Reading assignment: Read Sections 6.3 of Sipser’s text.

Problems:

1. (20 points) Prove that the language $E_{TM}$ is Turing-recognizable.

2. (20 points) A language $B$ is called r.e.-complete iff (a) $B$ is Turing-recognizable (equivalently, recursively enumerable) and (b) For all Turing-recognizable languages $A$, $A \leq_m B$. Prove that $A_{TM}$ is r.e.-complete.

3. (20 points) Show that $A$ is decidable if and only if $A \leq_m \{1^n \mid n \text{ is even}\}$.

4. (20 points) Let $J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither $J$ nor $\overline{J}$ is Turing-recognizable.

5. (20 points) Which of the following problems are decidable? Justify each answer:

   (a) Given a Turing machine $M$, does $M$ accept 1010?

   (b) Given Turing machines $M$ and $N$, is $L(N)$ the complement of $L(M)$?

   (c) Given a Turing machine $M$, integers $a$ and $b$ and an input $x$, does $M$ run for more than $a|x|^2 + b$ steps on input $x$?

6. (Extra Credit) Show that the following problem is undecidable: Given a Turing machine $M$ and integers $a$ and $b$, does there exist an input $x$ on which $M$ runs for more than $a|x|^2 + b$ steps on input $x$?

7. (Extra Credit) We showed previously that neither $EQ_{TM}$ nor its complement is Turing-recognizable. Your problem is to show that, despite this, if you had a magic black box that decided $A_{TM}$ that you could call repeatedly on different inputs, then you could recognize $\overline{EQ_{TM}}$. 