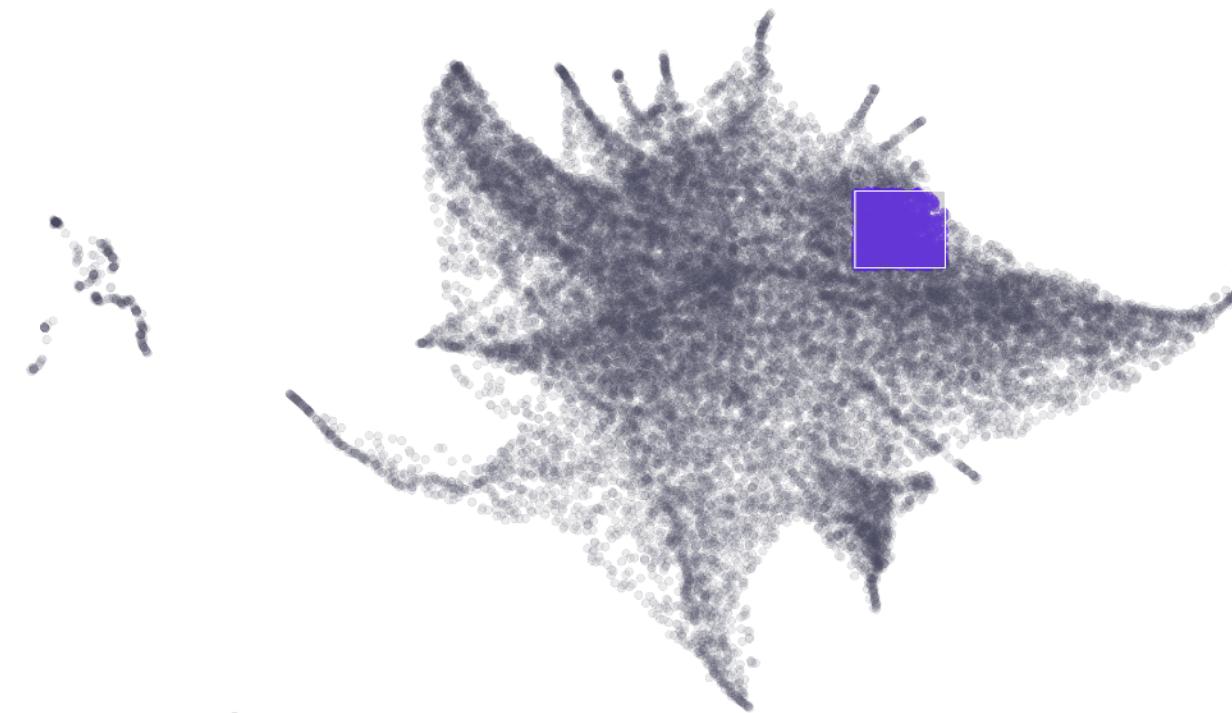
Identify variations in usage

Represent reader sessions as a feature vector with:

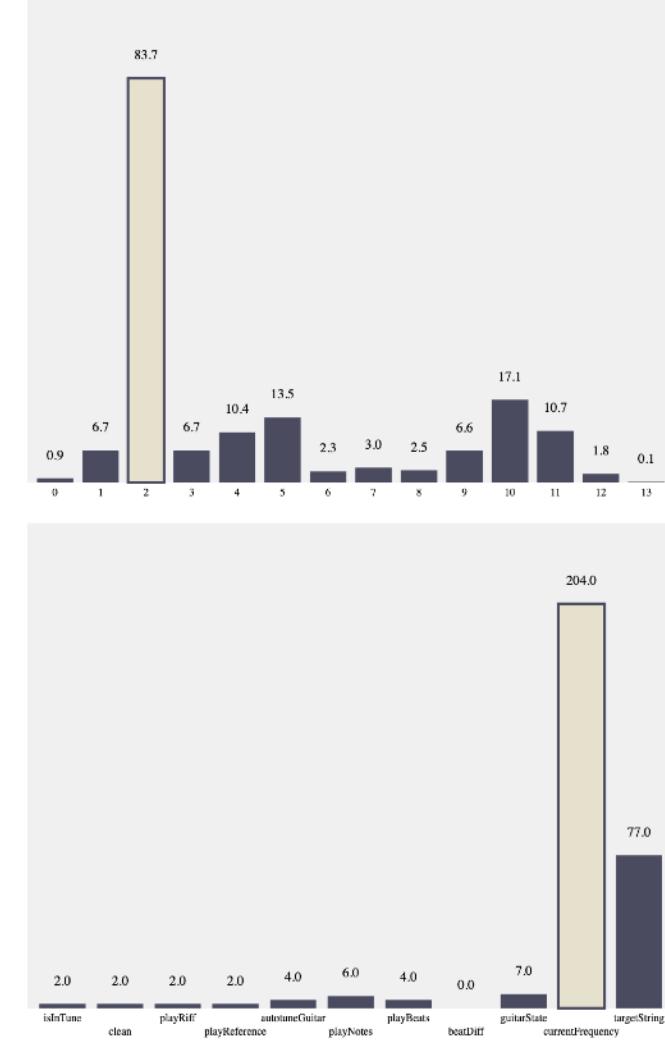
- time spent in each section
- count of variable changes



