CSE 431: Introduction to Theory of Computation

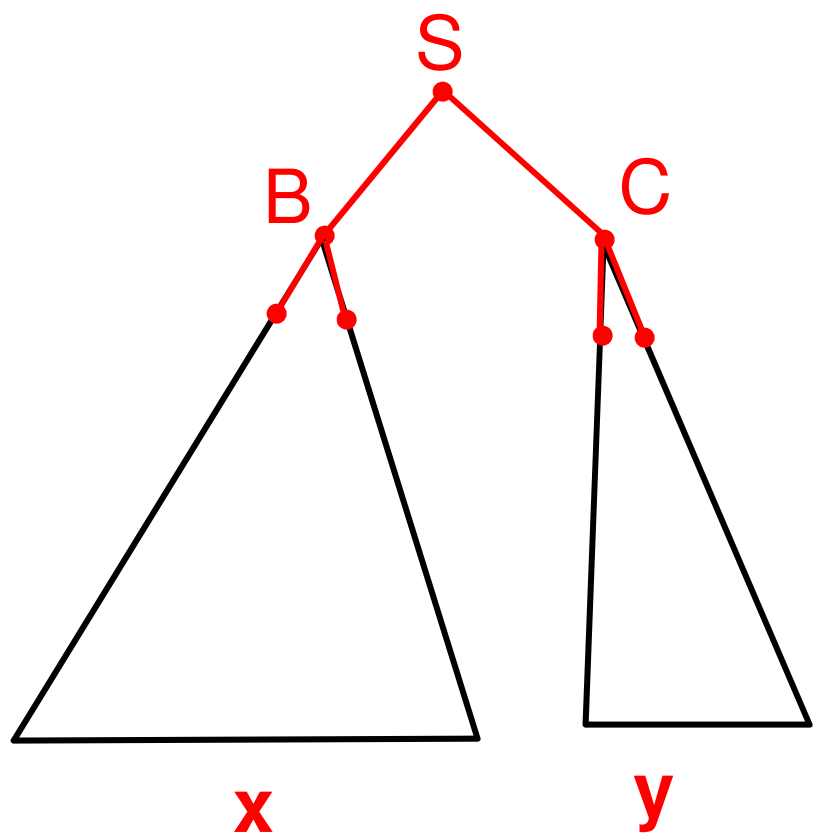
Cocke-Kasami-Younger Algorithm

Paul Beame

Determining whether $w \in L(G)$

- Assume $G=(V,\Sigma,R,S)$ is in Chomsky Normal Form
  - Grammar rules allowed
    - $A \rightarrow BC$ where $B,C \in V$ $B,C \neq S$
    - $A \rightarrow a$ where $a \in \Sigma$
    - $S \rightarrow \varepsilon$
  - If $w = \varepsilon$ check whether $S \rightarrow \varepsilon$ is in R
  - If $w = a \in \Sigma$ then check whether $S \rightarrow a$ is in R
  - Otherwise, parse tree must be a binary tree and first rule is some $S \rightarrow BC$

Parse Tree for w with |w|=n



w=xy so $x=w_1 \ldots w_k$ and $y=w_{k+1}..w_n$ for some k

Recursive Algorithm (Exponential Time)

```
Generates(A,w)
  if |w|≤1 output true iff A → w is a rule in R
  else
     n←|w|
     for k=1 to n-1
        x←w[1..k];  y←w[k+1..n]
        for each rule A → BC in R
              if Generates(B,x) and Generates(C,y)
                    output true
        endfor
     endfor
     output false
  endif
```

Dynamic Programming

- All the recursive calls are subproblems of the type Generates(A,x) where
  - $A \in V$
  - $x = w[i..j]$
  - Intervals in w get shorter the deeper the call
- **CKY Algorithm:** Create a table whose (i,j)th entry is the list of all variables that can generate the string w[i..j]
- Fill out table starting with short intervals first
- Answer is whether S is in table(1,n) where $n=|w|$

CKY algorithm: $O(n^3)$ time

- Base

  for all $i=1$ to n

  table(i,i)$\leftarrow${variables A with rule $A \rightarrow w_i$ }

- Iteration for $d=1$ to $n-1$
  - Entries table(i,j) with $j-i<d$ already computed

  for every (i,j) with $j=i+d$ do

  for $k=i$ to $j-1$

  for every rule $A \rightarrow BC$

  if $B \in$ table(i,k) and $C \in$ table(k+1,j)

  Add A to table(i,j)

Grammar S→AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	
4				B,T		
3			A			
2		A				
1	A					
	a	a	a	b	b	b

Grammar $S \to AT \mid AU \mid \varepsilon$, $T \to UB \mid b$,
$U \to AT \mid UT$, $A \to a$, $B \to b$

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	
3			A	S,U		
2		A	∅			
1	A	∅				
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	
3			A	S,U		
2		A	∅	S		
1	A	∅	∅			
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	
3			A	S,U	U,T	
2		A	∅	S		
1	A	∅	∅			
	a	a	a	b	b	b

Grammar $S \rightarrow AT \mid AU \mid \varepsilon$, $T \rightarrow UB \mid b$,
$U \rightarrow AT \mid UT$, $A \rightarrow a$, $B \rightarrow b$

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	
2		A	∅	S	S,U	
1	A	∅	∅	∅		
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	
1	A	∅	∅	∅		
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	
1	A	∅	∅	∅		
	a	a	a	b	b	b

Grammar $S \to AT \mid AU \mid \varepsilon$, $T \to UB \mid b$,
$U \to AT \mid UT$, $A \to a$, $B \to b$

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	
1	A	∅	∅	∅	S	
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	S,T,U
1	A	∅	∅	∅	S	
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	S,T,U
1	A	∅	∅	∅	S	
	a	a	a	b	b	b

Grammar S→ AT | AU | ε, T→UB | b,
U→ AT | UT, A→a, B→b

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	S,T,U
1	A	∅	∅	∅	S	S,U
	a	a	a	b	b	b

Grammar $S \rightarrow AT \mid AU \mid \varepsilon$, $T \rightarrow UB \mid b$,
$U \rightarrow AT \mid UT$, $A \rightarrow a$, $B \rightarrow b$

Input aaabbb

	1	2	3	4	5	6
6						B,T
5					B,T	∅
4				B,T	∅	∅
3			A	S,U	U,T	U,T
2		A	∅	S	S,U	S,T,U
1	A	∅	∅	∅	S	S,U
	a	a	a	b	b	b