CSE 431 Spring 2017
Assignment #4

Due: Friday, April 28, 2017

Reading assignment: Read Sections 7.1 and 7.2 of Sipser's text. We will start Chapter 7 this week.

Problems:

1. A language B is called *r*.e.-complete if and only if (a) B is Turing-recognizable (equivalently, recursively enumerable) and (b) For all Turing-recognizable languages A, $A \leq_m B$. Prove that A_{TM} is r.e.-complete.

2. Show that A is decidable if and only if $A \leq_m \{0^n 1^n \ : \ n \geq 0\}$.

3. Let $J = \{w \mid w = 0x$ for some $x \in A_{TM}$ or $w = 1y$ for some $y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

4. Show that there is an undecidable language contained in 1^*.

5. Which of the following problems are decidable? Justify each answer:

   (a) Given a Turing machine M, does M accept 0101?

   (b) Given Turing machines M and N, is $L(N)$ the complement of $L(M)$?

   (c) Given a Turing machine M, integers a and b and an input x, does M run for more than $a|x|^2 + b$ steps on input x?

6. (Bonus) Show that the following problem is undecidable: Given a Turing machine M and integers a and b, does there exist an input x on which M runs for more than $a|x|^2 + b$ steps on input x?

7. (Bonus) We showed previously that neither EQ_{TM} nor its complement is Turing-recognizable. Your problem is to show that, despite this, if you had a magic black box that decided A_{TM} that you could call repeatedly on different inputs, then you could decide $\overline{EQ_{TM}}$.