# CSE 431 Spring 2009
# Assignment #7

Due: Friday, May 29, 2009

**Reading assignment:** Read Sections 7.5 to 8.2 of Sipser's text.

**Problems:**

1. Sipser's text: Problem 7.24 2nd Edition (Problem 7.22 1st Edition).

2. Sipser's text: Problem 7.25 2nd Edition (Problem 7.23 1st Edition).

3. A subset of the nodes of a graph $G$ is a *dominating set* iff every other node of $G$ is adjacent to some node in the subset. Let

   $$DOMINATING\text{-}SET = \{\langle G, k\rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}.$$

   Show that $DOMINATING\text{-}SET$ is NP-complete using the NP-completeness of $VERTEX\text{-}COVER$.

4. This problem is inspired by the single-player game *Minesweeper*, generalized to an arbitrary graph. Minesweeper begins with an undirected graph $G$ in which each node either contains a single, hidden mine or is empty. The player chooses nodes, one by one. If the player chooses a node containing a mine, the player loses. If the player chooses an empty node, the player learns the number of neighboring nodes containing mines. The player wins if and when the player has chosen all the empty nodes. (In the actual game it suffices for the player to learn the number of neighboring nodes associated with each empty node.)

   We are interested in the related problem of *Mine-Consistency* in which the input is an undirected graph $G$ together with numbers for some of $G$'s nodes. The goal is to determine whether there is a placement of mines on the remaining nodes so that any node $u$ numbered $k$ has exactly $k$ neighboring nodes containing mines. Formulate *Mine-Consistency* as a language, and prove that it is $NP$-complete.

   Hint: One possibility is a reduction from 3SAT. The reduction from 3SAT to SUBSET-SUM in the text might inspire you in the right direction.

5. (Bonus) Sipser's text: Problem 9.25 2nd Edition (Problem 9.16 1st Edition).

6. (Bonus) Sipser's text: Problem 9.26 2nd Edition (Problem 9.17 1st Edition).