CSE 431 Spring 2015
Assignment #3

Due: Friday, April 24, 2015

Reading assignment: Read Chapter 5 of Sipser's text. We will cover section 5.3 before we cover computation histories in section 5.1. Before Friday, begin reading Chapter 7.

Problems:

1. Suppose that $A \subseteq \{\langle M\rangle \mid M \text{ is a decider TM}\}$ and that A is Turing-recognizable. (That is, A only contains descriptions of TMs that are deciders but it might not contain all such descriptions.)
   Prove that there is a decidable language D such that $L(M) \neq D$ for any M with $\langle M\rangle \in A$. (Intuitively, this means that one couldn't come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for A.)

2. Let $ODD_{TM} = \{\langle M\rangle \mid M \text{ is a TM that accepts an odd number of strings}\}$. Show that ODD_{TM} is undecidable.

3. A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

4. Show that for all Turing-recognizable problems A, $A \leq_m A_{TM}$.

5. Show that A is decidable if and only if $A \leq_m \{0^n1^n : n \geq 0\}$.

6. (Bonus) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the *busy beaver function* $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of k, consider all k-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.