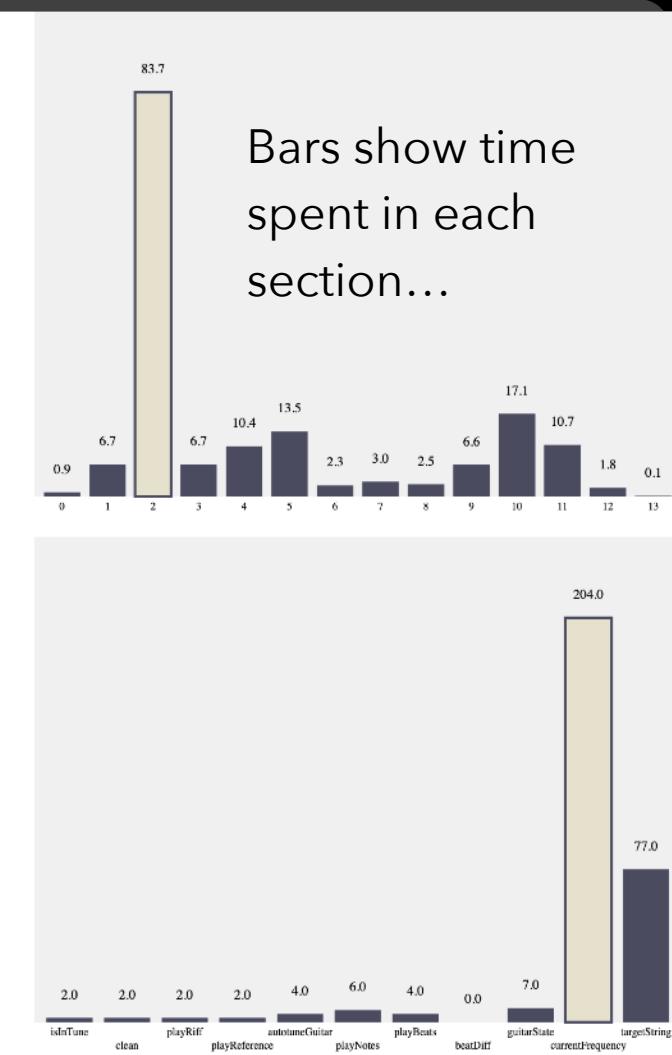
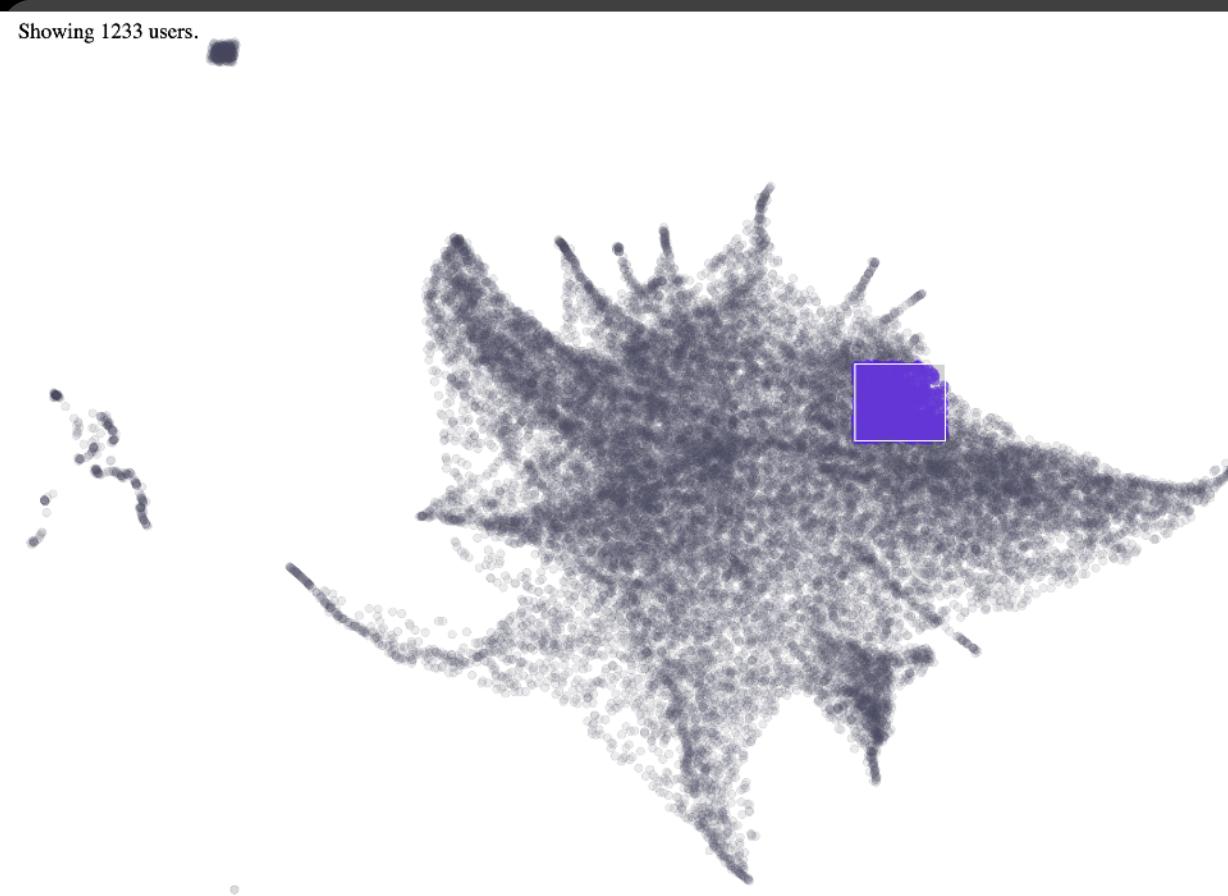
Showing 1233 users.



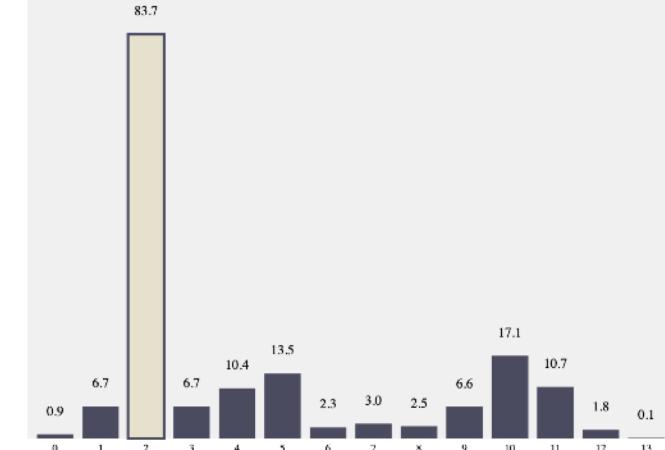
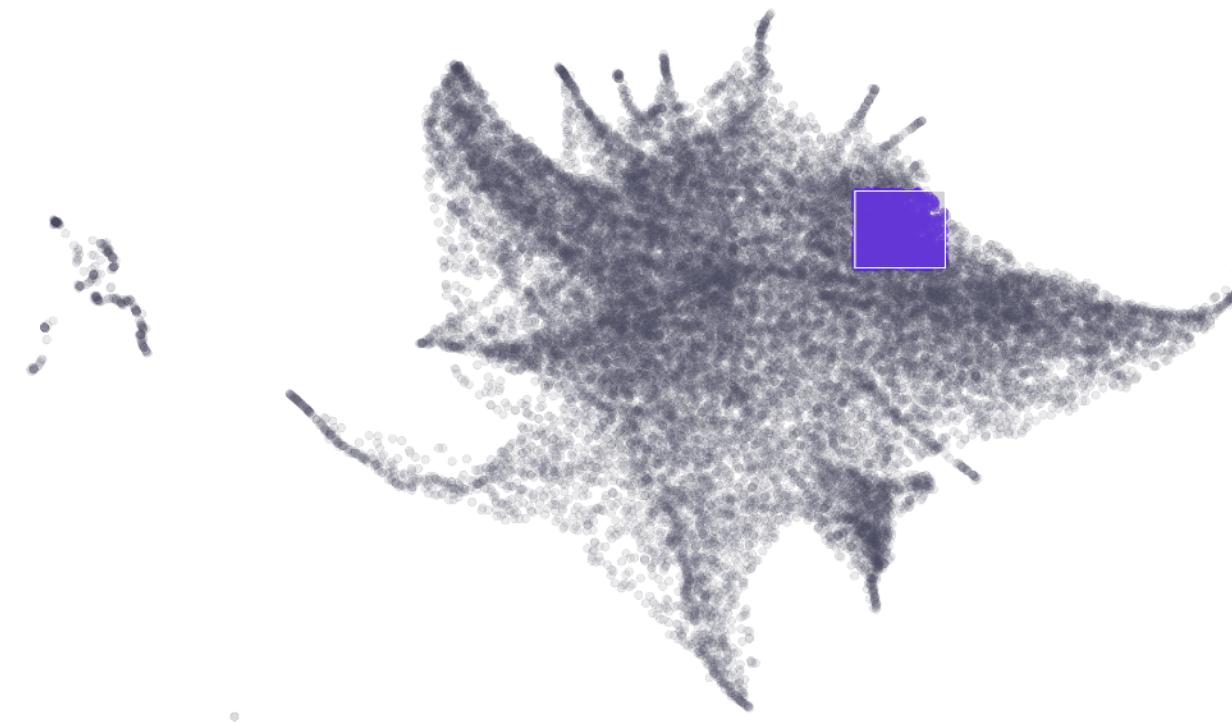
Each point represents a readers session, projected via UMAP.



