(1) A graph \( G = (V, E) \) is called **planar** if it can be drawn in the plane without edge crossings:

\[ \begin{array}{c}
\text{Define the language:} \\
\text{NONPLANAR} \\
= \{ <G> : \text{G is an undirected graph that is not planar} \} \\
\text{Use Kuratowski's theorem (see Wikipedia) to prove that NONPLANAR} \in NP \\
\text{Extra credit: Can you show that} \\
\text{PLANAR} = \{ <G> : \text{G is a planar graph} \} \end{array} \]
is in NP? Use as simple a certificate as possible.

[Make sure the length of the certificate is polynomial in the input size!]

(2) Recall from class the language

\[ 3\text{COL} = \{ \langle G \rangle : \text{G is a 3-colorable graph} \} \]

(a) Prove that \(3\text{COL} \leq_p 4\text{COL}\)

(b) Consider the language:

\[ \text{TEN-3COL} = \{ \langle G \rangle : \text{G is an undirected graph and has at least ten distinct proper 3-colorings} \} \]

Prove that \(3\text{COL} \leq_p \text{TEN-3COL}\)
Extra credit: Consider the language:

\[ \text{PLANAR-3col} = \{ \langle G \rangle : G \text{ is a planar, 3-colorable graph} \} \]

Show that 3col \( \leq_p \) PLANAR-3col.

Hint: Use the following gadget to replace edge crossings:

(3) Show that if 2col is NP-complete, then P = NP.
Extra credit: Two undirected graphs $H$ and $G$ are isomorphic if the vertices of $H$ can be reordered to obtain $G$.

Show that if $P=NP$, there is a polynomial-time algorithm that, given isomorphic graphs $H$ and $G$, outputs the ordering.