Lecture 8

Announcements

re HW#1, Aeron says "If I made a comment, even if I didn't take off points *this* time, people should pay attention because I will take off points for the same mistake in the future..."

EQ_{TM} is undecidable

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_i \text{ are TMs s.t. } L(M_1) = L(M_2) \}$

EQ_{TM} is undecidable

EQ_{TM} = { <M_1, M_2> | M_i are TMs s.t. $L(M_1) = L(M_2)$}

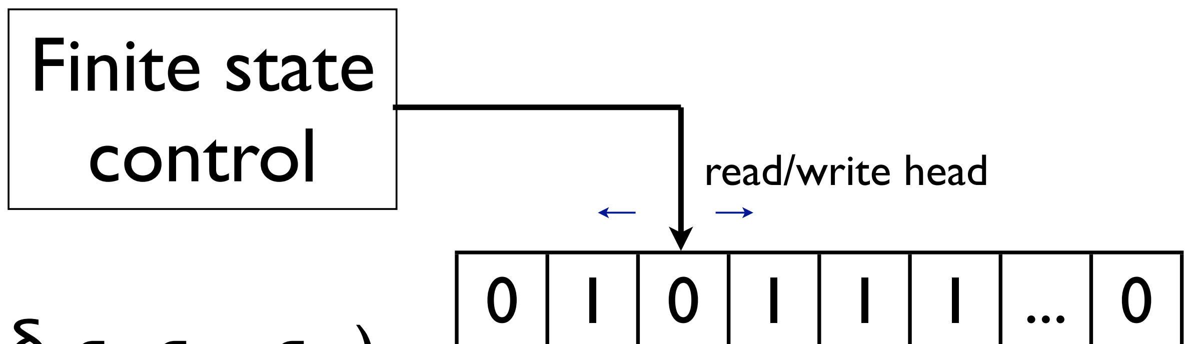
Pf: Will show $EMPTY_{TM} \leq_T EQ_{TM}$

Suppose EQ_{TM} were decidable. Let $M_\varnothing$ be a TM that accepts nothing, say one whose start state = q_{reject}. Consider the TM E that, given <M>, builds <M, $M_\varnothing$>, then calls the hypothetical subroutine for EQ_{TM} on it, accepting/rejecting as it does. Now, <M, $M_\varnothing$> $\in EQ_{TM}$ if and only if M accepts $\varnothing$, so, E decides whether M $\in$ $EMPTY_{TM}$, which we know to be impossible. Contradiction

Linear Bounded Automata

Like a (1-tape) TM, but tape only long enough for input

(head stays put if try to move off either end of tape)

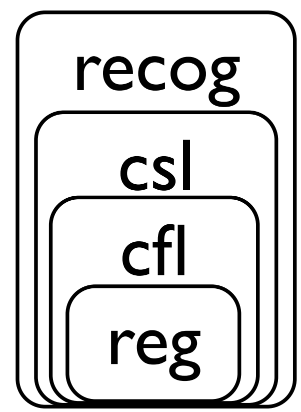


$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

$L(M) = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$

An Aside: The Chomsky Hierarchy

TM	= phrase structure grammars	$\alpha A\beta \rightarrow \alpha\gamma\beta$
LBA	= context-sensitive grammars	$\alpha A\beta \rightarrow \alpha\gamma\beta,\ \gamma \neq \varepsilon$
PDA	= context-free grammars	$A \rightarrow \gamma$
DFA	= regular grammars	$A \rightarrow abcB$



A_{LBA} is decidable

A_{LBA} = { <M, w> | M is an LBA and w $\in$ L(M) }

Key fact: the number of distinct configurations of an LBA on any input of length n is *bounded*, namely

$$\leq n\,|Q|\,|\Gamma|^n$$

If M runs for more than that many steps, it is looping

Decision procedure for A_{LBA}:

Simulate M on w and count steps; if it halts and accepts/rejects, do the same; if it exceeds that time bound, halt and reject.

$EMPTY_{LBA}$ is undecidable

Why is this hard, when the acceptance problem is not?

Loosely, it's about infinitely many inputs, not just one

Can we exploit that, say to decide A_{TM}?

An idea. An LBA is a TM, so can it simulate M on w?

Only if M doesn't use too much tape.

What about simulating M on w############ ?

Given M, build LBA M' that, on input w # # # # ... #, simulates M on w, treating # as a blank. If M halts, do the same. if M tries to move off the right end of the tape, reject.

L(M') = { $w\#^k$ | M accepts w using ≤ | $w\#^k$ | tape cells }

Key point:

if M rejects w, M' rejects $w\#^k$ for all k, ∴ L(M') = ∅

if M accepts w, some k will be big enough, ∴ L(M') ≠ ∅