Showing 1233 users.



Showing 1233 users.

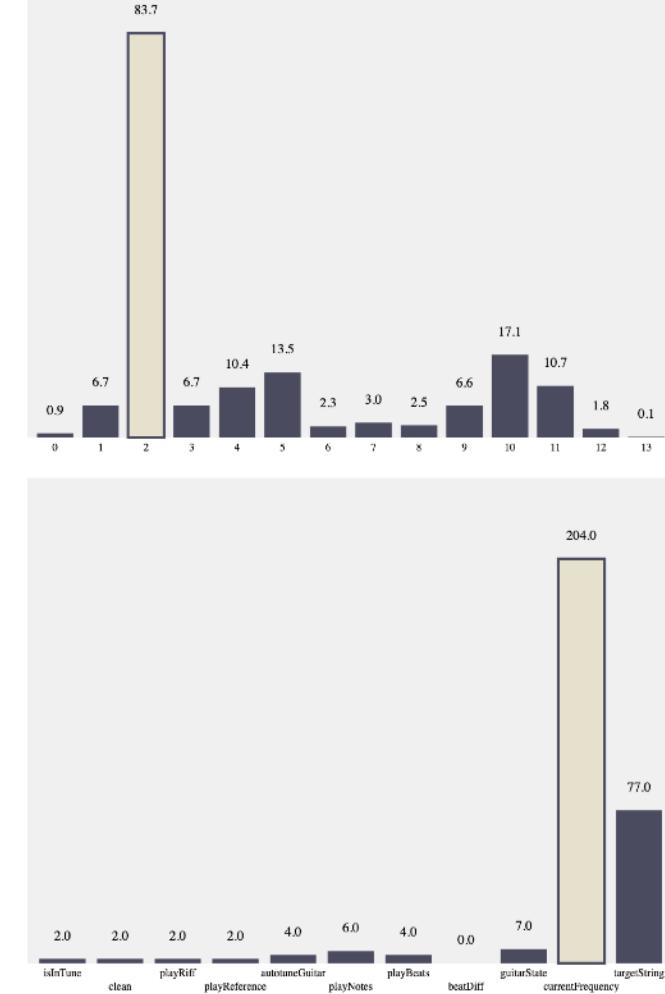
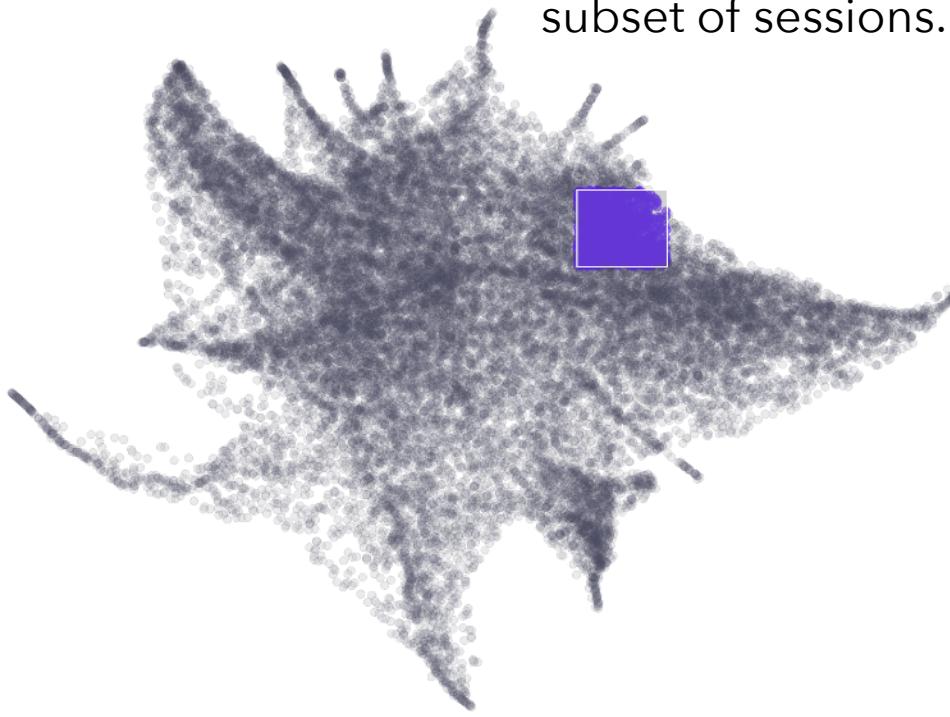


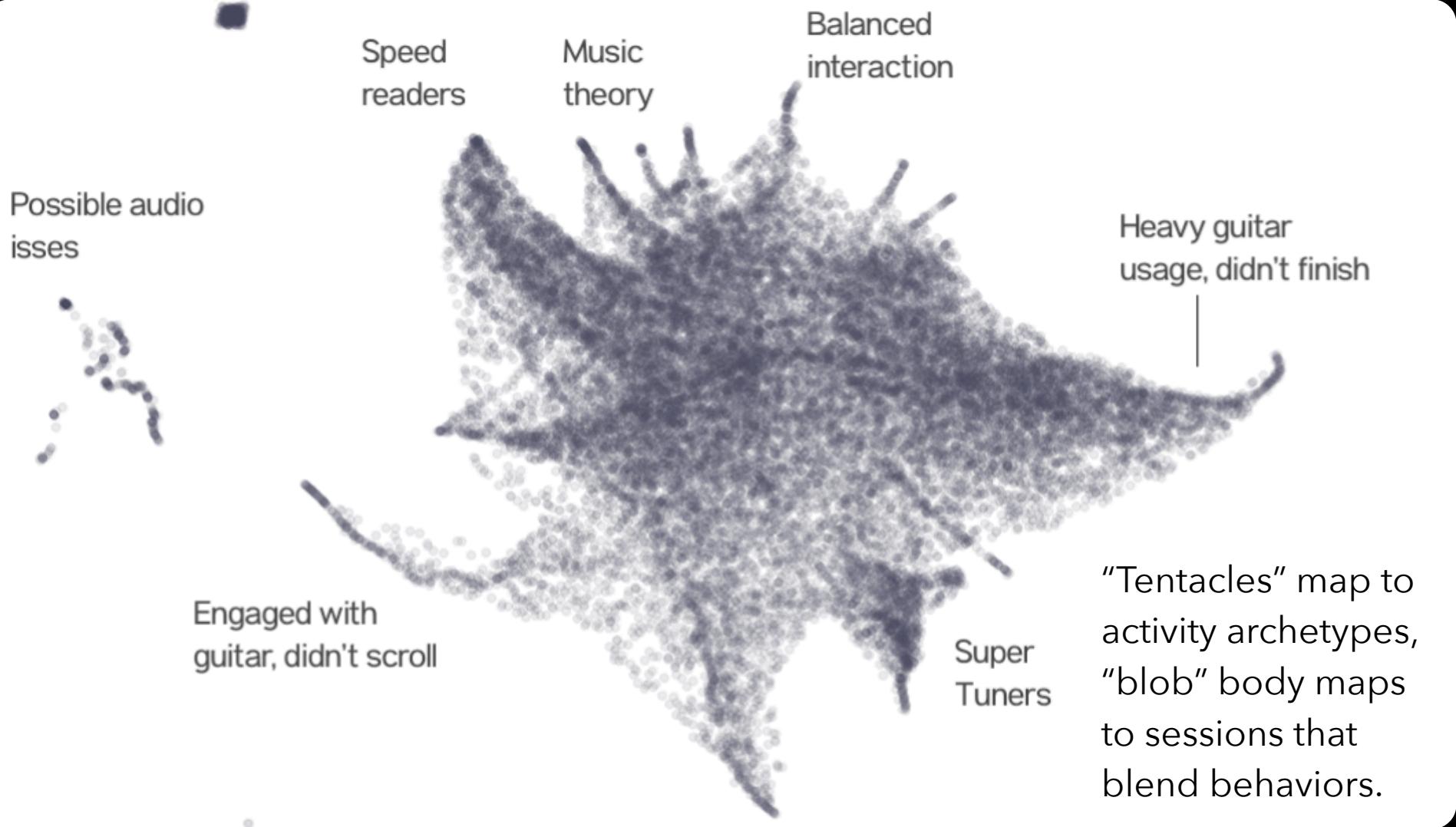
... and the count
of times each
variable changed



Showing 1233 users.

