Lecture 1  Intro to the Theory of Computation

Prehistory of Computation
A lot of the development was inspired need to remove bugs in reasoning

\[ \text{infinity} \]

Ancient Greece
Zenon's Paradox of Infinity

- Motion is impossible

\[ \text{Arrow} \quad \text{can never reach the Target} \]

- Achilles & Tortoise

\[ \text{Achilles can never overtake Tortoise} \]

Tom Stoppard play 'Jumpers'
Euclid's Elements:
- 5 postulates (axioms) of geometry
- Foundational reasoning for millennia

Why GCD ≠ (integers ≠ lengths)

[Diagram of a rectangle with dimensions 9 x 15, 3 x 6]

17th and 18th Century Math
- Calculus
- Newton & Leibniz
- Infinitesimals (infinitely small)
- No good logical foundation

- Fourier: infinite series of trig functions can represent any function
19th Century

Paradox of Series

\[
\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots
\]

\[
= 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots \right)
\]

\[
= 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots
\]

\[
= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \left( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots \right)
\]

\[= 0\]

Fourier did this kind of thing all the time in his work!

Non-Euclidean Geometry (1830)

- Coach in foundations
- Parallel postulate
- Doesn't follow from others
- World where it shifts true!

Boole Laws of Thought (1854)

Predicate Logic

Frege: Predicate Logic
Cantor (1875)  Continuity of Real

| aleph 0, 1, 2 |

Set A are same size if there is 1-1 correspondence

\[ |\text{natural numbers}| = |\text{rational numbers}| 
eq |\text{real numbers}|\]

Kronecker: "God made integers, the rest is the work of man."

Frege 1893  Foundations of Arithmetic

Reichen 1889  Axioms for natural numbers

+,*  Induction

\[ \forall x (P(x) \rightarrow P(x+1)) \]

\[ P(0) \]

\[ \therefore \forall x P(x) \]

A = "The set of all infinite sets"

\[ A \in A \]
Russell's Paradox (1902)

Why Frege noting

\[ S = \text{"The set of all sets that are not members of themselves."} \]

Is \( S \in S? \)
Yes \( \times \)
No \( \times \)

Logicomix: An epic search for the truth

Zermelo-Frankl set theory

\[ \forall x \ (x \not\in x) \]

ZFC