CSE 431 Fall 2019
Assignment #6

Due: Monday, November 18, 2019, 8:59 PM

Reading assignment: Read sections 9.3 and 7.5.

Problems:

1. Show that if $P = NP$ then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.

2. Let $U = \{\langle M, x, 1^t \rangle \mid M \text{ is an NTM that accepts input } x \text{ within } t \text{ steps}\}$. Show that U is NP-complete.

3. Let ϕ be a 3CNF-formula. An NAE assignment to the variables of ϕ is one that satisfies ϕ but does not set all three literals to true in any clause.

   (a) Show that the negation of an NAE assignment for ϕ is also an NAE assignment for ϕ.

   (b) Let *NAESAT* be the set of all 3CNF formulas ϕ that have an NAE assignment. Prove that *NAESAT* is NP-complete. For the hardness part use a reduction from 3SAT.
   (Hint: Use the function that replaces each clause C_i of ϕ of the form $(y_1 \vee y_2 \vee y_3)$ where y_1, y_2, y_3 are literals by the two clauses $(y_1 \vee y_2 \vee z_i)$ and $(\overline{z_i} \vee y_3 \vee w)$ where w is a single new variable for all clauses and there is one z_i variable per original clause.)

4. For any set of people V, an *influential subset* is a set $S \subseteq V$ of people so that everyone in V is either in S, has a friend in S, or both. We can represent the friendship relationships between pairs of people by edges in an undirected graph G with vertices V so we carry over the definition of influential subset to subsets of vertices of such graphs.
   Let $INFLUENTIAL\text{-}SUBSET = \{\langle G, k \rangle \mid G \text{ has an influential subset } S \subseteq V \text{ of size } \leq k\}$.
   Show that $INFLUENTIAL\text{-}SUBSET$ is NP-complete, using the NP-hardness of $VERTEX\text{-}COVER$.
   (Hint: In the reduction from $VERTEX\text{-}COVER$, add vertices and edges to the original graph using precisely one extra vertex per original edge.)

5. Let $01ROOT = \{\langle p \rangle \mid p \text{ is a polynomial in } n \text{ variables with integer coefficients such that } p(x_1, \ldots, x_n) = 0 \text{ for some assignment } (x_1, \ldots, x_n) \in \{0,1\}^n\}$.

   (a) Show that $01ROOT \in NP$.

   (b) Show that $3SAT \leq_m^P 01ROOT$. (HINT: First figure out how to convert each clause into a polynomial that evaluates to 0 iff the clause is satisfied. Then create a polynomial q that evaluates to 0 if and only if all of its inputs are 0. Finally, figure out how to combine the individual polynomials for the clauses using the polynomial q.

6. (Extra credit) Recall that a 2-CNF formula is a CNF formula in which each clause has 2 literals and that 2-$SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable 2-CNF formula$\}$. Show that $2SAT \in P$.