Brushing to select a subset of sessions.





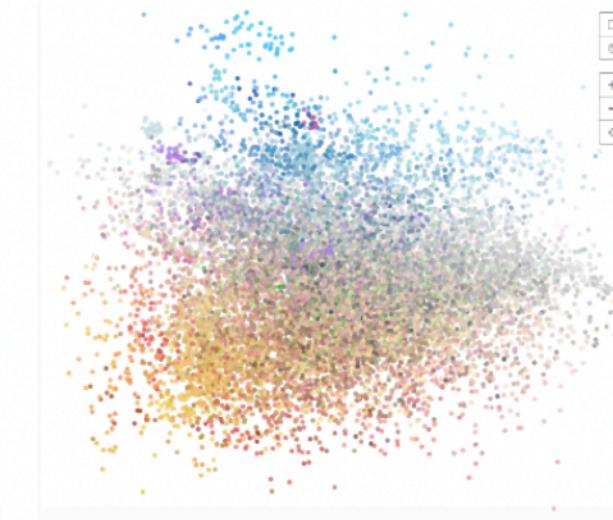
Mapping Emoji Images



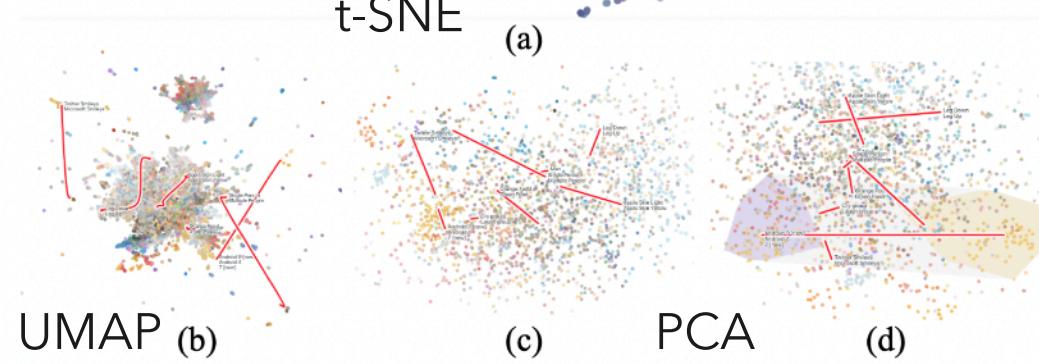
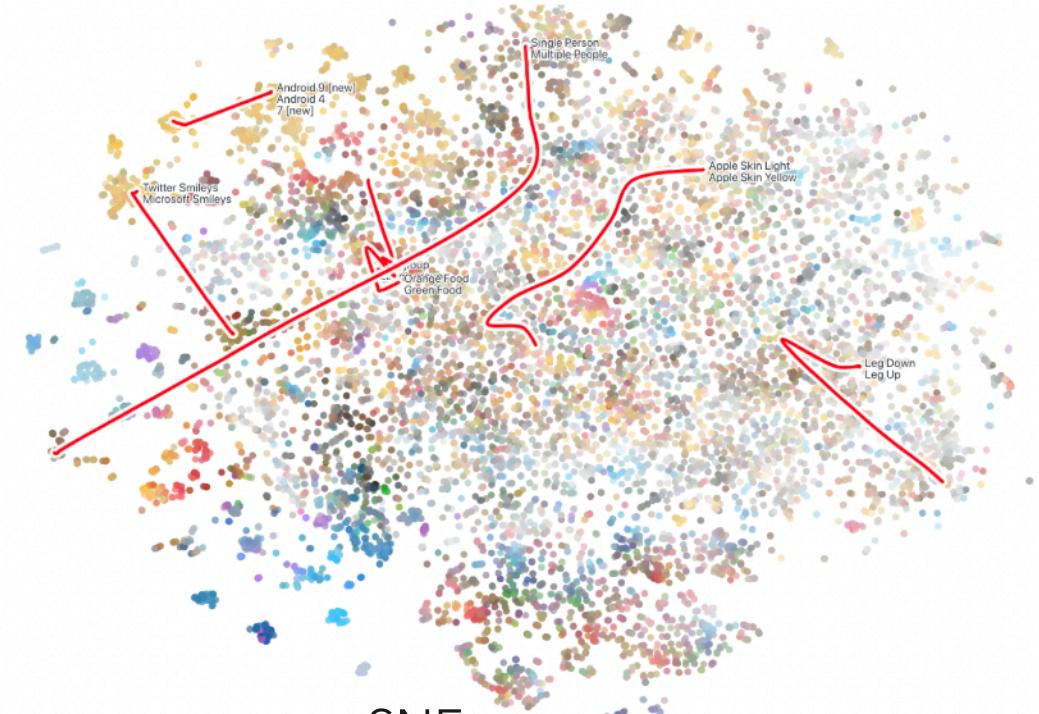
t-SNE



UMAP



PCA



Dimensionality Reduction Issues

Reproducible?

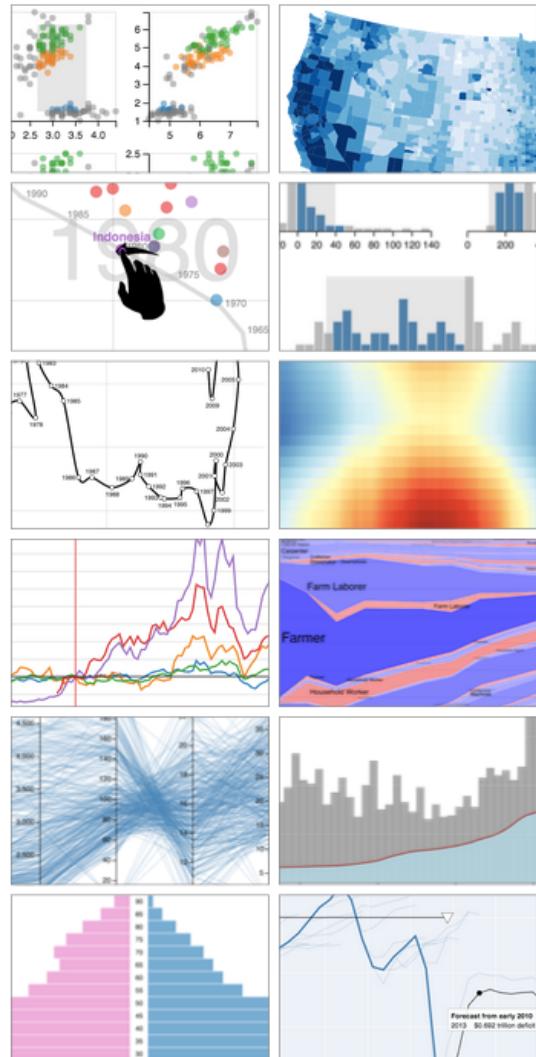
Projections are *data-dependent*. Fitting a new projection with different data can give rise to different results.

