Lecture 11

Mapping Reducibility

Defn: A is *mapping reducible* to B ($A \le_m B$) if there is computable function *f* such that $w \in A \Leftrightarrow f(w) \in B$

A special case of $\le_T$:

Call subr only once; its answer is *the* answer

Facts:

$A \le_m B$ & B decidable $\Rightarrow$ A is too

$A \le_m B$ & A *un*decidable $\Rightarrow$ B is too

$A \le_m B$ & $B \le_m C \Rightarrow A \le_m C$

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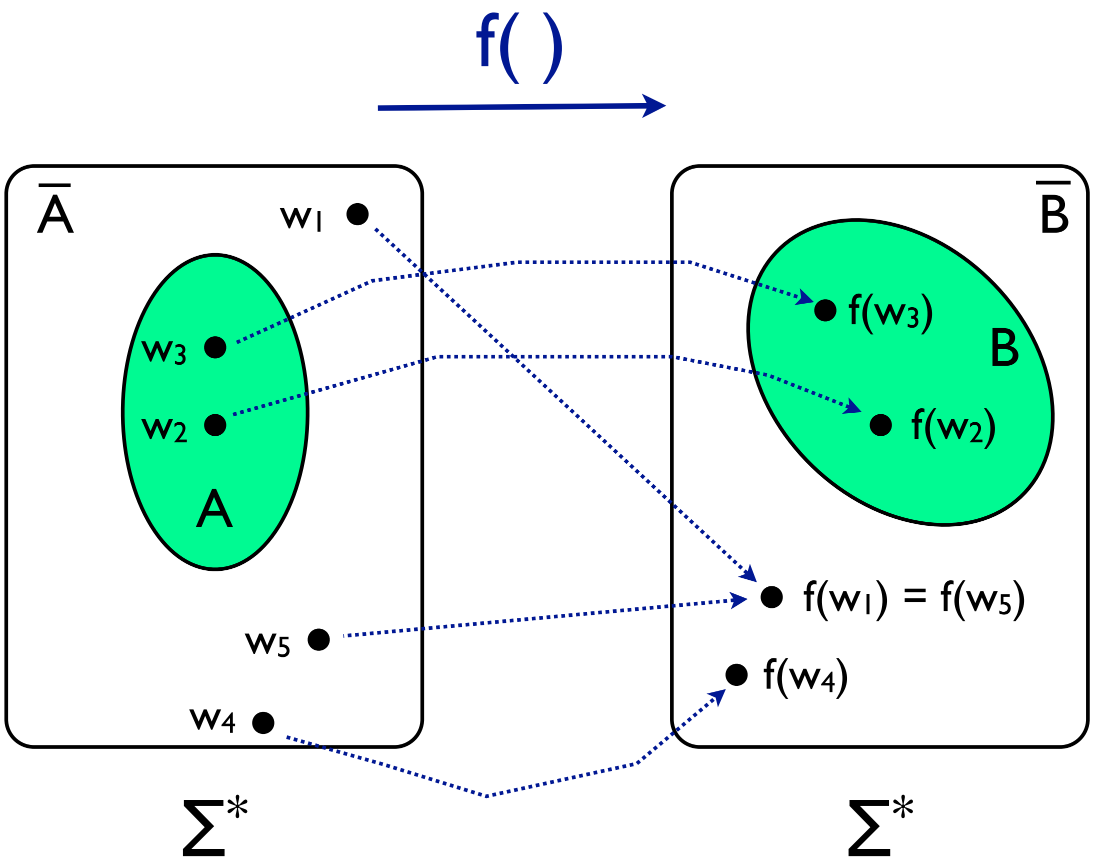
Theorem:

$A \leq_m B$ & B decidable (recognizable) $\Rightarrow$ A is too

$A \leq_m B$ & A *un*decidable (*un*recognizable) $\Rightarrow$ B is too

$A \leq_m B$ & $B \leq_m C \Rightarrow A \leq_m C$

Most reductions we've seen were actually $\leq_m$ reductions.



$$w \in A \Leftrightarrow f(w) \in B$$

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pf: To decide (recognize) w in A compute f(w), then use decider (recognizer, resp) for B on f(w).

