CSE 431

Introduction to Theory of Computation

Homework #2

Due: Friday, April 16, 2010

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1. 3.15(b)
2. 3.16(b) Prove it two different ways: first using an ordinary (deterministic) TM, then using a nondeterministic TM. [You may do 3.15(b) using either model.]
3. 4.7
4. Let $\mathcal{F} = \{f : \mathcal{N} \to \mathcal{N}\}$, and $\mathcal{F}_2 = \{b : \mathcal{N} \to \{0, 1\}\}$, i.e., the set of all functions mapping natural numbers to natural numbers and the set of all $\{0, 1\}$-valued functions on $\mathcal{N}$, resp. Show that both sets are uncountably infinite.

   Extra credit: Show that both have the same cardinality as the reals.
5. Let L be a language. Prove
   (a) L is recognizable if and only if there is a decidable language D such that
   $$L = \{x \mid \exists y \text{ s.t. } \langle x, y\rangle \in D\}.$$
   (b) L is co-recognizable if and only if there is a decidable language D such that
   $$L = \{x \mid \forall y \text{ s.t. } \langle x, y\rangle \in D\}.$$
6. (a) 4.28

   (b) Read definition 7.1 ("time complexity"). Suppose the set A in 4.28 included TMs deciding every language decidable in time n^2, say. What can you say about the time complexity of the decidable language D built from that A?

   (c) **Extra Credit:** Show that such a set A is Turing enumerable, i.e., it is possible to ennumerate a series of TMs, each of which is a decider, and every language decidable in time n^2 will be decided by at least one of the machines in the list.