Reusable?

PCA and UMAP provide reusable projection functions that can map new points from high-D to low-D. t-SNE (and others, like MDS) do not provide this.

Interpretable?

DR plots are hard to interpret! Try multiple methods and hyperparameter settings. Inspect via interaction!



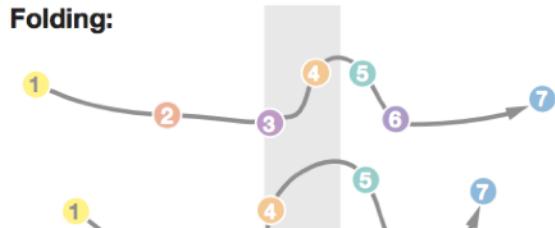
Time Curves [Bach et al. '16]

Timeline:



Circles are data cases with a time stamp.
Similar colors indicate similar data cases.

Folding:

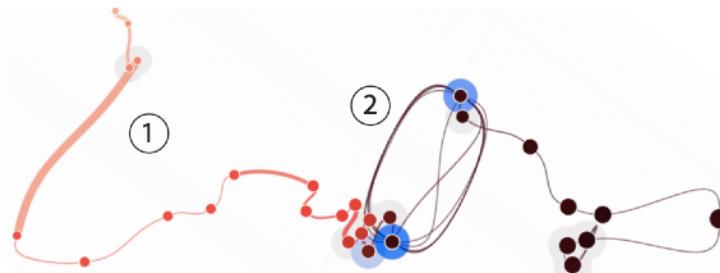


Time curve:

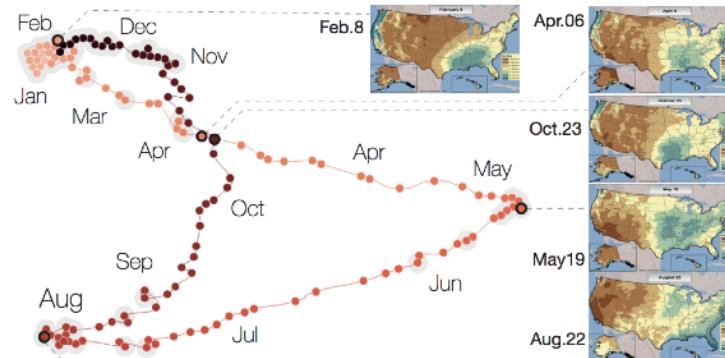


The temporal ordering of data cases is preserved.
Spatial proximity now indicates similarity.

(a) Folding time

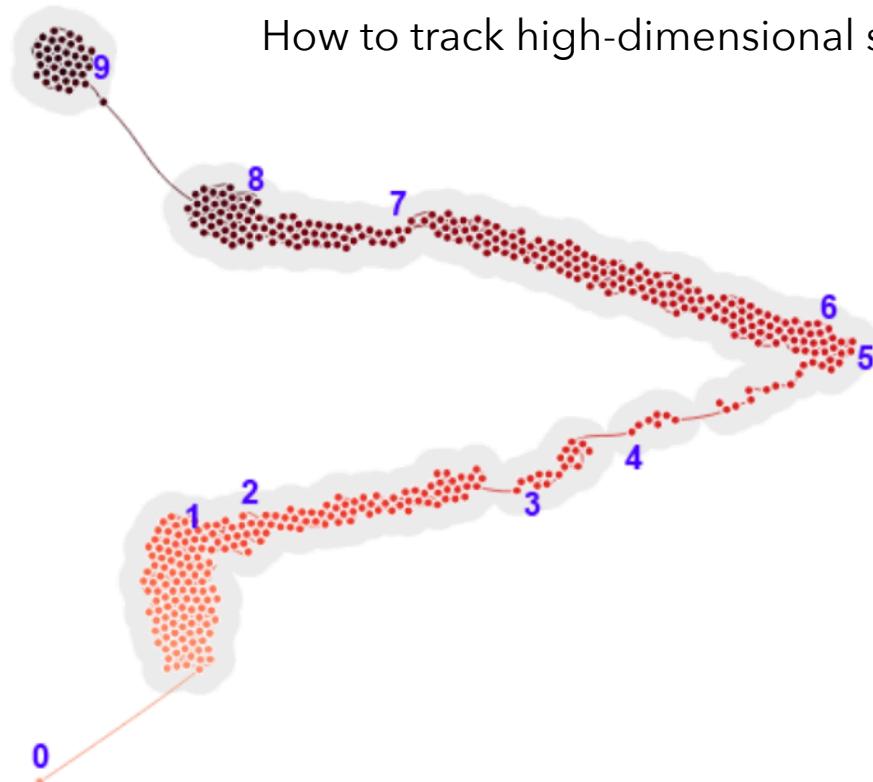


Wikipedia "Chocolate" Article

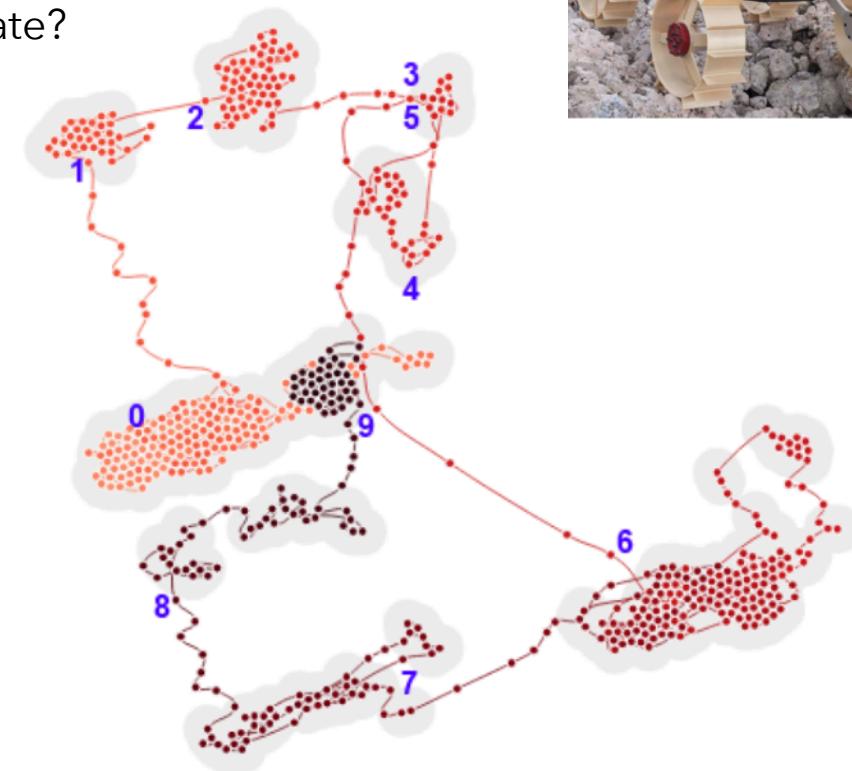


Rover Telemetry [Guy '16]

How to track high-dimensional state?



