CSE 431 Winter 2022
Assignment #8

Due: Thursday March 10, 2022, 11:59 PM

Reading assignment: Read section 9.1 of Sipser's text.

Problems:

1. (20 points) Show that $TQBF$ restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

2. (20 points) Let

$$TREE = \{\langle G \rangle \mid G \text{ is an undirected graph that is a tree}\}.$$

   Show that $TREE \in L$. You can use the fact, which we stated but did not prove, that $UPATH \in L$ where

$$UPATH = \{\langle G, s, t \rangle \mid G \text{ is an undirected graph with a path from } s \text{ to } t\}.$$

3. (30 points) Recall that a directed graph $G = (V, E)$ is *strongly connected* iff for every pair of vertices $u, v \in V$, there exists a path in G from u to v.
   Let $STRONGLY\text{-}CONNECTED = \{\langle G \rangle \mid G \text{ is a strongly connected directed graph}\}$. In this problem you will show that $STRONGLY\text{-}CONNECTED$ is NL-complete.

   (a) Prove that $PATH \leq_L STRONGLY\text{-}CONNECTED$.
   Hint: for your reduction, add a number of "backward" edges.

   (b) Prove that $STRONGLY\text{-}CONNECTED \in NL$.

4. (30 points) Recall that $EXP = \bigcup_k TIME(2^{n^k})$ and $NEXP = \bigcup_k NTIME(2^{n^k})$. Your goal in this problem is to show that if $EXP \neq NEXP$ then $P \neq NP$.

   To do this it will be helpful to define a padding function that maps any string x into a potentially much longer string that can be easily decoded to figure out what x was. In particular, define

$$pad : \Sigma^* \times \mathbb{N} \rightarrow (\Sigma \cup \{0, 1\})^*$$

   by $pad(x, m) = x01^j$ where j is the smallest natural number such that $|x01^j| \geq m$.

   For a language $A \in \Sigma^*$ and a function $g : \mathbb{N} \rightarrow \mathbb{N}$, define the "padded" language

$$pad(A, g(n)) = \{pad(x, g(|x|)) \mid x \in A\}.$$

   (a) Prove that if $A \in TIME(n^6)$ then $pad(A, n^2) \in TIME(n^3)$. (Recall that the running time is expressed as a function of the input length.)

   (b) Prove that if $A \in NTIME(2^{n^3})$ then $pad(A, 2^{n^3}) \in NTIME(n)$.

   (c) Using padded languages with a suitable bounding function $g(n)$ prove that if $EXP \neq NEXP$ then $P \neq NP$. (Hint: Prove the contrapositive.)

5. (Extra Credit) Let

$$ACYCLIC = \{\langle G \rangle \mid G \text{ is an undirected graph that does not have a cycle}\}.$$

Show that $ACYCLIC \in L$ without using the fact that $UPATH \in L$.