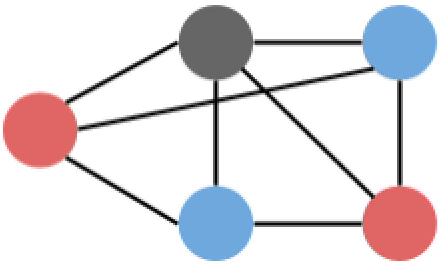
CSE 431 - Theory of Computation
Lecture 14: May 15, 2014
Lecturer: James Lee
Scribe: Yi Wang

Graph Coloring

Input: An undirected graph G = (V, E) and a number K
Output: Decide whether G has a K-coloring (No edge between same color nodes)

Example:

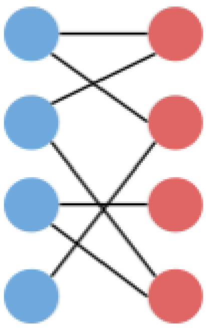


3 COL: {<G>: G has a proper 3-Coloring}

Planar Graph: Graph that can be drawn in the Euclidean Plane without edge crossings

Facts

- 1-Coloring ⇔ Graph has no edge
- 2-Coloring ⇔ Graph is bipartite (Graph has no odd cycle)
  Example:



- 4-Coloring: Every planar graph!

THM: 3 COL is NP-Complete

Proof:

1. **3 COL $\in$ NP**

(Easy! ☺)

2. **3 SAT $\leq_p$ 3 COL**

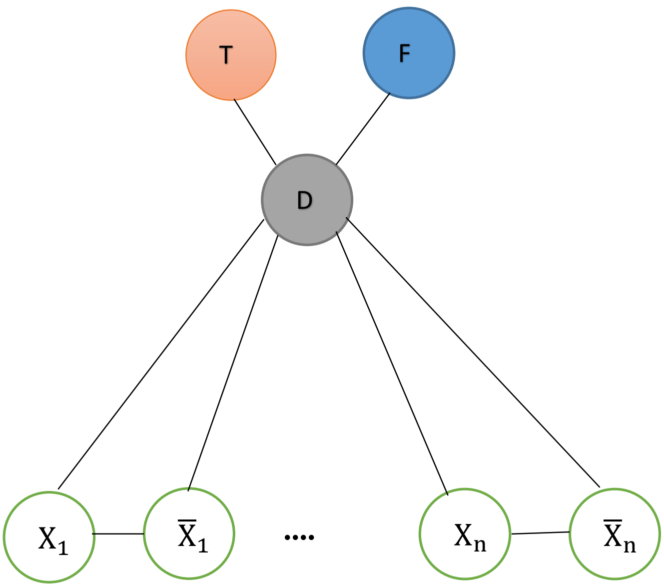
Goal:

Convert Ø => (Poly-time) G, such that

Ø is satisfiable ⇔ G has a 3-Coloring

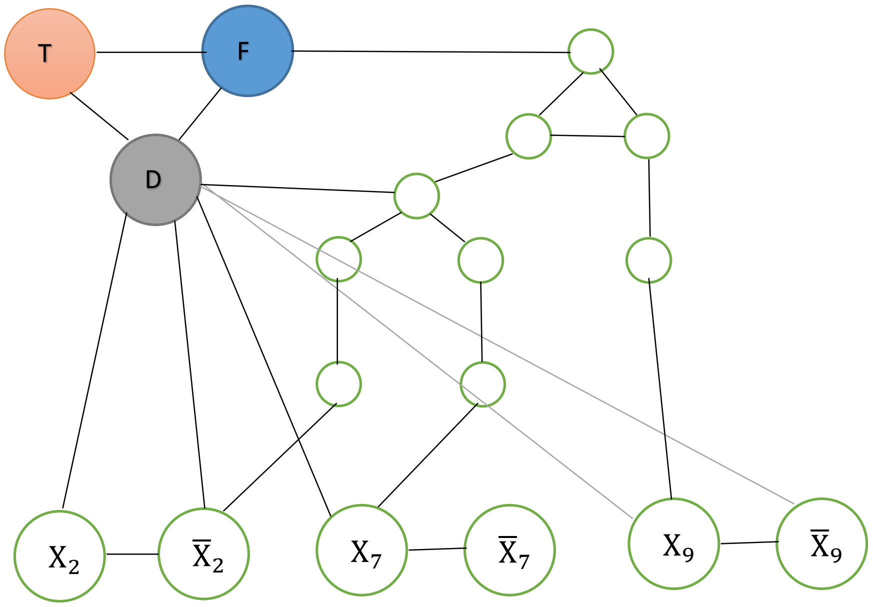
Ø = $C_1 \wedge C_2 \wedge \dots \wedge C_m$
Eg: Where $C_i = X_7 \vee \overline{X}_9 \vee X_5$
n varibles: $X_1, X_2, \dots, X_n$