Using Raw Multi-D Data



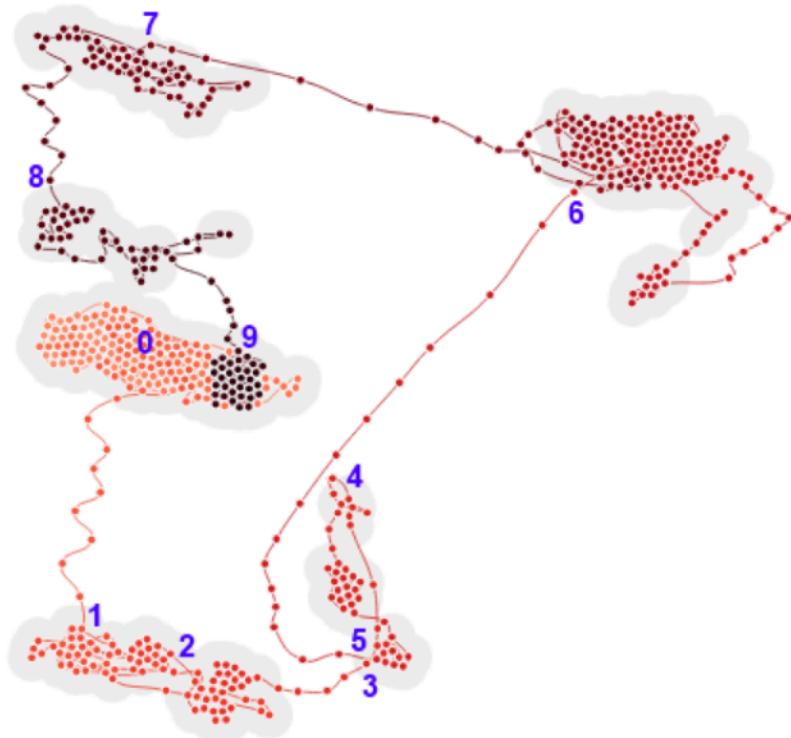
Using Pearson Correlation Matrix



Rover Telemetry [Guy '16]

How to track high-dimensional state?

| Event Number | Timestamp | Event Description |
|--------------|-----------|---|
| 0 | 0s | Rover begins traveling forward along smooth terrain. |
| 1 | 188s | Rover begins descending into crater. |
| 2 | 223s | Rover loses line of sight with lander and packet drops begin. |
| 3 | 287s | Rover enters shade, causing temp, comms, and power drops. |
| 4 | 300s | Rover begins traversing smooth bottom of crater. |
| 5 | 330s | Rover begins climbing out of crater. |
| 6 | 343s | Rover exits shade; continues uphill. |
| 7 | 534s | Rover emerges from crater and enters smooth terrain. |
| 8 | 594s | Rover enters choppy terrain. |
| 9 | 643s | Rover wheel has fault; rover stops moving. |

