CSE 431 Spring 2017
Assignment #8

Due: Friday, June 2, 2017

Reading assignment: Read section 9.1 of Sipser's text.

Problems:

1. Show that the language A of properly nested parentheses is in L. (For definiteness recall that A is the language generated by the grammar $S \to (S) \mid SS \mid \varepsilon$ though this grammar is not necessarily useful.)

2. Let
$$TREE = \{\langle G \rangle \mid G \text{ is an undirected graph that is a tree}\}.$$
Show that $TREE \in L$. You can use without proof the fact that $UPATH \in L$ where
$$UPATH = \{\langle G, s, t \rangle \mid G \text{ is an undirected graph with a path from } s \text{ to } t\}.$$

3. Recall that $EXP = \bigcup_k TIME(2^{n^k})$ and $NEXP = \bigcup_k NTIME(2^{n^k})$. Your goal in this problem is to show that if $EXP \neq NEXP$ then $P \neq NP$.

   To do this it be helpful to define a padding function that maps any string x into a potentially much longer string that can be easily decoded to figure out what x was. In particular, define
   $$pad : \Sigma^* \times \mathbb{N} \to (\Sigma \cup \{0,1\})^*$$
   by $pad(x, m) = x01^j$ where j is the smallest natural number such that $|x01^j| \geq m$.

   For a language $A \in \Sigma^*$ and a function $g : \mathbb{N} \to \mathbb{N}$, define the "padded" language
   $$pad(A, g(n)) = \{pad(x, g(|x|)) \mid x \in A\}.$$

   (a) Prove that if $A \in TIME(n^6)$ then $pad(A, n^2) \in TIME(n^3)$. (Recall that the running time is expressed as a function of the input length.)

   (b) Prove that if $A \in NTIME(2^{n^3})$ then $pad(A, 2^{n^3}) \in NTIME(n)$.

   (c) Using padded languages with a suitable bounding function $g(n)$ prove that if $EXP \neq NEXP$ then $P \neq NP$.

4. (Bonus) Let
$$ACYCLIC = \{\langle G \rangle \mid G \text{ is an undirected graph that does not have a cycle}\}.$$
Show that $ACYCLIC \in L$ without using the fact that $UPATH \in L$.

5. (Bonus) Show that the language generated by the following grammar, $S \to (S) \mid [S] \mid SS \mid \varepsilon$ and consisting of all strings with two kinds of balanced parentheses, is also in L.