CSE 431 Winter 2022
Assignment #3

Due: Thursday January 27, 2022, 11:59 PM

Reading assignment: Read Chapter 5 of Sipser's text. We will cover the first part of section 5.3 before we cover computation histories in section 5.1.

Problems:

1. (25 points) Define

   $INFINITE_{CFG} = \{\langle G\rangle \mid$ the language that context-free grammar G generates is infinite$\}$.

   Prove that $INFINITE_{CFG}$ is decidable.

2. (25 points) A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

3. (25 points) A version of Turing's proof of the undecidability of the Halting Problem for TMs also can be used to show that there is no "nice" way to describe the set of decidable languages precisely: In particular, suppose that $A \subseteq \{\langle M\rangle \mid M$ is a decider TM$\}$ and that A is Turing-recognizable. (In particular, A only contains descriptions of TMs that are deciders but it might not contain all TMs that decide each decidable language.)
   Prove that there is a decidable language D such that $L(M) \neq D$ for every M with $\langle M\rangle \in A$. (Hint: You may find it helpful to consider an enumerator for A.)

4. (25 points) Define $SUBSET_{TM} = \{\langle M_1, M_2\rangle \mid M_1, M_2$ are TMs and $L(M_1) \subseteq L(M_2)\}$. Prove that $SUBSET_{TM}$ is undecidable.

5. (Extra Credit) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of k, consider all k-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.