⇓



Inset Or-Gadget:

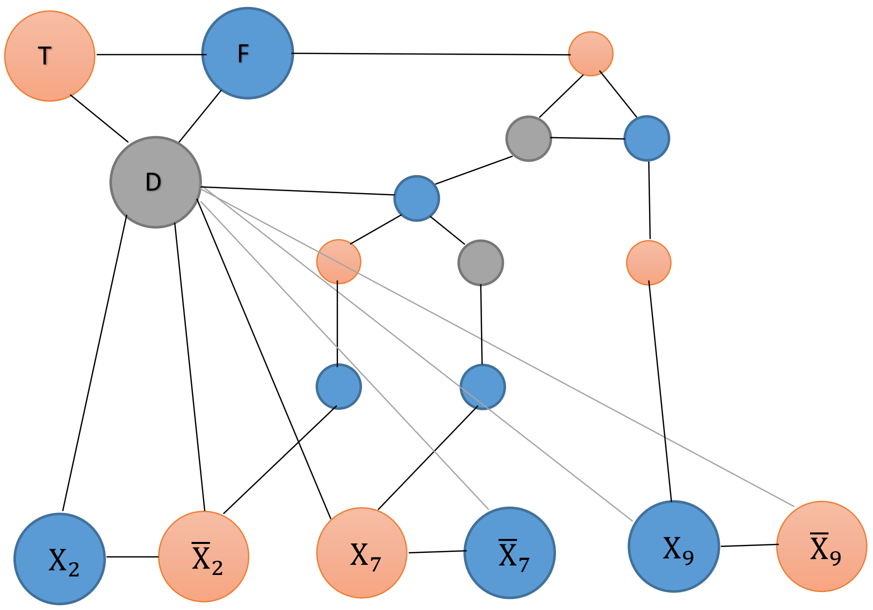
Example:



Assume Ø has a satisfy assignment

$X_2 = F$, $X_7 = T$, $X_9 = F$

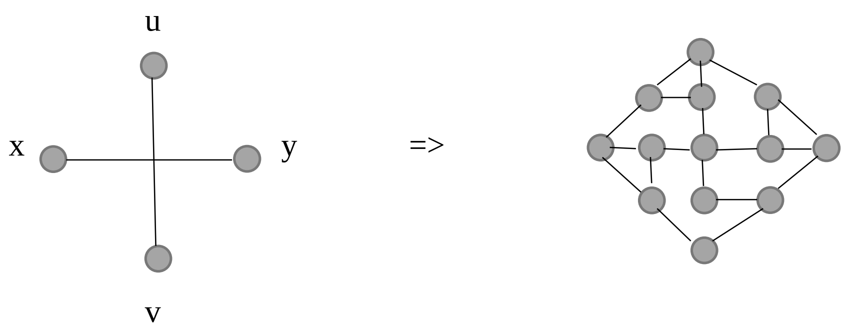
Then G has 3 coloring as follow:



3 COL $\leq_p$ PLANAR-3 COL

PLANAR-3COL = {<G>: G is planar and has a 3-Coloring}

Goal: Put a gadget in every across, make u and v have different color
x and y have different color



TBC…