

# CSE 431 Spring 2012

## Assignment #2

Due: Friday, April 13, 2012

**Reading assignment:** Read Chapter 4 of Sipser's text.

### Problems:

1. Show that the decidable languages are closed under the concatenation, intersection, and star operations.
2. Show that the Turing recognizable languages are closed under the concatenation, intersection, and star operations.
3. Let  $ALL_{DFA} = \{ \langle M \rangle \mid M \text{ is a DFA with alphabet } \Sigma \text{ and } L(M) = \Sigma^* \}$ . Prove that  $ALL_{DFA}$  is decidable.
4. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)
5. Use the result of question 4 to show that any infinite Turing-recognizable language contains an infinite decidable subset.
6. (Bonus) Let  $C$  be a language. Prove that  $C$  is Turing-recognizable iff there is a decidable language  $D$  such that  $C = \{ x \mid \exists y \text{ such that } \langle x, y \rangle \in D \}$ .