CSE 431: Theory of Computation (Spring 2008) **Instructor: James R. Lee**
Problem Set #6 **Due: Tuesday, May 27**

Reading assignment: Sipser, Sections 7.1–7.5.

Instructions: Same as homework #1. This problem set has **five** regular problems worth 10 points each.

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1. Show that if $\mathsf{P} = \mathsf{NP}$, then every language $A \in \mathsf{P}$, except $A = \emptyset$ and $A = \Sigma^*$ is NP-complete.

2. Define the language

   $U = \left\{ \langle M, x, 1^t \rangle : M \text{ is a nondeterministic Turing machine that accepts } x \text{ within } t \text{ steps} \right\}.$

   Show that U is NP-complete.

3. A subset of the nodes of a graph G is a *dominating set* if every other node of G is adjacent to some node in the subset. Let

   $$\text{DOMINATING-SET} = \{\langle G, k\rangle : G \text{ has a dominating set with } k \text{ nodes}\}.$$

   Show DOMINATING-SET is NP-complete by giving a reduction from VERTEX-COVER.

4. Let HALF-INDSET be the language

   $$\{\langle G\rangle : G \text{ is an undirected graph that has an independent set of size at least } n/2\},$$

   where n is the number of vertices in G. Prove that INDSET $\leq_p$ HALF-INDSET.

5. This problem investigates *resolution,* a method for proving the unsatisfiabiliy of CNF-formulas. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a formula in CNF, where the C_i's are its clauses. Let $\mathcal{C} = \{C_i : C_i \text{ is a clause of } \phi\}$. In a *resolution step,* we take two clauses $C_a, C_b \in \mathcal{C}$ which both have some variable x, occurring positively in one of the clauses and negatively in the other. Thus $C_a = (x \vee y_1 \vee y_2 \vee \cdots \vee y_k)$ and $C_b = (\bar{x} \vee z_1 \vee z_2 \vee \cdots \vee z_\ell)$, where $\{y_i\}$ and $\{z_i\}$ are literals. We form the new clause $(y_1 \vee y_2 \vee \cdots \vee y_k \vee z_2 \vee z_2 \vee \cdots \vee z_\ell)$ and remove repeated literals. Add this new clause to $\mathcal{C}$. Repeat the resolution steps until no additional clauses can be obtained. If the empty clause () is in $\mathcal{C}$ then declare ϕ to be unsatisfiable.

   We say that resolution is *sound* if it never declares satisfiable formulas to be unsatisfiable. We say that resolution is *complete* if all unsatisfiable formulas are declared to be unsatisfiable.

   (a) Show that resolution is sound and complete.

   (b) Define 2-SAT $= \{\langle\phi\rangle : \phi \text{ is a satisfiable 2-CNF formula}\}$. (A 2-CNF formula is an AND of clauses where each clause is an OR of at most two literals.) Use part (a) to show that 2-SAT $\in \mathsf{P}$.