CSE 431 Winter 2022
Assignment #4

Due: Thursday February 3, 2022, 11:59 PM

Reading assignment: Read Sections 6.3 of Sipser's text.

Problems:

1. (20 points) Prove that the language $\overline{E_{TM}}$ is Turing-recognizable.
2. (20 points) A language B is called *r.e.-complete* iff (a) B is Turing-recognizable (equivalently, recursively enumerable) and (b) For all Turing-recognizable languages A, $A \leq_m B$. Prove that A_{TM} is r.e.-complete.
3. (20 points) Show that A is decidable if and only if $A \leq_m \{1^n \mid n \text{ is even}\}$.
4. (20 points) Let $J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.
5. (20 points) Which of the following problems are decidable? Justify each answer:
   (a) Given a Turing machine M, does M accept 1010?
   (b) Given Turing machines M and N, is $L(N)$ the complement of $L(M)$?
   (c) Given a Turing machine M, integers a and b and an input x, does M run for more than $a|x|^2 + b$ steps on input x?
6. (Extra Credit) Show that the following problem is undecidable: Given a Turing machine M and integers a and b, does there exist an input x on which M runs for more than $a|x|^2 + b$ steps on input x?
7. (Extra Credit) We showed previously that neither EQ_{TM} nor its complement is Turing-recognizable. Your problem is to show that, despite this, if you had a magic black box that decided A_{TM} that you could call repeatedly on different inputs, then you could recognize $\overline{EQ_{TM}}$.