CSE 431 Winter 2025
Assignment #2

Reading assignment: Read Chapter 4 of Sipser's text.

Problems:

1. (15 points) Prove that if there is an enumerator that enumerates a language in lexicographic order then that language is decidable. (Hint: Handle the case where the language is finite separately from the case where it is infinite.)

2. (10 points) Use the result of question 1 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

3. (20 points) Given languages A and B, the language

$$AB = \{x \mid \exists y \in A \text{ and } z \in B \text{ such that } x = yz\}.$$

   (a) Suppose that A and B are decidable languages. Prove that AB is decidable.

   (b) Suppose that A and B are Turing-recognizable languages. Prove that AB is Turing recognizable.

4. (20 points) Given a language A, the language

$$A^* = \{x \mid \exists k \geq 0 \text{ and } y_1, \ldots, y_k \in A \text{ such that } x = y_1 \cdots y_k\}.$$

   (a) Suppose that A is a decidable language. Prove that A^* is decidable.

   (b) Suppose that A is a Turing-recognizable language. Prove that A^* is Turing recognizable.

5. (15 points) Define

$$ALL_{DFA} = \{\langle M \rangle \mid M \text{ is a DFA with alphabet } \Sigma \text{ and } L(M) = \Sigma^*\}.$$

   Prove that ALL_{DFA} is decidable.

6. (20 points) Consider the language

$$LEFTY_{TM} = \{\langle M, w \rangle \mid \text{TM } M \text{ on input } w \text{ makes an } L\text{-move at some point during its computation.}\}$$

   Prove that $LEFTY_{TM}$ is decidable.

A. (Extra credit) Let C be a language. Prove that C is Turing-recognizable if and only if there is a decidable language D such that $C = \{x \mid \exists y \text{ such that } \langle x, y \rangle \in D\}$.