CSE 431 Winter 2025
Assignment #6

Reading assignment: Read sections 7.5, 8.1-8.5.

Problems:

1. (20 points) For any set of people V, an *influential subset* is a set $S \subseteq V$ of people so that everyone in V is either in S, has a friend in S, or both. We can represent the friendship relationships between pairs of people by edges in an undirected graph G with vertices V so we carry over the definition of influential subset to subsets of vertices of such graphs.
   Let
   $$IS = \{\langle G, k\rangle \mid G \text{ has an influential subset } S \subseteq V \text{ of size } \leq k\}.$$
   Show that IS is NP-complete, using the NP-hardness of $VERTEX\text{-}COVER$.
   (Hint: In the reduction from $VERTEX\text{-}COVER$, add vertices and edges to the original graph using precisely one extra vertex per original edge.)

2. (20 points) This problem is inspired by the single-player game *Minesweeper*, generalized to an arbitrary graph. Minesweeper begins with an undirected graph G in which each node either contains a single, hidden mine or is empty. The player chooses nodes, one by one. If the player chooses a node containing a mine, the player loses. If the player chooses an empty node, the player learns the number of neighboring nodes containing mines. The player wins if and when the player has chosen all the empty nodes. (In the actual game it suffices for the player to learn the number of neighboring nodes associated with each empty node.)

   We are interested in the related problem of *Mine-Consistency* in which the input is an undirected graph G together with numbers for some of G's nodes. The goal is to determine whether there is a placement of mines on the remaining nodes so that any node u numbered k has exactly k neighboring nodes containing mines. Formulate *Mine-Consistency* as a language, and prove that it is NP-complete.

   Hint: One possibility is a reduction from 3SAT. The reduction from 3SAT to SUBSET-SUM in the text might inspire you in the right direction.

3. Let A be the language of properly nested parentheses. For example, $(()(()))\in A$ but $(()))(()) \notin A$. Show that $A \in SPACE(\log n)$.

4. (20 points) Prove that if every NP-hard language is $PSPACE$-hard then $NP = PSPACE$.

5. (20 points) Let $A_{LBA} = \{\langle M, w\rangle \mid M \text{ is an LBA that accepts input } w\}$. Show that A_{LBA} is PSPACE-complete.