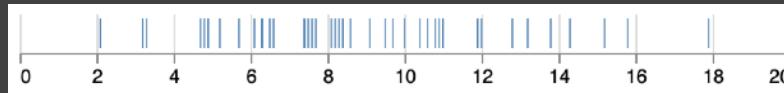


Using Spearman Correlation Matrix

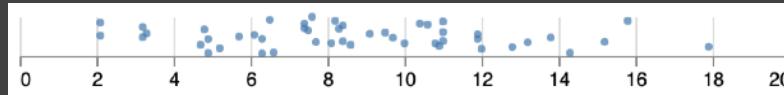
Visualizing Distributions *(Review)*

Distribution Visualizations

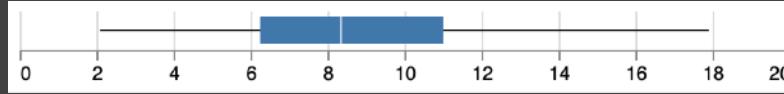
Strip Plot



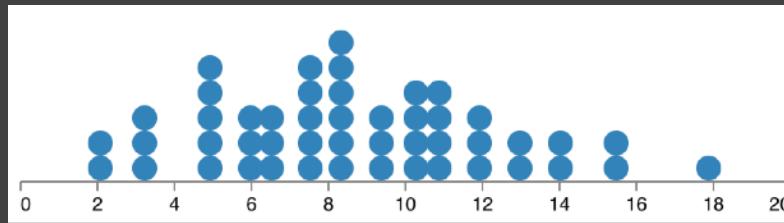
Jittered Plot



Box Plot



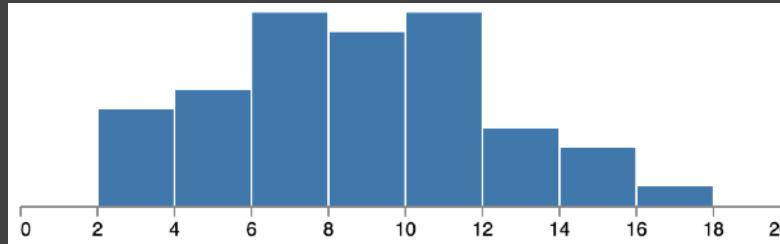
Dot Plot



Distribution Visualizations

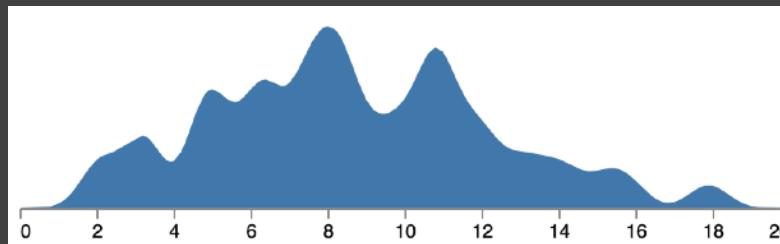
Histogram

bin size = 2



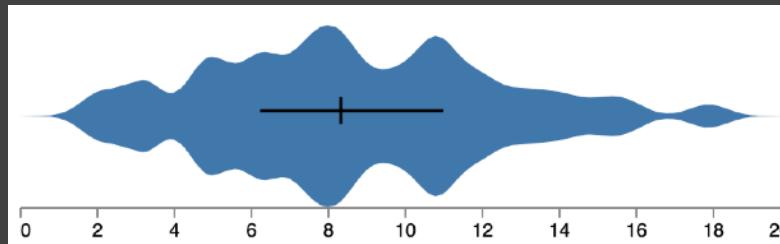
Density Plot

kde, $\sigma = 0.5$



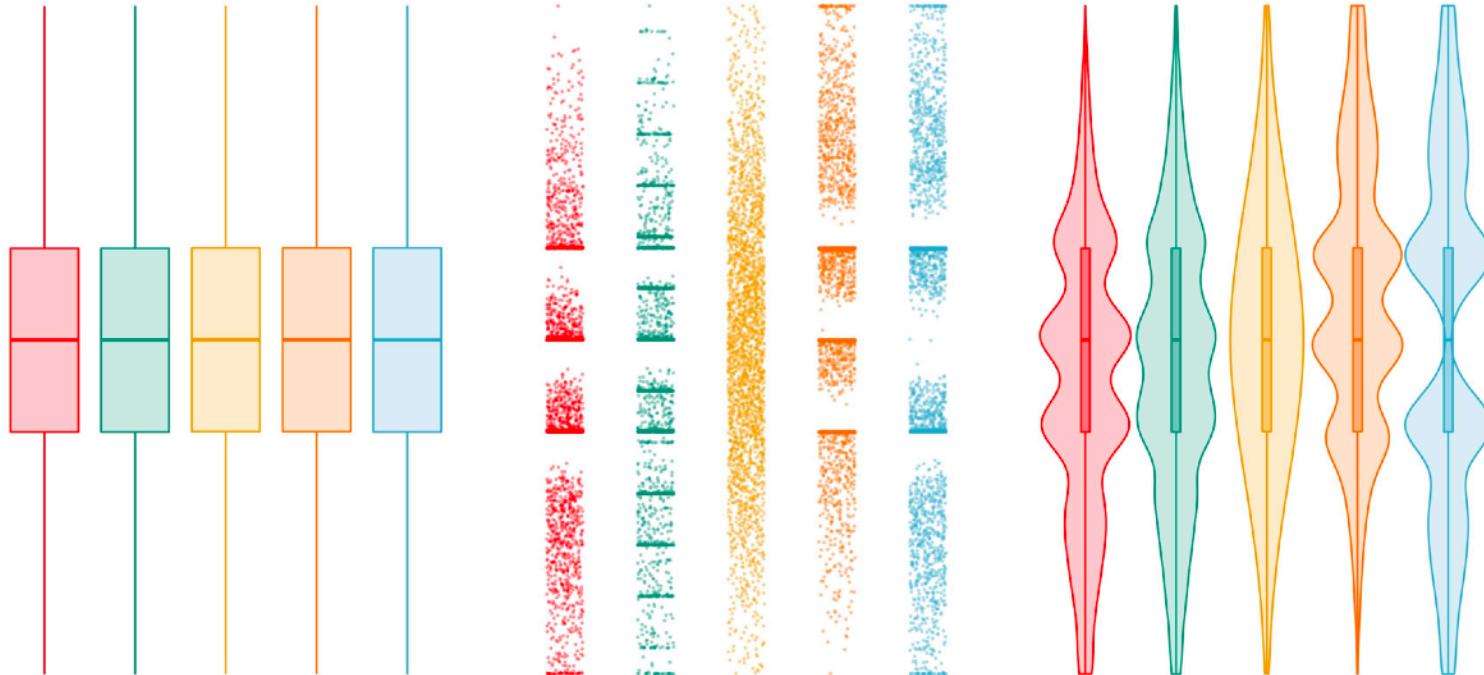
Violin Plot

kde, $\sigma = 0.5$

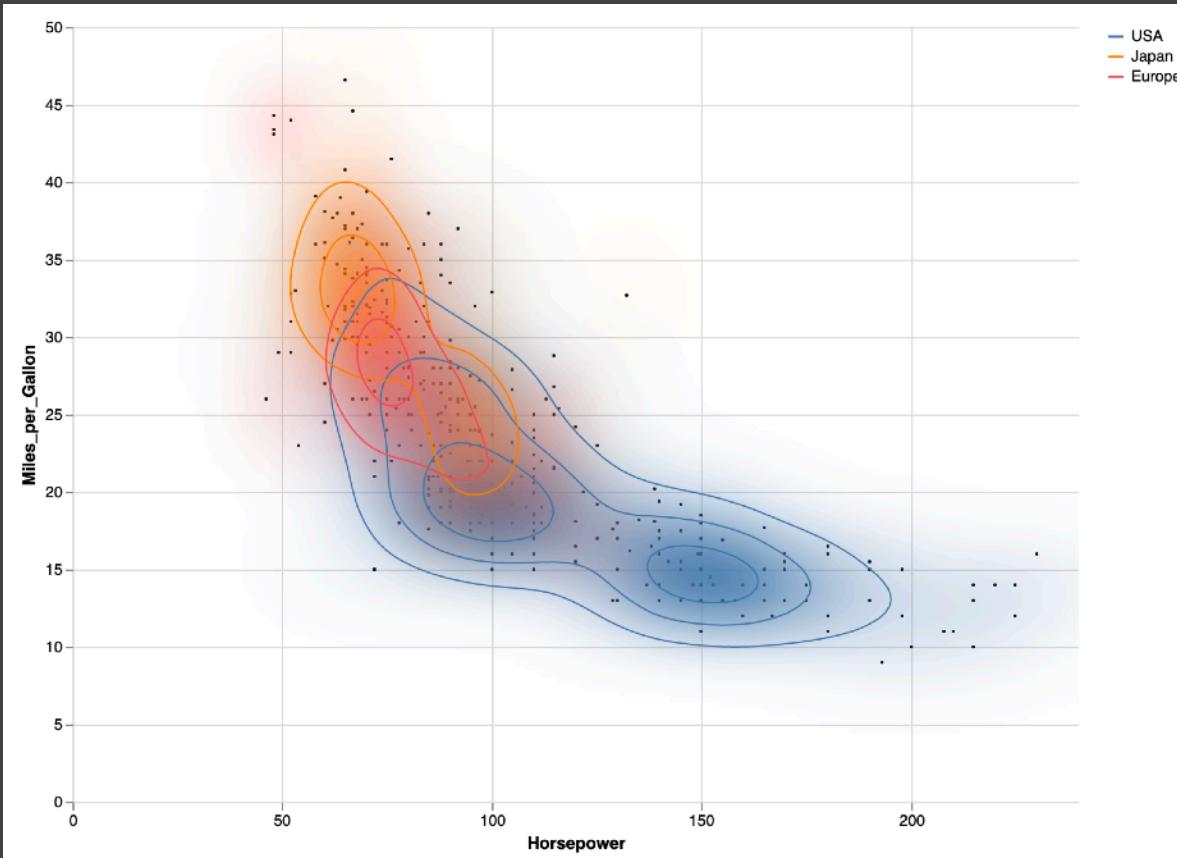


Identical boxplots, different distributions

Boxplots are great. They show medians and ranges and enable comparison of different groups. However, boxplots can be misleading. Different datasets can have the same descriptive statistics (left), but quite different underlying distributions (middle). Therefore, it is crucial to visualize the distribution in addition to descriptive statistics. Violin plots with integrated boxplots are great for this.

