CSE 431
Introduction to the Theory of Computation
Sample Midterm

1. (18 points) For each of the following questions answer True, False, or Unknown and briefly JUSTIFY your answer.

   (a) It is undecidable to tell, given two Turing machines M_1 and M_2, whether or not $L(M_1)$ is the complement of $L(M_2)$.

   (b) There is a regular language whose complement is undecidable.

   (c) There is no Turing machine that decides whether or not two context-free grammars generate the same language.

2. (20 points) Show that the language

   $$BH = \{\langle M \rangle \mid \text{Turing Machine } M \text{ halts when started at the left end of a blank tape}\}$$

   is undecidable.

3. (20 points)

   (a) Give the full formal definition of what it means for a set A to be mapping reducible to a set B.

   (b) Prove that if a set C is co-Turing-recognizable but not decidable then it cannot satisfy $C \leq_m A_{TM}$.

4. (22 points)

   (a) State the Church-Turing Thesis.

   (b) Why can't we ever prove the Church-Turing Thesis?

   (c) What is the evidence for it?

   (d) A queue automaton is a bit like a Turing machine except that it has a single queue of symbols instead of a tape. The input followed by a single blank symbol is initially in the queue with the first character at the head of the queue. In each step the queue automaton pops (reads and removes the symbol) at the head of the queue,o and then, based on that symbol and the current state, changes state and pushes one or two symbols onto the tail of the queue.

   Give the rough ideas for why a queue automaton is equivalent to a TM. (The tricky part is seeing how you can simulate TM moves in each direction on the queue automaton.)

5. (20 points) If language L is defined over alphabet Σ then the set of prefixes of strings in L,

$$Prefix(L) = \{x \mid \text{there exists a } y \in \Sigma* \text{ such that } xy \in L\}.$$

(a) Show that if L is Turing-recognizable then $Prefix(L)$ is Turing-recognizable.

(b) Intuitively, why doesn't the method you used for part (a) also show that if L is decidable then $Prefix(L)$ is decidable?

(c) (BONUS - Do not attempt unless you are finished early and have already checked over your paper) Actually come up with an example where you can prove that L is decidable but $Prefix(L)$ is not decidable.