CSE 431: Theory of Computation (Spring 2008) **Instructor: James R. Lee**
Problem Set #7 **Due: Thursday, June 5th**

Reading assignment: Sipser, Sections 7.4–7.5, 8.1–8.2.

Instructions: Same as homework #1. This problem set has **five** regular problems worth 10 points each, and an extra credit problem on the back.

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1. Show that 3COLOR is NP-complete. (Problem 7.27 in Sipser; the problem has a detailed hint.)

2. This problem is inspired by the single-player game *Minesweeper*, and the television show "Numbers," where the main character tries to solve P vs. NP by finding an algorithm for Minesweeper.

   Let $G = (V, E)$ be an undirected graph, where each node is either empty, or contains a single hidden *mine.* The player chooses nodes one by one. If the player chooses a node containing a mine, the player losers. If the player chooses an empty node, the player learns the number of neighboring nodes containing mines. (A neighboring node is one connected to the chosen node by an edge.) The player wins if and when all empty nodes have been chosen.

   In the problem *Mine-Consistency,* we are given a graph G along with numbers labeling some of G's nodes. The goal is to determine whether a placement of mines on the remaining nodes is possible, so that any node u which is labeled k has exactly k neighboring nodes containing mines. Formulate *Mine-Consistency* as a language, and prove that it is NP-complete.

   [Hint: One possibility is a reduction from 3SAT. The reduction to SUBSET-SUM in the text might inspire you in the right direction.]

3. For any k, define

   $$\text{DEG-}k\text{-SPANNING-TREE} = \{\langle G\rangle : G \text{ has a spanning tree of max. degree at most } k\}.$$

   (a) Show that DEG-2-SPANNING-TREE is NP-complete.
   (b) Use a reduction from DEG-2-SPANNING-TREE to show that DEG-3-SPANNING-TREE is NP-complete.
   (c) Generalize part (b) to show that for any k, DEG-k-SPANNING-TREE is NP-complete.

4. Show that $\text{LADDER}_{\mathsf{DFA}}$ is in PSPACE (Problem 8.9 in Sipser).

5. Show that balanced parentheses is in L. (Problem 8.17 in Sipser.)

6. **(Extra credit)** A bipartite graph is one whose vertices can be partitioned into two disjoint parts each of which is an independent set. Formally, a bipartite graph $H = (X, Y, E)$ has vertex set $X \cup Y$ for some disjoint sets X and Y and $E \subseteq X \times Y$ (i.e. edges go only between X and Y, but not within the the sets). A k-bipartite-clique of H is a pair of subsets $S \subseteq X$ and $T \subseteq Y$ with $|S| = |T| = k$ such that $(s, t) \in E$ for every $s \in S$ and $t \in T$. Define the language

   $$\text{BIPARTITE-CLIQUE} = \{\langle H, K\rangle : H \text{ is a bipartite graph that has a } k\text{-bipartite-clique}\}.$$

   Prove that BIPARTITE-CLIQUE is NP-complete.

   [Hint: This problem is tricky. The most obvious reduction from CLIQUE does not work, but there exists a reduction from CLIQUE that does.]