# CSE 431 Spring 2015
# Assignment #4

Due: Monday, May 4, 2015

**Reading assignment:** Read Sections 7.1 and 7.2 of Sipser's text.

**Problems:**

1. Let $J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither $J$ nor $\overline{J}$ is Turing-recognizable.

2. Show that there is an undecidable language contained in $1^*$.

3. Which of the following problems are decidable? Justify each answer:

   (a) Given a Turing machine $M$, does $M$ accept $0101$?

   (b) Given Turing machines $M$ and $N$, is $L(N)$ the complement of $L(M)$?

   (c) Given a Turing machine $M$, integers $a$ and $b$ and an input $x$, does $M$ run for more than $a|x|^2 + b$ steps on input $x$?

4. Prove that if $K$ and $L$ are decidable by Turing machines running in polynomial time then so are $K \cup L$, $KL$, and $\overline{L}$.

5. Let $TRI = \{\langle G \rangle \mid G \text{ is an undirected graph that contains a triangle}\}$. Prove that there is a polynomial-time Turing machine that decides $TRI$.

6. (Bonus) Show that the following problem is undecidable: Given a Turing machine $M$ and integers $a$ and $b$, does there exist an input $x$ on which $M$ runs for more than $a|x|^2 + b$ steps on input $x$?

7. (Bonus) We showed previously that neither $EQ_{TM}$ nor its complement is Turing-recognizable. Your problem is to show that, despite this, if you had a magic black box that decided $A_{TM}$ that you could call repeatedly on different inputs, then you could decide $\overline{EQ_{TM}}$.