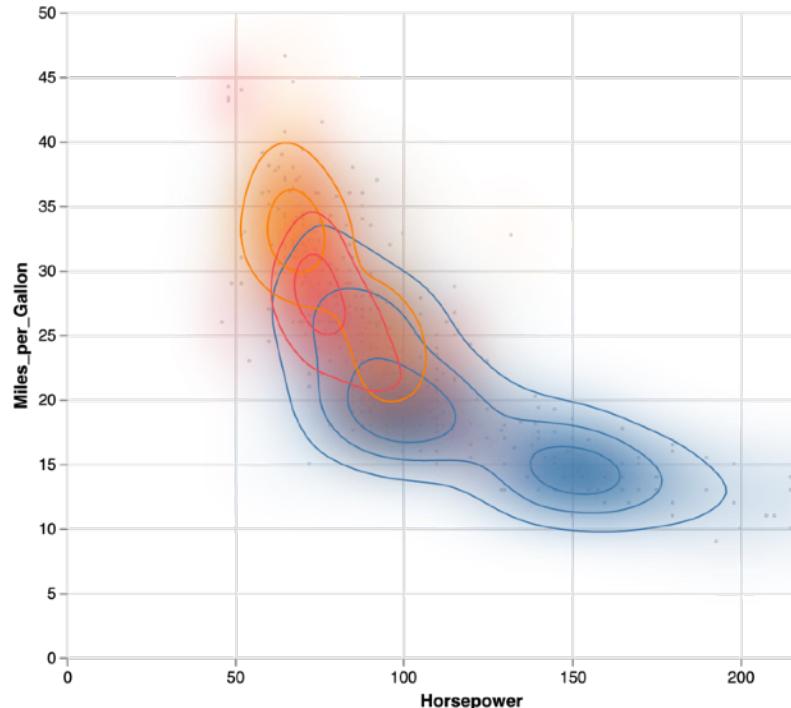
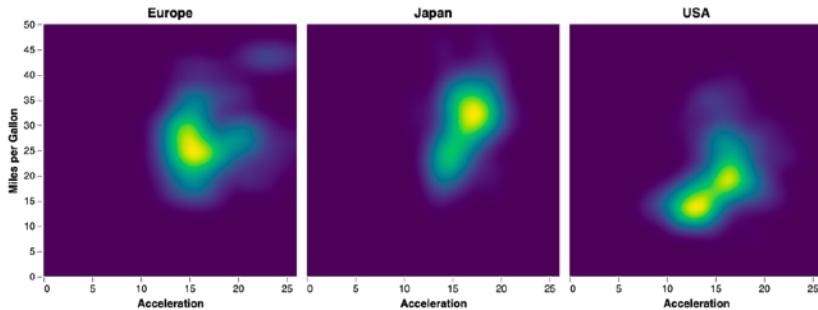
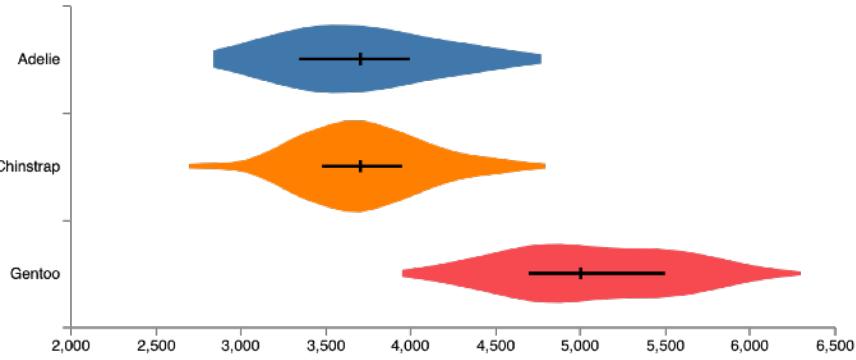


Now in 2D! Heatmaps, Contours



Kernel Density Estimation (KDE)

Enables violin plots, heat maps, contour plots...



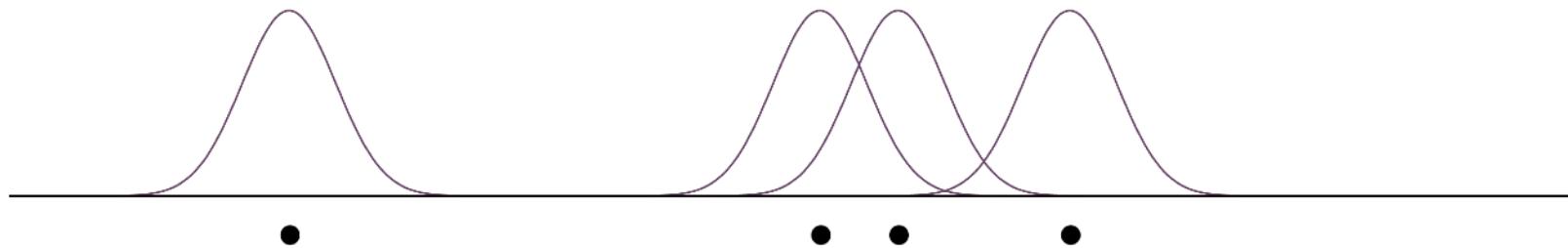
Kernel Density Estimation

For a set of input data points...



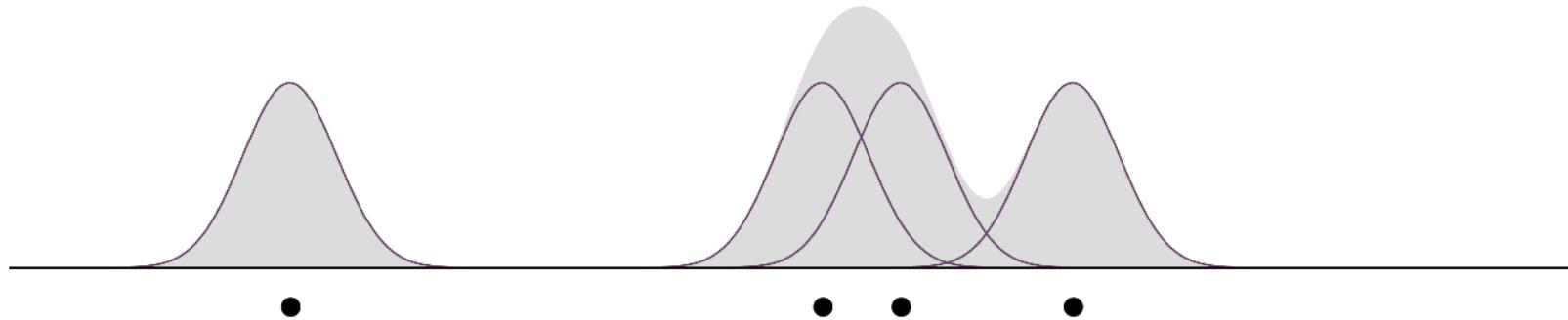
Kernel Density Estimation

Represent each point with a “kernel” distribution



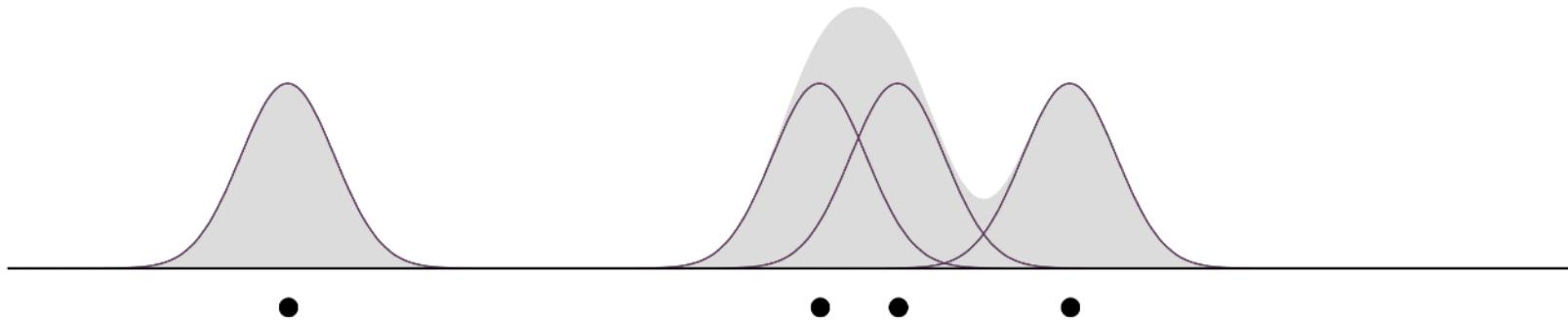
Kernel Density Estimation

Sum the kernels to form a density estimate



Kernel Density Estimation

Sized by bandwidth (standard deviation)



1D Distribution Exercise

This week you will think through how to communicate a distribution of values.

One column of numbers, a couple categories – *simple, right?* But there are many ways to transform and visualize distributions, which can lead to different insights as well as misleading omissions.

See Ed for links to the exercise document and online submission form.

We're here to help!