$A \leq_m B$ & A *un*decidable (*un*recognizable) $\Rightarrow$ B is too

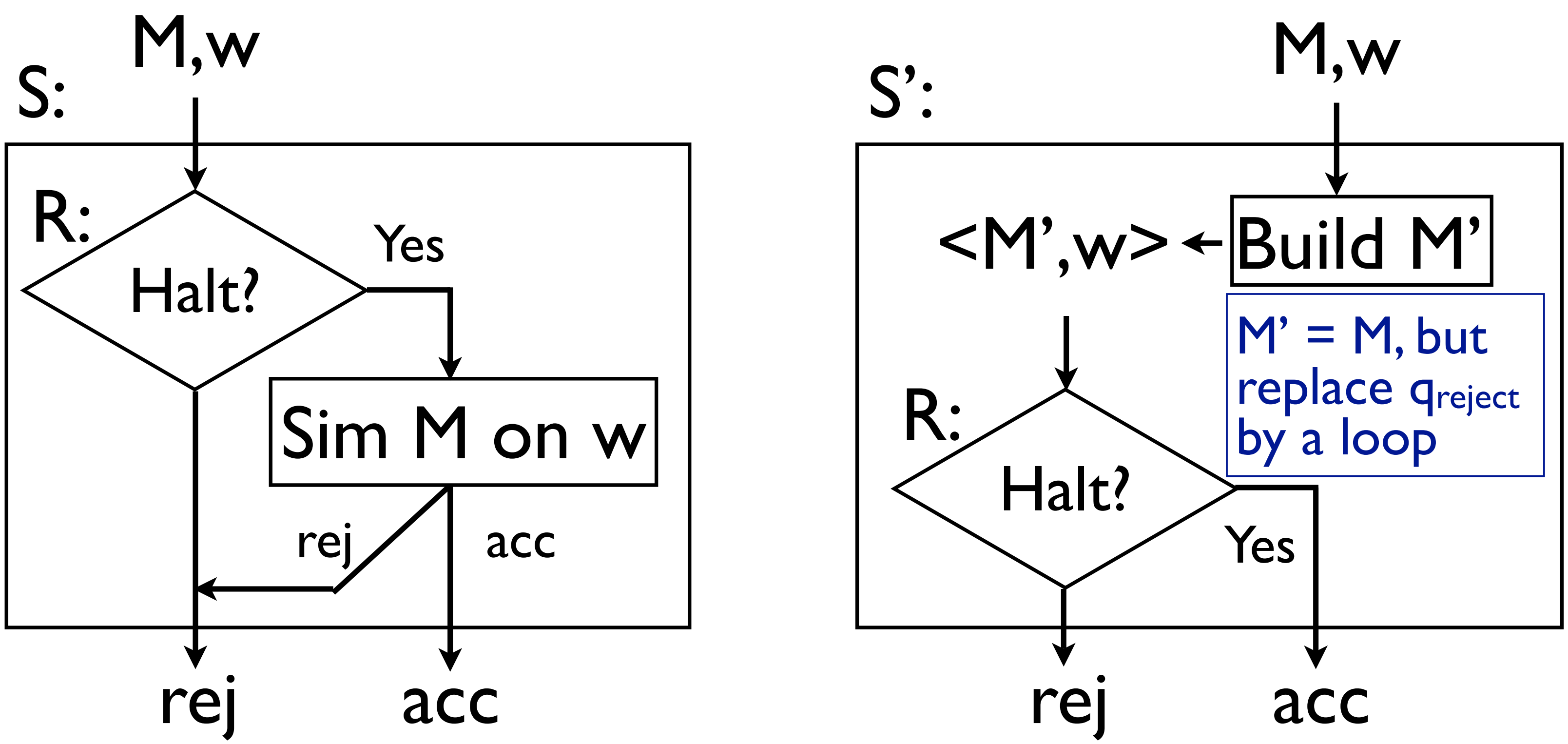
pf: Contrapositive

$A \leq_m B$ & $B \leq_m C \Rightarrow A \leq_m C$

pf: If f for $A \rightarrow B$, g for $B \rightarrow C$; then $w \in A \Leftrightarrow g(f(w)) \in C$

A_{TM} ($\leq_T$ vs $\leq_m$) $HALT_{TM}$

$f(\langle M,w \rangle) = \langle M',w \rangle$



From Lecture 07

Other Examples of $\leq_m$

$A_{TM} \leq_m REGULAR_{TM}$ $f(<M,w>) = <M_2>$

Build M_2 so $L(M_2) = \Sigma^* \;/\; \{0^n 1^n\}$, as M accept/rejects w

$EMPTY_{TM} \leq_m EQ_{TM}$ $f(<M>) = <M, M_{reject}>$

$L(M_{reject}) = \varnothing$, so equiv to M iff $L(M) = \varnothing$

$\left.\begin{array}{l} A_{TM} \leq_m MPCP \\ MPCP \leq_m PCP \end{array}\right\}$ 5.2

$A_{TM} \leq_m \overline{EMPTY_{TM}}$ $f(<M,w>) = <M_1>$

Build M_1 so $L(M_1) = \{w\} \;/\; \varnothing$, as M accept/rejects w